

$(\tau_1, \tau_2)^*$ - rg^* CONTINUOUS FUNCTIONS

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Abstract: The aim of the present paper is to introduce the different types of $(\tau_1, \tau_2)^*$ -regular generalized star continuous functions and study their basic properties in bitopological spaces.

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Key Words: $(\tau_1, \tau_2)^*$ - rg^* continuous function, $(\tau_1, \tau_2)^*$ - g^*r continuous function, $(\tau_1, \tau_2)^*$ - g continuous functions, $(\tau_1, \tau_2)^*$ - gs continuous functions, $(\tau_1, \tau_2)^*$ - rg continuous functions, $(\tau_1, \tau_2)^*$ - g^*r irresolute functions

1. Introduction

In 1963, the concept of bitopological space was introduced by Kelly [1].

Considerable effort had been expended in obtaining appropriate generalizations of standard topological properties to bitopological category. Kelly initiated the study of separation properties for bitopological spaces. This is the initiative of the study of bitopological spaces and generalized closed sets in bitopological spaces. There are several works dedicated to the investigation of pairs of topologies on the same set (i.e. bitopologies), most of them deal with the theory itself but very few with applications. Maheshwari and Prasad [2]

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introduced semi open sets in bitopological spaces in 1977 and further properties of this notion were studied by Bose [3] in 1981. Fukutake [4] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989.

Fukutake also introduced generalized closed sets and pairwise generalized closure operator [5] in bitopological spaces in 1986. For any subset $A \subseteq X$, the intersection (resp. union) of all τ_i -closed sets containing A (resp. τ_i -open sets contained in A) is called the τ_i -closure (resp. τ_i -interior) of A , denoted by $\tau_i\text{-cl}(A)$ {resp. $\tau_i\text{-int}(A)$ }. He defined a set A of a bitopological space (X, τ_1, τ_2) to be $\tau_i\tau_j$ -generalized closed set (briefly $\tau_i\tau_j$ - g closed) if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i -open in X , $i, j = 1, 2$ and $i \neq j$. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$ -space. Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by Khedr and Al-saadi. In 2002, Fukutake, Sundaram and Sheik John [6] introduced the concept of w -closed sets, w -open sets and w -continuous maps in bitopological spaces. The g^* -closed sets in bitopological spaces were introduced by Sheik and Sundaram [7] in 2004. Lellis Thivagar and Ravi [8] introduced a new type of generalized sets called $(1, 2)^*$ -semi generalized closed sets and a new class of generalized functions called $(1, 2)^*$ -semi generalized continuous maps in 2006. On the other hand, Kannan et. al. [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] introduced the various concepts closed sets in a bitopological space.

In the present paper, we introduce the concepts of $(\tau_1, \tau_2)^*$ - rg^* continuous functions, $(\tau_1, \tau_2)^*$ - rg^* homeomorphisms and $(\tau_1, \tau_2)^*$ - RG^*O compact spaces and study their basic properties in bitopological spaces.

2. Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. A subset A of X is $\tau_1\tau_2$ -open if $A \in \tau_1 \cup \tau_2$ and A is $\tau_1\tau_2$ -closed if A^C is $\tau_1\tau_2$ -open. For any subset $A \subseteq X$, the intersection (resp. union) of all $\tau_1\tau_2$ -closed sets containing A (resp. $\tau_1\tau_2$ -open sets contained in A) is called the $\tau_1\tau_2$ -closure (resp. $\tau_1\tau_2$ -interior) of A , denoted by $\tau_1\tau_2\text{-cl}(A)$ {resp. $\tau_1\tau_2\text{-int}(A)$ }.

Definition 2.1. A set A of a bitopological space (X, τ_1, τ_2) is called

- (a) $\tau_1\tau_2$ -semi open [4] if $A \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)]$.
- (b) $\tau_1\tau_2$ -semi closed [4] if $X - A$ is $\tau_1\tau_2$ -semi open.

Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ -semi closed if $\tau_2\text{-int}[\tau_1\text{-cl}(A)] \subseteq A$

- (c) $(\tau_1, \tau_2)^*$ -semi open [8] if $A \subseteq \tau_1\tau_2-cl[\tau_1\tau_2-int(A)]$.
- (d) $(\tau_1, \tau_2)^*$ -semi closed [8] if $X - A$ is $(\tau_1, \tau_2)^*$ -semi open.

Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ -semi closed if $\tau_1\tau_2-int[\tau_1\tau_2-cl(A)] \subseteq A$

- (e) $(\tau_1, \tau_2)^*$ -generalized closed $\{(\tau_1, \tau_2)^*-g \text{ closed}\}$ [20] if $\tau_1\tau_2-cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -open in X .
- (f) $(\tau_1, \tau_2)^*$ -generalized open $\{(\tau_1, \tau_2)^*-g \text{ open}\}$ [20] if $X - A$ is $(\tau_1, \tau_2)^*-g$ closed.

For any subset $A \subseteq X$, the intersection (resp. union) of all $(\tau_1, \tau_2)^*$ -semi closed sets containing A (resp. $(\tau_1, \tau_2)^*$ -semi open sets contained in A) is called the $(\tau_1, \tau_2)^*$ -semi closure (resp. $(\tau_1, \tau_2)^*$ -semi interior) of A , denoted by $(\tau_1, \tau_2)^*-scl(A)$ {resp. $(\tau_1, \tau_2)^*-sint(A)$ }.

Definition 2.2. A set A of a bitopological space (X, τ_1, τ_2) is called

- (a) $(\tau_1, \tau_2)^*$ -semi generalized closed $\{(\tau_1, \tau_2)^*-sg \text{ closed}\}$ [8] if $(\tau_1, \tau_2)^*-scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -semi open in X .
- (b) $(\tau_1, \tau_2)^*$ -semi generalized open $\{(\tau_1, \tau_2)^*-sg \text{ open}\}$ [8] if $X - A$ is $(\tau_1, \tau_2)^*-sg$ closed.
- (c) $\tau_1\tau_2$ -regular closed [8] if $\tau_1-cl[\tau_2-int(A)] = A$.
- (d) $\tau_1\tau_2$ -regular open [8] if $\tau_1-int[\tau_2-cl(A)] = A$.
- (e) $(\tau_1, \tau_2)^*$ -regular closed [8] if $\tau_1\tau_2-cl[\tau_1\tau_2-int(A)] = A$.
- (f) $(\tau_1, \tau_2)^*$ -regular open [8] if $X - A$ is $(\tau_1, \tau_2)^*$ -regular closed.

The intersection (resp. union) of all $(\tau_1, \tau_2)^*$ -regular closed sets containing A (resp. $(\tau_1, \tau_2)^*$ -regular open sets contained in A) is called the $(\tau_1, \tau_2)^*$ -regular closure (resp. $(\tau_1, \tau_2)^*$ -regular interior) of A , denoted by $(\tau_1, \tau_2)^*-rcl(A)$ {resp. $(\tau_1, \tau_2)^*-rint(A)$ }. The set of all $(\tau_1, \tau_2)^*$ -regular closed sets in X is denoted by $(\tau_1, \tau_2)^*-R.C(X, \tau_1, \tau_2)$. The set of all $(\tau_1, \tau_2)^*$ -regular open sets in X is denoted by $(\tau_1, \tau_2)^*-R.O(X, \tau_1, \tau_2)$. A^C denotes the complement of A in X unless explicitly stated.

