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THE MODIFIED JUNGCK MANN AND MODIFIED JUNGCK ISHIKAWA ITERATION SCHEMES FOR ZAMFIRESCU OPERATORS

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Abstract: The purpose of this paper is to establish some strong convergence results of the modified Jungck Mann and modified Jungck Ishikawa iteration schemes to a common fixed point for Zamfirescu operators.

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1. Introduction

Let $(E, \|\cdot\|)$ be a Banach space and $T : E \to E$ be a self mapping of E.

There are many iteration schemes for which the fixed points of operators have been approximated in last few decades by various authors.

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Now, we shall state some of these iteration schemes as follows:

Let (X, d) be a metric space and $T : X \to X$ be a self mapping. Choose $x_0 \in X$ and define $x_1 = Tx_0$, $x_2 = Tx_1$, $\cdots$, and obtain a relation

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \dots,$$

where x_n is the n-th Picard iterate of $x_0 \in X$.

Let x_0 be an initial point in a given set and $\{\alpha_n\}$ with $0 \leq \alpha_n < 1$, be a sequence of real numbers. Then Mann iteration scheme [7] is defined by

$$x_{n+1} = (1 - \alpha_n)x_n - \alpha_n Tx_n, \quad n = 0, 1, 2, \dots. \tag{1.1}$$

Let x_0 be an initial point in a given set and $\{\alpha_n\}$ and $\{\beta_n\}$ be the sequences of real numbers with $0 \leq \alpha_n \leq \beta_n < 1$. Then

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n Ty_n \\ y_n &= (1 - \beta_n)x_n + \beta_n Tx_n, \quad n = 0, 1, 2, \dots \end{aligned} \tag{1.2}$$

is called Ishikawa iteration scheme [3].

Berinde [1] proved the strong convergence results in an arbitrary Banach space for the Ishikawa iteration scheme by using the following contractive definitions:

Let $(E, \|\cdot\|)$ be a Banach space and $T : E \to E$ a self mapping of E, there exist a, b and c satisfying $0 \leq a < 1$, $0 \leq b < \frac{1}{2}$ and $0 \leq c < \frac{1}{2}$ respectively such that for each $x, y \in E$, at least one of the following is true:

$$d(Tx, Ty) \leq ad(x, y), \tag{1.3}$$

$$d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)], \tag{1.4}$$

$$d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)]. \tag{1.5}$$

(1.3), (1.4) and (1.5) are called Zamfirescu contraction conditions and the mapping satisyfing these conditions is called Zamfirescu operator [11]. The mapping satisfying (1.3) is called Kannan operator [6] and the mapping satisfying (1.4) is called Chattererjea operator [2].

2. Preliminaries

We shall use the following fundamental results and iteration schemes in establishing our main results:

Let $(X, \|\cdot\|)$ be a normed linear space and $S, T : S \longrightarrow E$ are two non-self mappings such that $T(Y) \subseteq S(Y)$. Then the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$Sx_{n+1} = Tx_n, \quad n = 0, 1, 2 \dots$$

is called Jungck iteration scheme [4].

The following iteration scheme is introduced by Singh et al. [10] to establish some stability results:

Let $(X, \|\cdot\|)$ be a normed linear space and $S, T : S \longrightarrow E$ are two non-self mappings such that $T(Y) \subseteq S(Y)$. Then the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tx_n, \quad n = 0, 1, 2, \dots, \tag{2.1}$$

where $\{\alpha_n\}$ is the sequence in $[0, 1)$, and is called Jungck Mann iteration scheme.

Let $(X, \|\cdot\|)$ be a normed linear space and $S, T : S \longrightarrow E$ are two non-self mappings such that $T(Y) \subseteq S(Y)$. Then the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} Sx_{n+1} &= (1 - \alpha_n)Sx_n + \alpha_n Tz_n \\ Sz_n &= (1 - \beta_n)Sx_n + \beta_n Tx_n, \quad n = 0, 1, 2, \dots, \end{aligned} \tag{2.2}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are the sequences in $[0, 1)$. This is called Jungck Ishikawa iteration scheme [8]. If $X = Y$ or S is the identity mapping, then Jungck Picard, Jungck Mann and Jungck Ishikawa iteration schemes becomes Picard, Mann and Ishikawa iteration schemes.

Olatinwo and Imoru [9] used the the following definitions and contractive conditions:

Definition 2.1. Let (X, d) be a metric space and $T : X \to X$ be a self mapping. Then T has a fixed point if there is an $x \in X$ such that $Tx = x$. The point x is called a fixed point of T.

Definition 2.2. Let (X, d) be a metric space and T, $S : X \to X$. Then x is called a coincidence (common fixed) point of T and S, respectively, if there exists $x \in X$ such that

$$x = Tx = Sx.$$

Definition 2.3. If an operator T satisfied above three conditions (1.3)-(1.5), then it is called Zamfirescu operator. For two non-self mappings S, T :

$Y \to E$ with $T(Y) \subseteq S(Y)$, there exist real numbers α, β and γ satisfying $0 \leq \alpha < 1$, $0 \leq \beta < 1/2$ and $0 \leq \gamma < \frac{1}{2}$, respectively such that for each $x, y \in Y$, at least one of the following is true.

$$d(Tx, Ty) \leq \alpha d(Sx, Sy), \tag{2.3}$$

$$d(Tx, Ty) \leq \beta[d(Sx, Tx) + d(Sy, Ty)], \tag{2.4}$$

$$d(Tx, Ty) \leq \gamma[d(Sx, Ty) + d(Sy, Tx)]. \tag{2.5}$$

The above three conditions will be called the generalized Zamfirescu contraction for the pair (S, T). Condition (2.4) is called generlized Kannan condition for the pair of (S, T) and condition (2.5) is called of Chatterjea condition for the pair of (S, T).

In this paper, we purposed the modified Jungck Mann and modified Jungck Ishikawa iteration schemes for Zamfirescu operators for solving nonlinear functions by following the approach of [5]. We also provided the applications of the two iteration schemes and compared them with the Jungck iteration scheme, Jungck Mann iteration scheme and the Jungck Ishikawa iteration scheme by using some significant nonlinear functions.

