LOW COST ELLIPTIC CURVE SCALAR MULTIPLICATION FOR CONSTRAINED DEVICES

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Abstract: This article proposes the technique to reduce the computation cost of scalar multiplication (SM) on elliptic curve for low processor constraints devices. Also the algorithm using the proposed technique secure against Side Channel Attack. There is no such algorithm is found to solve ECDLP in sub-exponential time. Equivalent strength of security is provided by ECC with shorter key size as compared to other Public key Cryptosystem. Therefore, ECC has acquired broad attention to research and development especially for low processor devices. Scalar multiplication on ECC operates on point doubling and adding. The secret the key stream consist of bit is '0' or '1'. While SM operate, doubling of points is followed by addition of points. This is a serious threat for low processor devices. For implementation, it is a challenge to develop algorithms that are not only efficient, but low cost.

Key Words: ECDLP, constraint device, scalar multiplication, side channel attack

1. Introduction

Due to smaller size in key as compared to the earlier conventional public key cryptography like RSA, Elliptic curve based cryptography (ECC) [1] [6] is having more attractiveness in many cryptographic applications. They are more

compatible for implementation on mobile devices such as Smart Phone, PDA, wireless sensor network etc. Due to the physical characteristics of these low processor devices, consumption of time in power of computations using the secret key can be clearly monitored. Therefore there exists a serious threat known as Side Channel Attack (SCA) against these devices [16]. It is very important to scalar multiplication algorithm on elliptic curve which is not only secure against SCA but also of low cost.

Miller and Koblitz introduced Elliptic curve cryptography (ECC) in 1985,. Security of ECC relies on ECDLP. Attractiveness of ECDLP is there does not exist any sub-exponential algorithms to solve this problem till date [3] [5] [4]. This cryptosystem provides equivalent strength of security as compared to the other conventional cryptosystem with shorter key. Because of this advantage, ECC has gained large interest in both cryptology research community and industry.

But it has been observed that, the attacker analyzes the power traces of cryptographic computations of the SM on Elliptic Curve and differentiates the consumption of power generated by the secret key. The operation SM on elliptic curve contains doubling and addition of points on elliptic curve. During the operation, the key bit is '0' or '1' that is doubling of points is followed by addition of points. This is a serious threat for low processor devices. Therefore it is essential to develop an efficient scalar multiplication algorithm on elliptic which is not only SCA-resistant but of low cost.

2. Introduction to Elliptic Curve

Let E denotes the elliptic curve on field F . *Weierstrass equation* is given by the curve $\mathcal{E}$ is

$$\mathcal{E} : v^2 + \alpha_1 uv + \alpha_3 v = u^3 + \alpha_2 u^2 + \alpha_4 u + \alpha_6, \quad (1)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_6 \in K$ and $\Delta \neq 0$.

Let Δ denotes the discriminant of $\mathcal{E}$. It is given by:

$$\begin{aligned} \Delta &= -t_2^2 t_8 - 8t_4^3 - 27t_6^2 + 9t_2 t_4 t_6, \\ t_2 &= \alpha_1^2 + 4\alpha_2, \\ t_4 &= 2\alpha_4 + \alpha_1 \alpha_3, \\ t_6 &= \alpha_3^2 + 4\alpha_6, \end{aligned}$$

$$t_8 = \alpha_1^2 \alpha_6 + 4\alpha_2 \alpha_6 - \alpha_1 \alpha_3 \alpha_4 + \alpha_2 \alpha_3^2 - \alpha_4^2.$$

Let L be the extension of field F .

$$\mathcal{E}(L) = \left\{ (u, v) \in L \times L : v^2 + \alpha_1 uv + \alpha_3 v - u^3 - \alpha_2 u^2 - \alpha_4 u - \alpha_6 \right\} \cup \{\mathcal{O}\}.$$

The point $\mathcal{O}$ called the point at infinity.