Definition 2.3. A set A of a bitopological space (X, τ_1, τ_2) is called

- (a) $(\tau_1, \tau_2)^*$ -regular generalized closed $((\tau_1\tau_2)^*$ - rg closed) [21] in X if $(\tau_1, \tau_2)^*$ - $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -regular open in X .
- (b) $(\tau_1, \tau_2)^*$ -regular generalized open $((\tau_1\tau_2)^*$ - rg open) [21] in X if $F \subseteq (\tau_1, \tau_2)^*$ - $\text{int}(A)$ whenever $F \subseteq A$ and F is $(\tau_1, \tau_2)^*$ -regular closed in X .
- (c) $(\tau_1, \tau_2)^*$ -regular generalized star closed $((\tau_1\tau_2)^*$ - rg^* closed) [22] in X if $(\tau_1, \tau_2)^*$ - $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -regular open in X .
- (d) $(\tau_1, \tau_2)^*$ -regular generalized star open $((\tau_1\tau_2)^*$ - rg^* open) [22] in X if $F \subseteq (\tau_1, \tau_2)^*$ - $\text{rint}(A)$ whenever $F \subseteq A$ and F is $(\tau_1, \tau_2)^*$ -regular closed in X .
- (e) $(\tau_1, \tau_2)^*$ -generalized star regular closed $((\tau_1\tau_2)^*$ - g^*r closed) [23] in X if $(\tau_1, \tau_2)^*$ - $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -open in X .
- (f) $(\tau_1, \tau_2)^*$ -generalized star regular open $((\tau_1\tau_2)^*$ - g^*r open) [23] in X if $F \subseteq (\tau_1, \tau_2)^*$ - $\text{rint}(A)$ whenever $F \subseteq A$ and F is $\tau_1\tau_2$ -closed in X .

Definition 2.4. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (a) $(\tau_1, \tau_2)^*$ - g continuous [8] in X if the inverse image of each $\sigma_1\sigma_2$ -closed set is $(\tau_1, \tau_2)^*$ - g closed in X .
- (b) $(\tau_1, \tau_2)^*$ - gs continuous [25] in X if the inverse image of each $\sigma_1\sigma_2$ -closed set is $(\tau_1, \tau_2)^*$ - gs closed in X .
- (c) $(\tau_1, \tau_2)^*$ - rg continuous [21] in X if the inverse image of each $\sigma_1\sigma_2$ -closed set is $(\tau_1, \tau_2)^*$ - rg closed in X .
- (d) almost $(\tau_1, \tau_2)^*$ - g continuous [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$ -regular closed set is $(\tau_1, \tau_2)^*$ - g closed in X .
- (e) almost $(\tau_1, \tau_2)^*$ - gs continuous in X if the inverse image of each $(\sigma_1, \sigma_2)^*$ -regular closed set is $(\tau_1, \tau_2)^*$ - gs closed in X .
- (f) almost $(\tau_1, \tau_2)^*$ - rg continuous [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$ -regular closed set is $(\tau_1, \tau_2)^*$ - rg closed in X .

Definition 2.5. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (a) $(\tau_1, \tau_2)^*-g$ irresolute [8] in X if the inverse image of each $(\sigma_1, \sigma_2)^*-g$ closed set is $(\tau_1, \tau_2)^*-g$ closed in X .
- (b) $(\tau_1, \tau_2)^*-gs$ irresolute [25] in X if the inverse image of each $(\sigma_1, \sigma_2)^*-gs$ closed set is $(\tau_1, \tau_2)^*-gs$ closed in X .
- (c) $(\tau_1, \tau_2)^*-rg$ irresolute [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*-rg$ closed set is $(\tau_1, \tau_2)^*-rg$ closed in X .
- (d) $(\tau_1, \tau_2)^*$ -generalized star regular continuous ($(\tau_1, \tau_2)^*-g^*r$ continuous) in X if the inverse image of each $\sigma_1\sigma_2$ -closed set is $(\tau_1, \tau_2)^*$ -generalized star regular closed in X .
- (e) $(\tau_1, \tau_2)^*-g^*r$ irresolute in X if the inverse image of each $(\sigma_1, \sigma_2)^*$ -generalized star regular closed set is $(\tau_1, \tau_2)^*$ -generalized star regular closed in X .

3. $(\tau_1, \tau_2)^*-rg^*$ Continuous Functions

Definition 3.1. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (a) $(\tau_1, \tau_2)^*$ -regular generalized star continuous ($(\tau_1, \tau_2)^*-rg^*$ continuous) in X if the inverse image of each $\sigma_1\sigma_2$ -closed set is $(\tau_1, \tau_2)^*$ -regular generalized star closed in X .
- (b) $(\tau_1, \tau_2)^*-rg^*$ irresolute in X if the inverse image of each $(\sigma_1, \sigma_2)^*$ -regular generalized star closed set is $(\tau_1, \tau_2)^*$ -regular generalized star closed in X .