3. Main Results

Consider the nonlinear equation

$$f(x) = 0, \quad x \in \mathbb{R}. \tag{3.1}$$

Let $S, T : Y \to X$, $T(Y) \subset S(Y)$, S is onto and T is differentiable. Suppose that ξ is simple zero of $f(x)$ and x_0 is an initial guess nearer to ξ. The equation (3.1) can be written as

$$Sx = Tx. \tag{3.2}$$

Following the approch of [5], if $T'x \neq -1$, we can modify (3.2) by multiplying $\theta \neq -1$ on both sides as follows

$$\theta Sx = \theta Tx,$$

which implies that

$$Sx = \frac{\theta Tx + Sx}{\theta + 1}, \tag{3.3}$$

where θ is an orbitrary number. In order (3.3) to be efficient, we can choose

$$\theta = -T' x. \tag{3.4}$$

For a given x_0, we can find the approximate solution of x_{n+1} by using the iteration scheme named as modified Jungck iteration scheme

$$Sx_{n+1} = \frac{\theta Tx_n + Sx_n}{\theta + 1}, \quad \theta \neq -1, \tag{3.5}$$

or

$$Tx_{n+1} = \frac{\theta Tx_n + Sx_n}{\theta + 1}, \quad \theta \neq -1. \tag{3.6}$$

Using (3.6) in Jungck Mann and Jungck Ishikawa iteration schemes, we develope modified Jungck Mann iteration scheme and modified Jungck Ishikawa iteration scheme. modified Jungck Mann iteration scheme and modified Jungck Ishikawa iteration scheme are as follows:

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n T_\theta x_n, \tag{3.7}$$

and

$$\begin{aligned} Sx_{n+1} &= (1 - \alpha_n)Sx_n + \alpha_n T_\theta z_n, \\ Sz_n &= (1 - \beta_n)Sx_n + \beta_n Tx_n, \end{aligned} \tag{3.8}$$

where $T_\theta x_n = \frac{\theta Tx_n + Sx_n}{1+\theta}$ and $T_\theta z_n = \frac{\theta Tz_n + Sz_n}{1+\theta}$, $\theta \neq -1$, $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are the sequences in $[0, 1)$.

Now we shall establish the following theorems as our main results:

Theorem 3.1. *Let $(E, \|\cdot\|)$ be any Banach space and Y be an orbitrary set. Suppose that $S, T : S \longrightarrow E$ are two non-self mappings such that $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of E, S is onto and T is differentiable. let l be the coincidence point of S and T (i.e, $Sl = Tl = w$). Suppose also that S and T satisfy the contractive conditions (2.1)-(2.3). For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be modified Jungck Mann iteration scheme defined by (3.7), where $\{\alpha_n\}_{n=0}^{\infty}$ is the sequence in $[0, 1)$ such that $\sum_{k=0}^{\infty} \alpha_k = \infty$. Then $\{Sx_n\}_{n=0}^{\infty}$ strongly converges to w.*

Proof. We shall first establish that the conditions (2.3)-(2.5) implies

$$\begin{aligned} &\|T_\theta x - T_\theta y\| \\ &\leq \beta[\|Sx - T_\theta x\| + \|Sy - T_\theta y\|] \\ &\leq \beta[\|Sx - T_\theta x\| + \|Sy - Sx\| + \|Sx - T_\theta x\| + \|T_\theta x - T_\theta y\|], \end{aligned}$$

which implies that

$$\|T_\theta x - T_\theta y\| \leq \left(\frac{2\beta}{1-\beta}\right)\|Sx - T_\theta x\| + \left(\frac{\beta}{1-\beta}\right)\|Sy - Sx\|. \tag{3.9}$$

Similary

$$\|T_\theta x - T_\theta y\| \leq \left(\frac{2\gamma}{1-\gamma}\right)\|Sx - T_\theta x\| + \left(\frac{\gamma}{1-\gamma}\right)\|Sy - Sx\|. \tag{3.10}$$

From (3.9) and (3.10)

$$\|T_\theta x - T_\theta y\| \leq 2\delta\|Sx - T_\theta x\| + \delta\|Sy - Sx\|, \tag{3.11}$$

where δ =max $\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}$ for $0 \leq \delta < 1$. Let $C(S,T)$ be the set of coincedence points S and T. We shall now use the (3.11) to establish that S and T have a unique coincedence point l (i.e, $Sl = Tl = w$). Injectivity of S is sufficient. Suppose that there exist $l_1, l_2 \in C(S,T)$ such that $S(l_1) = T(l_1) = w_1$ and $S(l_2) = T(l_2) = w_2$. If $w_1 = w_2$, then $S(l_1) = S(l_2)$ and since S is onto, it follows that $l_1 = l_2$.

If $w_1 \neq w_2$ by using (3.10) for S and T, then

$$\begin{aligned} 0 &< \|w_1 - w_2\| \\ &= \|T_\theta(l_1) - T_\theta(l_2)\| \\ &\leq 2\delta\|S(l_1) - T_\theta(l_1)\| + \delta\|S(l_2) - S(l_1)\| \\ &= \delta\|w_2 - w_1\|, \end{aligned}$$

which implies $(1-\delta)\|w_2 - w_1\| \leq 0$, it follows that $(1-\delta) > 0$ for $\delta \in [0,1)$, but $\|w_2 - w_1\| \leq 0$, which contradicts that norm is nonnegative. Therefore, we have $\|w_2 - w_1\| = 0$, i.e, $w_1 = w_2 = w$. Since $w_1 = w_2$, then we have $w_1 = S(l_1) = T(l_1) = S(l_2) = T(l_2) = w_2$, implies that $S(l_1) = S(l_2)$, leading to $l_1 = l_2 = l$ (since S is onto). Hence S and T have a unique coincedence point $l \in C(S,T)$. Using $\delta = \max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}$, $0 \leq \delta < 1$ and $\theta \neq -1$. Put $y = x_n$ and $x = w$ in (3.11), we have

$$\begin{aligned} \|T_\theta x_n - w\| &\leq 2\delta\|Sw - T_\theta w\| + \delta\|Sx_n - w\| \\ &\leq 2\delta\|Sw - \frac{\theta Tw + Sw}{1+\theta}\| + \delta\|Sx_n - w\| \\ &\leq 2\delta\left\|\frac{Sw + \theta Sw - \theta Tw - Sw}{1+\theta}\right\| + \delta\|Sx_n - w\|, \end{aligned}$$

which implies

$$\|T_\theta x_n - w\| \leq \delta \|Sx_n - w\|. \tag{3.12}$$

We now prove that $\{Sx_n\}_{n=0}^{\infty}$ strongly converges to w, where $Sl = Tl = w$, using again (3.11). Therefore, we have

$$\begin{aligned}
\|Sx_{n+1} - w\| &= \|(1-\alpha_n)Sx_n + \alpha_n T_\theta x_n - w\| \\
&\leq (1-\alpha_n)\|Sx_n - w\| + \alpha_n \|T_\theta x_n - w\| \\
&\leq (1-\alpha_n)\|Sx_n - w\| + \delta\alpha_n \|Sx_n - w\| \\
&\leq [1-(1-\delta)\alpha_n]\|Sx_n - w\| \\
&\leq \prod_{k=0}^{n}[1-(1-\delta)\alpha_k]\|Sx_0 - w\| \\
&\leq e^{-[(1-\delta)\sum_{k=0}^{n}\alpha_k]}\|Sx_0 - w\| \\
&\to 0 \quad \text{as } n \to \infty,
\end{aligned} \tag{3.13}$$

since $\sum_{k=0}^{n}\alpha_k = \infty$, $\delta \in [0,1)$ and $\theta \neq -1$. Therefore, from (3.13) we get $\|Sx_n - w\| \longrightarrow 0$ as $n \to \infty$, that is, $\{Sx_n\}_{n=0}^{\infty}$ strongly converges to w. This compleets the proof. □