Consider the following transformation

$$(u, v) \rightarrow \left(\frac{u - \alpha_1^2 - 12\alpha_2}{36}, \frac{v - 3\alpha_1 u}{216} - \frac{\alpha_1^3 + 4\alpha_1 \alpha_2 - 12\alpha_3}{24} \right).$$

This equation $\mathcal{E}$ can be changed to the curve $v^2 = u^3 + \alpha u + \beta$, where $\alpha, \beta \in K$ and $\Delta = -16(4\alpha^3 + 27\beta^2)$. Characteristics of $F \neq 2$ or 3 Where $4\alpha^3 + 27\beta^2 \neq 0$ means it does not have multiple roots.

2.1. Group Law in EC

Refere to [26] [25], here we described the group laws. Consider the collection of rational points over the elliptic curve form a group with respect to the operation addition. According to secant and tangent's rule, let (u, v) is any arbitrary point lies on the non-super singular curve over binary field. So opposite of the point (u, v) is $(u, -v)$. Consider the field characteristic > 3 . Opposite of the point (u, v) will be $(u, -v)$. For any arbitrary points lies on the elliptic curve over the field of characteristic > 3 , the sum with at infinity is equal to the point itself. Also the sum of opposite point with the point at infinity is the same point.

Consider two arbitrary points P and Q lies on the elliptic curve. Where $Q \neq \pm P$. R be the point of intersection of the line joining P and Q and the curve. Sum of the point P and Q is equal to opposite of the point R . When the two points P and Q are same, we take the tangent at the point of the curve. All the properties of group satisfies on these points. Here we consider the point of infinity is the identity element. in Fig 1 shows the elliptic curve with group laws.

Analytically, let the characteristic of the underlying field K be greater than 3 and let the equation of the elliptic curve E be $v^2 = u^3 + \alpha u + \beta$. Let P be any point on the curve, $P + \mathcal{O} = P$. If P has coordinates (u_1, v_1) then coordinates of $-P$ are $(u_1, -v_1)$ and $P + (-P) = \mathcal{O}$.

Let Q be another point on the curve with coordinates (u_2, v_2) . The sum of P and Q has coordinates (u_3, v_3) , where

$$u_3 = \left(\frac{v_2 - v_1}{u_2 - u_1} \right)^2 - u_1 - u_2$$

and

$$v_3 = \left(\frac{v_2 - v_1}{u_2 - u_1} \right) (u_1 - u_3) - v_1$$

If $P = Q$ and $P \neq -P$ then the sum is $2P$ and it has coordinates (u_4, v_4) with

$$u_4 = \left(\frac{3u_1^2 + \alpha}{2v_1} \right)^2 - 2u_1$$

and

$$v_4 = \left(\frac{3u_1^2 + \alpha}{2v_1} \right) (u_1 - v_4) - v_1$$

Let the E be a non-super-singular elliptic curve over a binary field $K = F_{2^n}$ with equation $v^2 + uv = u^3 + \alpha u^2 + \beta$. On the group $E(K)$, again the point at infinity $\mathcal{O}$ plays the role of identity. The inverse of the point $P(u_1, v_1)$ is now $-P = (u_1, u_1 + v_1)$. Let $Q(u_2, v_2)$ be another point on the curve with $Q \neq \pm P$. Then $P + Q$ is the point (u_3, v_3) , where

$$u_3 = \lambda^2 + \lambda + u_1 + u_2 + \alpha$$

and

$$v_3 = \lambda(u_1 + u_3) + u_3 + v_1$$

with $\lambda = \frac{v_1 + v_2}{u_1 + u_2}$.

The double of the point P is (u_4, v_4) where,

$$v_4 = \lambda^2 + \lambda + \alpha$$

and

$$v_4 = u_1^2 + \lambda u_4 + u_4$$

with $\lambda = u_1 + v_1/u_1$.