Example 3.2. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*-rg^*$ continuous and $(\tau_1, \tau_2)^*-rg^*$ irresolute.

Theorem 3.3. Every $(\tau_1, \tau_2)^*-g^*r$ continuous function is $(\tau_1, \tau_2)^*-rg^*$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ continuous and F be $\sigma_1\sigma_2$ -closed in Y . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ continuous, $f^{-1}(F)$ is $(\tau_1, \tau_2)^*-g^*r$ closed in X . Since every $(\tau_1, \tau_2)^*-g^*r$ closed set is $(\tau_1, \tau_2)^*-rg^*$ closed, $f^{-1}(F)$ is $(\tau_1, \tau_2)^*-rg^*$ closed in X . Therefore, f is $(\tau_1, \tau_2)^*-rg^*$ continuous. \square

Remark 3.4. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.5. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, c\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*-rg^*$ continuous, but not a $(\tau_1, \tau_2)^*-g^*r$ -continuous function.

Theorem 3.6. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ continuous, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous.

Proof. Let F be $\mu_1\mu_2$ -closed set in Z . Since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ continuous, $g^{-1}(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*-rg^*$ closed in X . Hence $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous. \square

Theorem 3.7. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$ -continuous, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous.

Proof. Let F be $\mu_1\mu_2$ -closed set in Z . Since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$ -continuous, $g^{-1}(F)$ is $\sigma_1\sigma_2$ -closed in Y . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*-rg^*$ closed in X . Hence $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-g^*r$ continuous. \square

Remark 3.8. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ continuous, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ need not be $(\tau_1, \tau_2)^*-rg^*$ continuous. The following example supports our claim.

Example 3.9. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b, c\}\}$, $Z = \{a, b, c\}$, $\mu_1 = \{\phi, Z, \{a\}\}$, $\mu_2 = \{\phi, Z, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be the function defined by $g(a) = b$, $g(b) = c$, $g(c) = a$. Then f and g are $(\tau_1, \tau_2)^*-rg^*$ continuous and $\sigma_1, \sigma_2)^*-rg^*$ continuous functions, but gof is not $(\tau_1, \tau_2)^*-rg^*$ continuous.

Theorem 3.10. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ irresolute, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute.

Proof. Let F be $(\mu_1, \mu_2)^*-rg^*$ closed set in Z . Since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ irresolute, $g^{-1}(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*-rg^*$ closed in X . Hence $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ irresolute. \square

Definition 3.11. Recall that a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (a) $(\tau_1, \tau_2)^*-g$ closed [25] in X if the image of each $\tau_1\tau_2$ -closed set is $(\sigma_1, \sigma_2)^*-g$ closed in Y .
- (b) $(\tau_1, \tau_2)^*-gs$ closed [25] in X if the image of each $\tau_1\tau_2$ -closed set is $(\sigma_1, \sigma_2)^*-gs$ closed in Y .
- (c) $(\tau_1, \tau_2)^*-rg$ closed [21] in X if the image of each $\tau_1\tau_2$ -closed set is $(\sigma_1, \sigma_2)^*-rg$ closed in Y .
- (d) almost $(\tau_1, \tau_2)^*-g$ closed [21] in X if the image of each $(\tau_1, \tau_2)^*$ -regular closed set is $(\sigma_1, \sigma_2)^*-g$ closed in Y .
- (e) almost $(\tau_1, \tau_2)^*-gs$ closed in X if the image of each $(\tau_1, \tau_2)^*$ -regular closed set is $(\sigma_1, \sigma_2)^*-gs$ closed in Y .
- (f) almost $(\tau_1, \tau_2)^*-rg$ closed [21] in X if the image of each $(\tau_1, \tau_2)^*$ -regular closed set is $(\sigma_1, \sigma_2)^*-rg$ closed in Y .
- (g) $(\tau_1, \tau_2)^*-g^*r$ closed in X if the image of each $\tau_1\tau_2$ -closed set is $(\sigma_1, \sigma_2)^*$ -generalized star regular closed in Y .