Theorem 3.2. *Let $(E, \|\cdot\|)$ be any Banach space and Y be an orbitrary set. Suppose that $S, T : S \longrightarrow E$ are two non-self mappings such that $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of E, S is onto and T is differentiable. let l be the coincidence point of S and T (i.e., $Sl = Tl = w$). Suppose also that S and T satisfy the contractive conditions* (2.3)-(2.5)*. For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be modified Jungck Ishikawa iteration scheme defined by* (3.8)*, where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are the sequences in $[0,1)$ such that $\sum_{k=0}^{\infty}\alpha_k = \infty$. Then $\{Sx_n\}_{n=0}^{\infty}$ strongly converges to w.*

Proof. Let us suppose that $Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n T_\theta z_n$, where $Sz_n = (1-\beta_n)Sx_n + \beta_n Tx_n$, $T_\theta x_n = \frac{\theta Tx_n + Sx_n}{1+\theta}$ and $T_\theta z_n = \frac{\theta Tz_n + Sz_n}{1+\theta}$, $\theta \neq -1$. Then using condition (3.11), we have

$$\begin{aligned}
\|Sx_{n+1} - w\| &= \|(1-\alpha_n)Sx_n + \alpha_n T_\theta z_n - w\| \\
&\leq (1-\alpha_n)\|Sx_n - w\| + \alpha_n \|T_\theta z_n - w\| \\
&\leq (1-\alpha_n)\|Sx_n - w\| + \delta\alpha_n \|Sz_n - w\|.
\end{aligned} \tag{3.14}$$

Also, we have

$$\begin{aligned}
\|Sz_n - w\| &\leq (1-\beta_n)\|Sx_n - w\| + \beta_n \|Tx_n - w\| \\
&\leq (1-\beta_n)\|Sx_n - w\| + \delta\beta_n \|Sx_n - w\| \\
&= [(1-(1-\delta)\beta_n]\|Sx_n - w\|.
\end{aligned} \tag{3.15}$$

Using (3.15) in (3.14) yields

$$\begin{aligned}
&\|Sx_{n+1} - w\| \\
&\leq (1-\alpha_n)\|Sx_n - w)\| + \delta\alpha_n[1-\beta_n(1-\delta)]\|Sx_n - w\| \\
&\leq [1-(1-\delta)\alpha_n - (1-\delta)\delta\alpha_n\beta_n]\|Sx_n - w\| \\
&\leq [1-(1-\delta)\alpha_n]\|Sx_n - w\| \quad \text{(because } 1+\delta\beta_n \geq 1) \\
&\leq \prod_{k=0}^{n}[1-(1-\delta)\alpha_k]\|Sx_0 - w\| \\
&\leq e^{-[(1-\delta)\sum_{k=0}^{n}\alpha_k]}\|Sx_0 - w\| \\
&\to 0 \quad \text{as } n \to 0,
\end{aligned} \tag{3.16}$$

since $\sum_{k=0}^{n}\alpha_k = \infty$, $\alpha \in [0,1)$ and $\theta \neq -1$. Hence, we obtain from (3.16) that $\|Sx_n - w\| \to 0$ as $n \to \infty$, which implies that $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to w. This compleets the proof. □

4. Applications

In this section we present some applications of nonlinear functions that shows the efficiency of modified Jungck Mann and modified Jungck Ishikawa Iteration schemes by comparing them with Jungck, Jungck Mann, Jungck Ishikawa iteration schemes.

Example 4.1. Consider the nonlinear equation $\ln(x) - \cos(x) = 0$. Let us take $Tx = \ln(x)$ and $Sx = \cos(x)$. The true graphical solution of given equation is 1.3029640012 ($10D$). If we choose initial guess $x_0 = 1.5$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, Jungck Ishikawa, modified Jungck Mann and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 101 iterations, Jungck Mann iteration scheme for $\beta = 0.9083$ is diverged, modified Jungck Mann iteration scheme for $\beta = 0.9083$ evaluate the solution in 5 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.96636945$ and $\beta = 0.3$ evaluate the solution in 23 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.96636945$ and $\beta = 0.3$ evaluate the solution in 3 iterations of the given equation (see

Tables 1-5).

Table 1: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.4054651081	0.4054651081	1.1533087441	0.3466912559
1	0.1426349800	0.1426349800	1.4276732194	0.2743644753
2	0.3560459998	0.3560459998	1.2067631452	0.2209100742
3	0.1879416885	0.1879416885	1.3817302578	0.1749671126
4	0.3233365238	0.3233365238	1.2415430316	0.1401872262
5	0.2163549863	0.2163549863	1.3527168518	0.1111738203
6	0.3021150529	0.3021150529	1.2638857195	0.0888311323
7	0.2341908799	0.2341908799	1.3344101071	0.0705243876
8	0.2884893269	0.2884893269	1.2781476192	0.0562624879
9	0.2454118573	0.2454118573	1.3228518075	0.0447041883
10	0.2797898664	0.2797898664	1.2872210998	0.0356307076
⋮	⋮	⋮	⋮	⋮
99	0.2646416701	0.2646416701	1.3029640012	0.0000000001
100	0.2646416702	0.2646416702	1.3029640012	0.0000000000

Table 2: Jungck Mann iteration scheme ($\beta = 0.9083$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.0707372017	0.1014317507	1.1067543806	0.3932456194
1	0.4475662666	0.4158257315	1.5156217208	0.4088673402
2	0.0551466162	0.0882208911	1.0922293591	0.4233923617
⋮	⋮	⋮	⋮	⋮