Definition 3.12. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*-rg^*$ closed in X if the image of each $\tau_1\tau_2$ -closed set is $(\sigma_1, \sigma_2)^*$ -regular generalized star closed in Y .

Example 3.13. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{b, c\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*-rg^*$ closed function.

Theorem 3.14. Every $(\tau_1, \tau_2)^*-g^*r$ closed function is $(\tau_1, \tau_2)^*-rg^*$ closed.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ closed and F be $\tau_1\tau_2$ -closed in X . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ closed, $f(F)$ is $(\sigma_1, \sigma_2)^*-g^*r$ closed in Y . Since every $(\sigma_1, \sigma_2)^*-g^*r$ closed set is $(\sigma_1, \sigma_2)^*-rg^*$ closed, $f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Therefore, f is $(\tau_1, \tau_2)^*-rg^*$ closed. \square

Remark 3.15. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.16. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{a, c\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*-rg^*$ closed, but not a $(\tau_1, \tau_2)^*-g^*r$ -closed function.

Theorem 3.17. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$ -closed and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ closed.

Proof. Let F be $\tau_1\tau_2$ -closed set in X . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$ -closed, $f(F)$ is $\sigma_1\sigma_2$ -closed in Y . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ closed, $(gof)(F) = g[f(F)]$ is $(\mu_1, \mu_2)^*-rg^*$ closed in Z . Hence $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ closed. \square

Corollary 3.18. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$ -closed and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-g^*r$ closed, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*-rg^*$ closed.

Definition 3.19. Recall that a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (a) $(\tau_1, \tau_2)^*-g$ open [25] in X if the image of each $\tau_1\tau_2$ -open set is $(\sigma_1, \sigma_2)^*-g$ open in Y .
- (b) $(\tau_1, \tau_2)^*-gs$ open [25] in X if the image of each $\tau_1\tau_2$ -open set is $(\sigma_1, \sigma_2)^*-gs$ open in Y .
- (c) $(\tau_1, \tau_2)^*-rg$ open in X if the image of each $\tau_1\tau_2$ -open set is $(\sigma_1, \sigma_2)^*-rg$ open in Y .

- (d) almost $(\tau_1, \tau_2)^*-g$ open in X if the image of each $(\tau_1, \tau_2)^*$ -regular open set is $(\sigma_1, \sigma_2)^*-g$ open in Y .
- (e) almost $(\tau_1, \tau_2)^*-gs$ open in X if the image of each $(\tau_1, \tau_2)^*$ -regular open set is $(\sigma_1, \sigma_2)^*-gs$ open in Y .
- (f) almost $(\tau_1, \tau_2)^*-rg$ open in X if the image of each $(\tau_1, \tau_2)^*$ -regular open set is $(\sigma_1, \sigma_2)^*-rg$ open in Y .
- (g) $(\tau_1, \tau_2)^*-g^*r$ open in X if and only if image of each $\tau_1\tau_2$ -open set is $(\sigma_1, \sigma_2)^*-g^*r$ open in Y .

Definition 3.20. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*-rg^*$ open in X if and only if image of each $\tau_1\tau_2$ -open set is $(\sigma_1, \sigma_2)^*-rg^*$ open in Y .

Example 3.21. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{b, c\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*-rg^*$ open function.

Theorem 3.22. Every $(\tau_1, \tau_2)^*-g^*r$ open function is $(\tau_1, \tau_2)^*-rg^*$ open.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ open and F be $\tau_1\tau_2$ -open in X . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*-g^*r$ open, $f(F)$ is $(\sigma_1, \sigma_2)^*-g^*r$ open in Y . Since every $(\sigma_1, \sigma_2)^*-g^*r$ open set is $(\sigma_1, \sigma_2)^*-rg^*$ open, $f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ open in Y . Therefore, f is $(\tau_1, \tau_2)^*-rg^*$ open. \square

Remark 3.23. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.24. In Example 3.16, f is $(\tau_1, \tau_2)^*-rg^*$ open, but not a $(\tau_1, \tau_2)^*-g^*r$ open function.