Table 3: Modified Jungck Mann iteration scheme ($\beta = 0.9083$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.0707372017	0.2536390701	1.2887065876	0.2112934124
1	0.2783633989	0.2646412508	1.3029634548	0.0142568672
2	0.2646421971	0.2646416728	1.3029640046	0.0000005498
3	0.2646416669	0.2646416701	1.3029640012	0.0000000034
4	0.2646416702	0.2646416702	1.3029640012	0.0000000000

Table 4: Jungck Ishikawa iteration scheme
($\alpha = 0.96636945$ and $\beta = 0.3$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.0707372017	0.2189661378	1.2447891247	0.2552108753
1	0.3202630995	0.2814748656	1.3250826907	0.0802935660
2	0.2432485880	0.2586692006	1.2952052808	0.0298774099
⋮	⋮	⋮	⋮	⋮
22	0.2646416702	0.2646416702	1.3029640012	0.0000000000

Table 5: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.96636945$ and $\beta = 0.3$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\| e_n \|$
0	0.0707372017	0.2642115591	1.3024037025	0.1975962975
1	0.2651819509	0.2646416701	1.3029640012	0.0005602987
2	0.2646416701	0.2646416701	1.3029640012	0.0000000000

Example 4.2. Consider the nonlinear equation $\ln(x) + \tan(x) = 0$. Let us take $Tx = -\ln(x)$ and $Sx = \tan(x)$. The true graphical solution of given equation is 0.5452257174 ($10D$). If we choose initial guess $x_0 = 0.5$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, Jungck Ishikawa, modified Jungck Mann and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 74 iterations, Jungck Mann iteration scheme for $\beta = 0.66$ evaluate the solution in 37 iterations, modified Jungck Mann iterative scheme for $\beta = 0.66$ evaluate the solution in 5 iterations, Jungck Ishikawa iteration scheme for $\beta = 0.2879415656$ and $\alpha = 0.86$ evaluate the solution in 15 iterations and modified Jungck Ishikawa iteration scheme for $\beta = 0.2879415656$ and $\alpha = 0.86$ evaluate the solution in 2 iteration of the given equation (see Tables 6-10).

Table 6: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.5463024898	0.5463024898	0.5790870369	0.0790870369
1	0.6538643799	0.6538643799	0.5200322865	0.0590547504
2	0.5726047020	0.5726047020	0.5640543302	0.0440220437
3	0.6326119075	0.6326119075	0.5312025356	0.0328517946
⋮	⋮	⋮	⋮	⋮
72	0.6065554097	0.6065554097	0.5452257174	0.0000000001
73	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Table 7: Jungck Mann iteration scheme ($\beta = 0.66$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6432199857	0.5715941783	0.0715941783
1	0.5593260180	0.5878499670	0.5314377478	0.0401564305
2	0.6321692136	0.6171006698	0.5528987210	0.0214609733
⋮	⋮	⋮	⋮	⋮
36	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Table 8: Modified Jungck Mann iteration scheme ($\beta = 0.66$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6109141538	0.5484059872	0.0484059872
1	0.6007394137	0.6065772237	0.5452416642	0.0031643231
2	0.6065261621	0.6065554144	0.5452257208	0.0000159434
3	0.6065554035	0.6065554098	0.5452257174	0.0000000034
4	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Table 9: Jungck Ishikawa iteration scheme
($\beta = 0.2879415656$ and $\alpha = 0.86$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6192644038	0.5544642057	0.0544642057
1	0.5897530267	0.6035011554	0.5429899033	0.0114743024
2	0.6106645535	0.6072725030	0.5457497763	0.0027598730
⋮	⋮	⋮	⋮	⋮
15	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Table 10: Modified Jungck Ishikawa iteration scheme
($\beta = 0.2879415656$ and $\alpha = 0.86$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\| e_n \|$
0	0.6931471806	0.6065554098	0.5452257174	0.0452257174
1	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Example 4.3. Consider the nonlinear equation $e^x - \tan(x) = 0$. Let us take $Tx = e^x$ and $Sx = \tan(x)$. The true graphical solution of given equation is 1.3063269404 ($10D$). If we choose initial guess $x_0 = 1.5$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, Jungck Ishikawa, modified Jungck Mann and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 19 iterations, Jungck Mann iterstion scheme $\beta = 0.9757$ evaluate the solution in 19 iterations, modified

Jungck Mann iterative scheme for $\beta = 0.9757$ evaluate the solution in 6 iterations, Jungck Ishikawa iteration scheme for $\beta = 0.27$ and $\alpha = 0.98015$ evaluate the solution in 17 iterations and modified Jungck Ishikawa iterative scheme for $\beta = 0.27$ and $\alpha = 0.98015$ evaluate the solution in 5 iterations of the given equation (see Tables 11-15).

Table 11: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	4.4816890703	4.4816890703	1.3512623360	0.1487376640
1	3.8622979725	3.8622979725	1.3174465040	0.0338158320
2	3.7338747596	3.7338747596	1.3091190649	0.0083274391
3	3.7029102514	3.7029102514	1.3070305691	0.0020884958
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
17	3.6925856857	3.6925856857	1.3063269404	0.0000000001
18	3.6925856854	3.6925856854	1.3063269404	0.0000000000

Table 12: Jungck Mann iteration scheme ($\beta = 0.9757$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	4.4816890703	4.7154485306	1.3618234407	0.1381765593
1	3.9033042605	3.9230393663	1.3212071100	0.0406163307
2	3.7479428270	3.7521976729	1.3103397589	0.0108673511
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
18	3.6925856854	3.6925856854	1.3063269404	0.0000000000

Table 13: Modified Jungck Mann iteration scheme ($\beta = 0.9757$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\| e_n \|$
0	4.4816890703	2.0196387593	1.1110458360	0.3889541640
1	3.0375334954	3.5002310790	1.2925141066	0.1814682707
2	3.6419312590	3.6908196870	1.3062062187	0.0136921121
3	3.6921399371	3.6925863471	1.3063269856	0.0001207669
4	3.6925858523	3.6925856853	1.3063269404	0.0000000452
5	3.6925856854	3.6925856854	1.3063269404	0.0000000000

Table 14: Jungck Ishikawa iteration scheme
($\beta = 0.27$ and $\alpha = 0.98015$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	4.4816890703	4.6032513462	1.3568822657	0.1431177343
1	3.8840649225	3.8637833045	1.3175397857	0.0393424800
2	3.7342230781	3.7286964189	1.3087720490	0.0087677367
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
16	3.6925856855	3.6925856855	1.3063269404	0.0000000000