4. $(\tau_1, \tau_2)^*-rg^*$ Homeomorphisms

Definition 4.1. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*$ -regular generalized star homeomorphism ($(\tau_1, \tau_2)^*-rg^*$ homeomorphism) if and

only if both f is $(\tau_1, \tau_2)^*-rg^*$ continuous and $(\tau_1, \tau_2)^*-rg^*$ open.

Example 4.2. In Example 3.9, f is $(\tau_1, \tau_2)^*-rg^*$ homeomorphism.

Definition 4.3. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*-rg^*c$ homeomorphism) if and only if f is $(\tau_1, \tau_2)^*-rg^*$ irresolute and f^{-1} is $(\sigma_1, \sigma_2)^*-rg^*$ irresolute.

Example 4.4. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*-rg^*c$ homeomorphism.

Theorem 4.5. For any bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent.

- (a) $f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ continuous.
- (b) f is $(\tau_1, \tau_2)^*-rg^*$ open.
- (c) f is $(\tau_1, \tau_2)^*-rg^*$ closed.

Proof. (a) \Rightarrow (b) : Let F be $\tau_1\tau_2$ -open in X . Then $X - F$ is $\tau_1\tau_2$ -closed in X . Since f^{-1} is $(\sigma_1, \sigma_2)^*-rg^*$ continuous, $[f^{-1}]^{-1}(X - F) = f(X - F) = Y - f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Then $f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ open in Y . Hence f is $(\tau_1, \tau_2)^*-rg^*$ open.

(b) \Rightarrow (c) : Let F be $\tau_1\tau_2$ -closed in X . Then $X - F$ is $\tau_1\tau_2$ -open in X . Since f is $(\tau_1, \tau_2)^*-rg^*$ open, $f(X - F) = Y - f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ open in Y . Then $f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Hence f is $(\tau_1, \tau_2)^*-rg^*$ closed.

(c) \Rightarrow (a) : Let F be $\tau_1\tau_2$ -closed in X . Since f is $(\tau_1, \tau_2)^*-rg^*$ closed, $[f^{-1}]^{-1}(F) = f(F)$ is $(\sigma_1, \sigma_2)^*-rg^*$ closed in Y . Hence f is $(\tau_1, \tau_2)^*-rg^*$ continuous. \square

Theorem 4.6. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a bijective and $(\tau_1, \tau_2)^*-g^*r$ continuous function. Then the following statements are equivalent.

- (a) f is $(\tau_1, \tau_2)^*-rg^*$ open.
- (b) f is $(\tau_1, \tau_2)^*-rg^*$ homeomorphism.
- (c) f is $(\tau_1, \tau_2)^*-rg^*$ closed.

Proof. (a) \Rightarrow (b) : obvious from the definitions.

(b) \Rightarrow (c) : Let F be $\tau_1\tau_2$ -closed in X . Then $X - F$ is $\tau_1\tau_2$ -open in X . Since f is $(\tau_1, \tau_2)^*-rg^*$ homeomorphism, f is $(\tau_1, \tau_2)^*-rg^*$ open. Hence by above theorem, f is $(\tau_1, \tau_2)^*-rg^*$ closed.

(c) \Rightarrow (a) : Follows from above Theorem. \square

Remark 4.7. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ homeomorphism and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*-rg^*$ homeomorphism, then $gof : (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)$ need not be a $(\tau_1, \tau_2)^*-rg^*$ homeomorphism.

Example 4.8. In Example 3.9, f and g are $(\tau_1, \tau_2)^*-g^*r$ homeomorphism and $(\sigma_1, \sigma_2)^*-rg^*$ homeomorphism, but gof is not $(\tau_1, \tau_2)^*-rg^*$ homeomorphism.

5. $(\tau_1, \tau_2)^*-RG^*O$ Compact Space

Definition 5.1. A subset B of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)^*-RG^*O$ compact relative to X if for every cover $\{A_i : i \in \Lambda\}$ of B by $(\tau_1, \tau_2)^*-rg^*$ open subsets of (X, τ_1, τ_2) , i.e. $B \subseteq \bigcup_{i \in \Lambda} A_i$ where $A_i, i \in \Lambda$ are $(\tau_1, \tau_2)^*-rg^*$ open sets of (X, τ_1, τ_2) , there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i \in \Lambda_0} A_i$.