Table 15: Modified Jungck Ishikawa iteration scheme ($\beta = 0.27$ and $\alpha = 0.98015$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	4.4816890703	2.5649928655	1.1990577549	0.3009422451
1	3.3170157877	3.6430207157	1.3028973663	0.1038396113
2	3.6799433803	3.6925103388	1.3063217920	0.0034244257
3	3.6925666745	3.6925856858	1.3063269404	0.0000051484
4	3.6925856855	3.6925856855	1.3063269404	0.0000000000

Example 4.4. Consider the nonlinear equation $1 - x - \sin(x) = 0$. Let us take $Tx = 1 - x$ and $Sx = \sin(x)$. The true graphical solution of given equation is 0.5109734294 ($10D$). If we choose initial guess $x_0 = 0.5$, then the comparision tables correct upto ten decimal places of Jungck,Jungck Mann, modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 147 iterations, Jungck Mann Iteration scheme for $\beta = 0.931$ diverged, modified Jungck Mann Iteration scheme for $\beta = 0.931$ evaluate the solution in 4 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.945625$ and $\beta = 0.1$ evaluate the solution in 86 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.945625$ and $\beta = 0.1$ evaluate the solution in 3 iterations of the given equation (see Tables 15-20).

Table 16: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	0.4794255386	0.4794255386	0.5205744614	0.0205744614
1	0.4973785844	0.4973785844	0.5026214156	0.0179530458
2	0.4817243973	0.4817243973	0.5182756027	0.0156541872
3	0.4953829348	0.4953829348	0.5046170652	0.0136585375
4	0.4834722701	0.4834722701	0.5165277299	0.0119106647
5	0.4938638460	0.4938638460	0.5061361540	0.0103915759
⋮	⋮	⋮	⋮	⋮
145	0.4890265706	0.4890265706	0.5109734294	0.0000000001
146	0.4890265706	0.4890265706	0.5109734294	0.0000000000

Table 17: Jungck Mann iteration scheme ($\beta = 0.931$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	0.5000000000	0.4985803622	0.5219602933	0.0219602933
1	0.4780397067	0.4794570119	0.5000358640	0.0219244292
2	0.4999641360	0.4985491444	0.5219242805	0.0218884165
⋮	⋮	⋮	⋮	⋮

Table 18: Modified Jungck Mann iteration scheme ($\beta = 0.931$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.5000000000	0.4890029504	0.5109463505	0.0109463505
1	0.4890536495	0.4890265508	0.5109734067	0.0000270562
2	0.4890265933	0.4890265706	0.5109734294	0.0000000227
3	0.4890265706	0.4890265706	0.5109734294	0.0000000000

Table 19: Jungck Ishikawa iteration scheme
($\alpha = 0.945625$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.5000000000	0.4966628720	0.5197496595	0.0197496595
1	0.4802503405	0.4829299528	0.5039976346	0.0157520249
2	0.4960023654	0.4938792797	0.5165454793	0.0125478447
⋮	⋮	⋮	⋮	⋮
85	0.4890265706	0.4890265706	0.5109734294	0.0000000000

Table 20: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.945625$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.5000000000	0.4890169916	0.5109624477	0.0109624477
1	0.4890375523	0.4890265706	0.5109734294	0.0000109817
2	0.4890265706	0.4890265706	0.5109734294	0.0000000000

Example 4.5. Consider the nonlinear equation $e^{x^2} - \frac{5}{e^{2x}} = 0$. Let us take $Tx = -e^{x^2}$ and $Sx = -\frac{5}{e^{2x}}$. The true graphical solution of given equation is 0.6153754710 $(10D)$. If we choose initial guess $x_0 = 0.5$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann and Jungck Ishikawa, modified Jungck Mann and modified Jungck Ishikawa iteration schemes are shown below.From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 47 iterations, Jungck Mann iteration scheme for $\beta = 0.96581$ evaluate the solution in 40 iterations, modified Jungck Mann iteration scheme for $\beta = 0.96581$ evaluate the solution in 5 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.96988718$ and $\beta = 0.1$ evaluate the solution in 31 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.96988718$ and $\beta = 0.1$ evaluate the solution in 4 iterations of

the given equcation (see Tables 21-25).

Table 21: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.2840254167	1.2840254167	0.6797189562	0.1797189562
1	1.5872736508	1.5872736508	0.5737100265	0.1060089297
2	1.3897768497	1.3897768497	0.6401473590	0.0664373325
3	1.5064993396	1.5064993396	0.5998246356	0.0403227233
4	1.4330278645	1.4330278645	0.6248241595	0.0249995238
5	1.4775794325	1.4775794325	0.6095163411	0.0153078184
⋮	⋮	⋮	⋮	⋮
41	1.4603658268	1.4603658268	0.6153754709	0.0000000004
42	1.4603658261	1.4603658261	0.615375471	0.0000000002
43	1.4603658266	1.4603658266	0.6153754710	0.0000000001
44	1.4603658263	1.4603658263	0.6153754711	0.0000000001
45	1.4603658265	1.4603658265	0.6153754710	0.0000000001
46	1.4603658264	1.4603658264	0.6153754711	0.000000000

Table 22: Jungck Mann iteration scheme ($\beta = 0.96581$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−1.2840254167	−1.3030135782	0.6723790968	0.1723790968
1	−1.5715991217	−1.5624161820	0.5816022274	0.0907768694
2	−1.4025067215	−1.4079740259	0.6336430512	0.0520408238
⋮	⋮	⋮	⋮	⋮
39	−1.4603658264	−1.4603658264	0.6153754710	0.0000000000

Table 23: Modified Jungck Mann iteration scheme ($\beta = 0.96581$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−1.2840254167	−1.5378549349	0.5895246832	0.0895246832
1	−1.4155800156	−1.4640066147	0.6141304894	0.0246058062
2	−1.4581321363	−1.4603658394	0.6153754666	0.0012449772
3	−1.4603658184	−1.4603658263	0.6153754710	0.0000000044
4	−1.4603658264	−1.4603658264	0.6153754710	0.0000000000

Table 24: Jungck Ishikawa iteration scheme
($\alpha = 0.96988718$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−1.2840254167	−1.3202838247	0.6657955901	0.1657955901
1	−1.5578143292	−1.5329492512	0.5911222087	0.0746733815
2	−1.4182524796	−1.4278460624	0.6266354269	0.0355132182
⋮	⋮	⋮	⋮	⋮
30	−1.4603658264	−1.4603658264	0.6153754710	0.0000000000

Table 25: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.96988718$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−1.2840254167	−1.5187560057	0.5957731650	0.0957731650
1	−1.4261031636	−1.4618889957	0.6148542402	0.0190810752
2	−1.4594296907	−1.4603658264	0.6153754710	0.0005212309
3	−1.4603658264	−1.4603658264	0.6153754710	0.0000000000

Example 4.6. Consider the nonlinear equation $\tan(x) - \frac{1}{2x} = 0$. Let us take $Tx = 2\tan(x)$ and $Sx = \frac{1}{x}$. The true graphical solution of given equation is 0.6532711871 (10D). If we choose initial guess $x_0 = 0.7$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, Jungck Ishikawa, modified Jungck Mann and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 71 iterations, Jungck Mann Iteration scheme for $\beta = 0.82$ evaluate the solution in 27 iterations, modified Jungck Mann Iterative scheme for $\beta = 0.82$ evaluate the solution in 5 iterations, Jungck Ishikawa iteration scheme for $\beta = 0.28289$ and $\alpha = 0.9$ evaluate the solution in 31 iteration and modified Jungck Ishikawa iteration scheme for $\beta = 0.28289$ and $\alpha = 0.9$ evaluate the solution in 4 iteration of the given equation (see Tables 26-30).

Table 26: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.4285714286	1.4285714286	0.6202494860	0.0797505140
1	1.6122544598	1.6122544598	0.6784659072	0.0582164212
2	1.4739134117	1.4739134117	0.6351010057	0.0433649016
3	1.5745526949	1.5745526949	0.6669343221	0.0318333164
⋮	⋮	⋮	⋮	⋮
69	1.5307578534	1.5307578534	0.6532711871	0.0000000001
70	1.5307578533	1.5307578533	0.6532711871	0.0000000000

Table 27: Jungck Mann iteration scheme ($\beta = 0.82$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.4285714286	1.4746523884	0.6353404081	0.0646595919
1	1.5739593881	1.5560841282	0.6612078185	0.0258674103
2	1.5123838105	1.5202498677	0.6499496370	0.0112581815
⋮	⋮	⋮	⋮	⋮
26	1.5307578534	1.5307578534	0.6532711871	0.0000000000

Table 28: Modified Jungck Mann iteration scheme ($\beta = 0.82$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.4285714286	1.5436879204	0.6573352986	0.0426647014
1	1.5212936262	1.5308651753	0.6533050245	0.0040302740
2	1.5306785689	1.5307579308	0.6532712115	0.0000338130
3	1.5307577961	1.5307578533	0.6532711871	0.0000000244
4	1.5307578534	1.5307578534	0.6532711871	0.0000000000

Table 29: Jungck Ishikawa iteration scheme
($\alpha = 0.9$ and $\beta = 0.28289$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.4285714286	1.4950384397	0.6419115180	0.0580884820
1	1.5578471051	1.5392794188	0.6559524933	0.0140409753
2	1.5245006464	1.5287327278	0.6526323584	0.0033201349
⋮	⋮	⋮	⋮	⋮
30	1.5307578533	1.5307578533	0.6532711871	0.0000000000

Table 30: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.9$ and $\beta = 0.28289$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.4285714286	1.5342361319	0.6543669587	0.0456330413
1	1.5281945194	1.5307594849	0.6532717015	0.0010952572
2	1.5307566479	1.5307578533	0.6532711871	0.0000005144
3	1.5307578533	1.5307578533	0.6532711871	0.0000000000

Example 4.7. Consider the nonlinear equation $2x + 3\cos(x) - e^x = 0$. Let us take $Tx = e^x - 3\cos(x)$ and $Sx = 2x$. The true graphical solution of given equation is -0.8109753311 $(10D)$. If we choose initial guess $x_0 = -0.9$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann , modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iterations are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 153 iterations, Jungck Mann Iterative scheme for $\beta = 0.8459$ evaluate the solution in 41 iterations, modified Jungck Mann Iterative scheme for $\beta = 0.8459$ evaluate the solution in 5 iterations, Jungck Ishikawa iterative scheme for $\alpha = 0.959$ and $\beta = 0.41026$ evaluate the solution in 13 iterations and modified Jungck Ishikawa iterative scheme for $\alpha = 0.959$ and $\beta = 0.41026$ evaluate the solution in 3 iterations of the given

equcation (see Tables 31-35).

Table 31: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	−1.4582602451	−1.4582602451	−0.7291301225	0.1708698775
1	−1.7549342714	−1.7549342714	−0.8774671357	0.1483370132
2	−1.5014689938	−1.5014689938	−0.7507344969	0.1267326388
3	−1.7215443016	−1.7215443016	−0.8607721508	0.1100376539
⋮	⋮	⋮	⋮	⋮
151	−1.6219506622	−1.6219506622	−0.8109753311	0.0000000001
152	−1.6219506621	−1.6219506621	−0.8109753311	0.0000000000

Table 32: Jungck Mann iteration scheme ($\beta = 0.8459$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n\mid$
0	−1.4582602451	−1.5109223413	−0.7554611707	0.1445388293
1	−1.7140723826	−1.6827669612	−0.8413834806	0.0859223100
2	−1.5681822910	−1.5858397887	−0.7929198943	0.0484635863
⋮	⋮	⋮	⋮	⋮
40	−1.6219506621	−1.6219506621	−0.8109753311	0.0000000000

Table 33: Modified Jungck Mann iteration scheme ($\beta = 0.8459$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n\mid$
0	−1.4582602451	−1.6091341275	−0.8045670638	0.0954329362
1	−1.6329882023	−1.6218782962	−0.8109391481	0.0063720843
2	−1.6220132735	−1.6219506558	−0.8109753279	0.0000361798
3	−1.6219506676	−1.6219506621	−0.8109753311	0.0000000032
4	−1.6219506621	−1.6219506621	−0.8109753311	0.0000000000

Table 34: Jungck Ishikawa iteration scheme
($\alpha = 0.959$ and $\beta = 0.41026$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n\mid$
0	−1.4582602451	−1.5974186041	−0.7987093021	1.6987093021
1	−1.6429867699	−1.6257780647	−0.8128890324	0.0141797303
2	−1.6186344964	−1.6213633194	−0.8106816597	0.0022073727
⋮	⋮	⋮	⋮	⋮
12	−1.6219506621	−1.6219506621	−0.8109753311	0.0000000000

Table 35: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.959$ and $\beta = 0.41026$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	−1.4582602451	−1.6219470030	−0.8109735015	0.0890264985
1	−1.6219538281	−1.6219506621	−0.8109753311	0.0000018296
2	−1.6219506621	−1.6219506621	−0.8109753311	0.0000000000

Example 4.8. Consider the nonlinear equation $4\sin^{-1}(x) - 4x\sqrt{(1-x^2)}$ $+8x^2\cos^{-1}(x) - \pi = 0$. Let us take $Tx = 4x\sqrt{(1-x^2)} - 8x^2\cos^{-1}(x) + \pi$, $Sx = 4\sin^{-1}(x)$ and $Sx = 4\sin^{-1}(x)$. The true graphical solution of given equation is 0.5793642365 ($10D$). If we choose initial guess $x_0 = 0.52$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iterationschemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 77 iterations, Jungck Mann iteration scheme for $\beta = 0.6950029203$ evaluate the solution in 17 iterations, modified Jungck Mann iteration scheme for $\beta = 0.6950029203$ evaluate the solution in 2 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.891$ and $\beta = 0.3991341$ evaluate the solution in 11 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.891$ and $\beta = 0.3991341$ evaluate the solution in 3 iterations of the given equation (see Tables 36-40).