Definition 5.2. A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)^*-RG^*O$ compact if for every cover $\{A_i : i \in \Lambda\}$ of X by $(\tau_1, \tau_2)^*-rg^*$ open subsets of (X, τ_1, τ_2) , i.e. $X \subseteq \bigcup_{i \in \Lambda} A_i$ where $A_i, i \in \Lambda$ are $(\tau_1, \tau_2)^*-rg^*$ open sets of (X, τ_1, τ_2) , there exists a finite subset Λ_0 of Λ such that $X \subseteq \bigcup_{i \in \Lambda_0} A_i$.

Theorem 5.3. Every $(\tau_1, \tau_2)^*-RG^*O$ compact space is $(\tau_1, \tau_2)^*-G^*RO$ compact space.

Proposition 5.4. A $(\tau_1, \tau_2)^*-rg^*$ closed subset of a $(\tau_1, \tau_2)^*-RG^*O$ compact space (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*-RG^*O$ compact relative to X .

Proof. Let (X, τ_1, τ_2) be a $(\tau_1, \tau_2)^*-RG^*O$ compact space and Y be a $(\tau_1, \tau_2)^*-rg^*$ closed subset of X . Let $\zeta = \{A_i : i \in \Lambda\}$ be a $(\tau_1, \tau_2)^*-rg^*$ open cover of Y .

Since Y is $(\tau_1, \tau_2)^*-rg^*$ closed subset, Y^C is $(\tau_1, \tau_2)^*-rg^*$ open. Also $\zeta \cup Y^C = Y^C \cup \left(\bigcup_{i \in \Lambda} A_i \right)$ be a $(\tau_1, \tau_2)^*-rg^*$ open cover of X . Since X is $(\tau_1, \tau_2)^*-RG^*O$ compact, $X = Y^C \cup \left(\bigcup_{i \in \Lambda_0} A_i \right)$ where Λ_0 is a finite subset Λ . Hence $Y = \bigcup_{i \in \Lambda_0} A_i$. Therefore, Y is $(\tau_1, \tau_2)^*-RG^*O$ compact. \square

Proposition 5.5. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*-rg^*$ continuous and a subset B of X is $(\tau_1, \tau_2)^*-RG^*O$ compact relative to X , then $f(B)$ is $(\sigma_1, \sigma_2)^*$ -compact in Y .

Proof. Let $\{A_i : i \in \Lambda\}$ be any collection of $\sigma_1\sigma_2$ -open subsets of Y such that $f(B) \subseteq \bigcup_{i \in \Lambda} A_i$. Then $B \subseteq \bigcup_{i \in \Lambda} f^{-1}(A_i)$ holds and there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i \in \Lambda_0} f^{-1}(A_i)$. Therefore, we have $f(B) \subseteq \bigcup_{i \in \Lambda_0} A_i$ which shows that $f(B)$ is $(\sigma_1, \sigma_2)^*$ -compact in Y . \square

Proposition 5.6. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is resp. $(\tau_1, \tau_2)^*-rg^*$ irresolute and a subset B of X is $(\tau_1, \tau_2)^*-RG^*O$ compact relative to X , then $f(B)$ is $(\tau_1, \tau_2)^*-RG^*R$ compact relative to Y .

Proof. Let $\{A_i : i \in \Lambda\}$ be any collection of $(\sigma_1, \sigma_2)^*-rg^*$ open subsets of Y such that $f(B) \subseteq \bigcup_{i \in \Lambda} A_i$. Then $B \subseteq \bigcup_{i \in \Lambda} f^{-1}(A_i)$ holds and there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i \in \Lambda_0} f^{-1}(A_i)$. Therefore, we have $f(B) \subseteq \bigcup_{i \in \Lambda_0} A_i$ which shows that $f(B)$ is $(\sigma_1, \sigma_2)^*-RG^*O$ compact in Y . \square

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