Table 36: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	2.70325983	2.703259837	0.62553335	0.10553335
1	2.29215848	2.29215848	2.70325984	0.08334474
2	2.61743109	2.61743109	0.60864980	0.06646119
3	2.35730769	2.35730769	2.61743109	0.05284819
4	2.56424910	2.56424910	0.59804715	0.04224554
5	2.39860595	2.39860595	0.56435480	0.03369235
⋮	⋮	⋮	⋮	⋮
75	2.47179384	2.47179384	0.57936423	0.00000001
76	2.47179386	2.47179386	0.57936424	0.00000000

Table 37: Jungck Mann iteration scheme ($\beta = 0.6950029203$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	2.7032598372	2.5459252531	0.5943694265	0.0743694265
1	2.4129803137	2.4535281320	0.5756362552	0.0187331713
2	2.4864290717	2.4763943812	0.5803012907	0.0046650355
⋮	⋮	⋮	⋮	⋮
16	2.4717938486	2.4717938486	0.5793642365	0.0000000000

Table 38: Modified Jungck Mann iteration scheme ($\beta = 0.6950029203$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	2.7032598372	2.4717938486	0.5793642365	0.0593642365
1	2.4717938486	2.4717938486	0.5793642365	0.0000000000

Table 39: Jungck Ishikawa iteration scheme ($\alpha = 0.891$ and $\beta = 0.3991341$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	2.7032598372	2.4971256681	0.5845143776	0.0645143776
1	2.4515855985	2.4694595702	0.5788884885	0.0000644928
2	2.4736612871	2.4720078118	0.5794078344	0.0005193458
⋮	⋮	⋮	⋮	⋮
10	2.4717938486	2.4717938486	0.5793642365	0.0000000000

Table 40: Modified Jungck Ishikawa iteration scheme ($\alpha = 0.891$ and $\beta = 0.3991341$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	2.7032598372	2.4714773552	0.5792997437	0.0592997437
1	2.4720469948	2.4717938485	0.5793642365	0.0000644928
2	2.4717938486	2.4717938486	0.5793642365	0.0000000000

Example 4.9. Consider the nonlinear equation $e^x - 2 - \cos(e^x - 2) = 0$. Let us take $Tx = \cos(e^x - 2)$ and $Sx = e^x - 2$. The true graphical solution of given equation is 1.0076239716 ($10D$). If we choose initial guess $x_0 = 1.5$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iteration schemes are shown below. From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 50 iterations, Jungck Mann iteration scheme for $\beta = 0.92$ evaluate the solution in 35 iterations, modified Jungck Mann iteration scheme for $\beta = 0.92$ evaluate the solution in 8

iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.9337912$ and $\beta = 0.1$ evaluate the solution in 31 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.9337912$ and $\beta = 0.1$ evaluate the solution in 6 iterations of the given equation (see Tables 41-45).

Table 41: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.7529375549	0.7529375549	1.0126685433	0.0126685433
1	0.7296833636	0.7296833636	1.0041856184	0.0084829249
2	0.7453855202	0.7453855202	1.0099215096	0.0057358912
3	0.7348264759	0.7348264759	1.0060679881	0.0038535215
4	0.7419471053	0.7419471053	1.0086682904	0.0026003023
5	0.7371542502	0.7371542502	1.0069187859	0.0017495045
⋮	⋮	⋮	⋮	⋮
48	0.7390851333	0.7390851333	1.0076239717	0.0000000001
49	0.7390851332	0.7390851332	1.0076239716	0.0000000000

Table 42: Jungck Mann iteration scheme ($\beta = 0.92$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−0.7900513426	−0.5283121095	0.3864099669	1.1135900331
1	0.8636591260	0.7523014271	1.0124374443	0.6260274774
2	0.7301181906	0.7318928495	1.0049947204	0.0074427239
⋮	⋮	⋮	⋮	⋮
34	0.7390851332	0.7390851332	1.0076239717	0.0000000000

Table 43: Modified Jungck Mann iteration scheme ($\beta = 0.92$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−0.7900513425	0.2748994024	0.8219358327	0.6780641673
1	0.9624525097	0.5163877552	0.9228244427	0.0691065510
2	0.8696083667	0.6964362497	0.9919309936	0.0691065510
3	0.7671331565	0.7376504595	1.0071000560	0.0151690624
4	0.7400507857	0.7390815202	1.0076226526	0.0005225966
5	0.7390875670	0.7390851279	1.0076239697	0.0000013171
6	0.7390851368	0.7390851332	1.0076239717	0.0000000020
7	0.7390851332	0.7390851332	1.0076239717	0.0000000000

Table 44: Jungck Ishikawa iteration scheme
($\alpha = 0.9337912$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	−0.7900513426	−0.3503313386	0.5005744565	0.9994255435
1	0.9392590460	0.8878089262	1.0604980576	0.5599236011
2	0.6311131359	0.6665057921	0.9807689232	0.0797291343
⋮	⋮	⋮	⋮	⋮
30	0.7390851332	0.7390851332	1.0076239717	0.0000000000

Table 45: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.9337912$ and $\beta = 0.1$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	−0.7900513426	0.2153125014	0.7953934777	0.7046065223
1	0.9769096751	0.5334802556	0.9295939526	0.1342004749
2	0.8610424636	0.7142683653	0.9985224384	0.0689284858
3	0.7555727115	0.7387680168	1.0075081904	0.0089857520
4	0.7392987094	0.7390851332	1.0076239717	0.0001157813
5	0.7390851332	0.7390851332	1.0076239717	0.0000000000

Example 4.10. Consider the nonlinear equation $x^3 + 4x^2 - 5x - 10 = 0$. Let us take $Tx = x^3+4x^2-10$ and $Sx = 5x$. The true graphical solution of given equation is ($10D$). If we choose initial guess $x_0 = -1$, then Jungck iteration cannot converges to the true solution. But the comparision tables correct upto ten decimal places of Jungck Mann, modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iteration schemes are shown below.From tables, we can see that Jungck Mann Iteration scheme for $\beta = 0.578841$ evaluate the solution in 15 iterations, modified Jungck Mann iteration scheme for $\beta = 0.578841$ evaluate the solution in 4 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.88063307794$ and $\beta = 0.4$ evaluate the solution in 8 iterations and modified Jungck Ishikawa iteration scheme for $\alpha = 0.88063307794$ and $\beta = 0.4$ evaluate the solution in 2 iterations of the given equation (see Tables 46-50).

Table 46: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	−7.0000000000	−7.0000000000	−1.4000000000	2.4000000000
1	−4.9040000000	−4.9040000000	−0.9808000000	0.4192000000
2	−7.0956242821	−7.0956242821	−1.4191248564	0.4383248564
⋮	⋮	⋮	⋮	⋮

Table 47: Jungck Mann iteration scheme ($\beta = 0.578841$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−7.0000000000	−6.1576820000	−1.2315364000	0.2315364000
1	−6.0052195912	−5.9512900794	−1.1902580159	0.0412783841
2	−5.9844336354	−5.9907141930	−1.1981428386	0.0078848227
⋮	⋮	⋮	⋮	⋮
14	5.9844338974	−5.9844338974	−1.1968867795	0.0000000000

Table 48: Modified Jungck Mann iteration scheme ($\beta = 0.578841$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−7.0000000000	−5.9647350000	−1.1929470000	0.1929470000
1	−6.0052195912	−5.9844341456	−1.1968868291	0.0039398291
2	−5.9844336354	−5.9844338973	−1.1968867795	0.0000000497
3	5.9844338974	−5.9844338974	−1.1968867795	0.0000000000

Table 49: Jungck Ishikawa iteration scheme
($\alpha = 0.88063307794$ and $\beta = 0.4$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−7.0000000000	−6.0378225598	−1.2075645120	2.2075645120
1	−5.9280369209	−5.9819989431	−1.1963997886	0.0111647233
2	−5.9870038848	−5.9845456964	−1.1969091393	0.0005093507
⋮	⋮	⋮	⋮	⋮
8	−5.9844338974	−5.9844338974	−1.1968867795	0.0000000000

Table 50: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.88063307794$ and $\beta = 0.4$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	−7.0000000000	−5.9844338974	−1.1968867795	0.1968867795
1	−5.9844338974	−5.9844338974	−1.1968867795	0.0000000000

Example 4.11. Consider the nonlinear equation $2x - e^{\cos(x)} = 0$. Let us take $Tx = e^{\cos(x)}$ and $Sx = 2x$. The true graphical solution of given equation is 0.9179048993 ($10D$). If we choose initial guess $x_0 = 1$, then the comparision tables correct upto ten decimal places of Jungck, Jungck Mann, modified Jungck Mann, Jungck Ishikawa and modified Jungck Ishikawa iteration schemes are shown below.From tables, we can see that Jungck iteration scheme evaluate the solution of given equation in 71 iterations, Jungck Mann iteration scheme

for $\beta = 0.98$ evaluate the solution in 59 iterations, modified Jungck Mann iteration scheme for $\beta = 0.98$ evaluate the solution in 5 iterations, Jungck Ishikawa iteration scheme for $\alpha = 0.9876$ and $\beta = 0.3$ evaluate the solution in 21 iterations and modified Jungck Ishikawa iterative scheme for $\alpha = 0.9876$ and $\beta = 0.3$ evaluate the solution in 4 iterations of the given equation (see Tables 51-55).

Table 51: Jungck iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.7165256996	1.7165256996	0.8582628498	0.1417371502
1	1.9227433021	1.9227433021	0.9613716511	0.1031088013
2	1.7725085479	1.7725085479	0.8862542740	0.0751173771
3	1.8819684975	1.8819684975	0.9409842488	0.0547299748
⋮	⋮	⋮	⋮	⋮
69	1.8358097987	1.8358097987	0.9179048993	0.0000000001
70	1.83580979876	1.83580979876	0.9179048993	0.0000000000

Table 52: Jungck Mann iteration scheme ($\beta = 0.98$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.7165256995	1.7221951856	0.8610975928	0.1389024072
1	1.9186182640	1.9146898024	0.9573449012	0.0962473084
2	1.7783624990	1.7810890451	0.8905445225	0.0668003787
⋮	⋮	⋮	⋮	⋮
58	1.8358097986	1.8358097986	0.9179048993	0.0000000000

Table 53: Modified Jungck Mann iteration scheme ($\beta = 0.98$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.7165256995	1.8358443705	0.9179221852	0.0820778148
1	1.8357845916	1.8358096186	0.9179048093	0.0000173760
2	1.8358099299	1.8358097996	0.9179048998	0.0000000905
3	1.8358097979	1.8358097986	0.9179048993	0.0000000005
4	1.8358097986	1.8358097986	0.9179048993	0.0000000000

Table 54: Jungck Ishikawa iteration scheme
($\alpha = 0.9876$ and $\beta = 0.3$)

n	Tx_n	Sx_{n+1}	x_{n+1}	$\mid e_n \mid$
0	1.7165256995	1.7809184378	0.8904592189	0.1095407811
1	1.8758372895	1.8541530315	0.9270765157	0.0366172968
2	1.8224376033	1.8296804230	0.9148402115	0.0122363042
⋮	⋮	⋮	⋮	⋮
20	1.8358097986	1.8358097986	0.9179048993	0.0000000000

Table 55: Modified Jungck Ishikawa iteration scheme
($\alpha = 0.9876$ and $\beta = 0.3$)

n	Tx_{n}	$Sx_{\mathsf{n+1}}$	$x_{\mathsf{n+1}}$	$\lvert e_{\mathsf{n}}\rvert$
0	1.7165256995	1.8359669942	0.9179834971	0.0820165029
1	1.8356951843	1.8358098261	0.9179049130	0.0000785841
2	1.8358097786	1.8358097986	0.9179048993	0.0000000137
3	1.8358097986	1.8358097986	0.9179048993	0.0000000000

5. Conclusions

The modified Jungck Mann and modified Jungck Ishikawa iteration schemes for solving nonlinear equations are established. The examples discussed above show that the efficiencies of both the iteration schemes are very good as compared to the Jungck, Jungck Mann and Jungck Ishikawa iteration schemes.

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