2.2. Computation of Addition and Doubling of Points

Equation of the curve in the simplified is given by

$$E : v^2 = u^3 + \alpha u + \beta \tag{2}$$

This form is known as *Weierstress equation*. Where the coefficient α denotes the arbitrary element of the field. But for efficient implementation of ECDBL, [NIST, ANSI, SEC2] recommends the specification of curves taking the coefficient $\alpha = -3$ in the above equation.

Figure 1: Addition and Doubling of points on Elliptic Curve

3. Algorithms for Scalar Multiplication

The most costly operation in ECC is Scalar Multiplication. From implementation point of view, we need to develop low cost algorithms with regards to its computational time and space. We can define scalar multiplication as $d \times P = P + \dots (d \text{ times}) \dots + P$, where P be any point on E and d is a positive integer. The computation is known as the scalar multiplication operation [12]. Two versions of Binary algorithm are defined below to compute the Scalar Multiplication. Where *ADD* and *DBL* denote two functions addition and doubling of points respectively. The function *ADD* computes $P + Q$ by taking as input two points P and Q . Similarly *DBL* returns $2P$ by taking as input P .

Algorithm 1: Left-to-right binary method DBL-AND-ADD

Input P, d ($d_{k-1}, \dots, d_1, d_0$)

Output dP

$E = d_{k-1}P$

for $j = k - 2$ **down to** 0 **do**

$E = \text{DBL}(E)$

if $d_j = 1$ **then**

$E = \text{ADD}(E, P)$

return E

end if

end for

Algorithm 2: Right-to-left binary method DBL-AND-ADD

Input P, d ($d_{k-1}, \dots, d_1, d_0$)

Output dP

$E_0 = P, E_1 = 0$

for $j = 0$ to $k - 1$ **do**

if $d_j = 1$ **then**

$E_1 = \text{ADD}(E_0, E_1)$

$E_0 = \text{DBL}(E_0)$

end if

end for

return (E_1)

3.1. Algorithms for Non-Adjacent Form(NAF) and w -NAF

Hamming weight of the multiplier in the Scalar multiplication has the important role to reduce the computation cost. It has been noted the the algorithm is more efficient if the hamming weight less [11] [19] [5]. It is observed that, there is no cost for negation of a point for curve over prime fields, it can be considered, both the addition and subtraction of points are almost same *i.e* the cost of $(P + Q)$ and $(P - Q)$ are same. Therefore the multiplier can be represented in signed binary with smaller hamming weight. One of the algorithm to represent signed binary multiplier is Non-Adjacent Form (NAF) [17] [8] [15].

Definition 1. Let binary representation of the multiplier d is $\Sigma_i d_i 2^i$. It can be represented in NAF $\iff d_i d_{i+1} = 0 \forall i$.

Definition 2. Let the width w be a positive integer. The multiplier d is represented by w -NAF as $\Sigma_{j=0}^{l-1} d_j 2^j$, where $d_j \in [-2^{w-1}, 2^{w-1} - 1]$, d are odd integers and $d_j \neq 0$.

To calculate pre-computation, it needs 1 doubling and $2^{w-2} - 1$ additions. On average the no of addition and doubling are $n/(w + 1)$ and n respectively.

4. Proposed Low Cost Scalar Multiplication Algorithm

Consider the 1's compliment of the binary number, this can be computed by the following equation

$$n = 2^d - c_1 - 1 \quad (3)$$

Algorithm 3: Non Adjacent Form Computation for multiplier

Input: d (integer)
 Output: NAF representation $\sum d_i 2^i$ of d
 $j \leftarrow 0$
while $d \geq 0$ **do**
 if d is odd **then**
 $d_j \leftarrow 2 - (d \bmod 4)$
 else $\{d_j = 0\}$
 $d \leftarrow d/2, j \leftarrow j + 1$
 end if
end while
return $d_0, d_1, \dots, d_{j-1}$

Algorithm 4: Window based NAF Computation

Input: d (integer)
 Output: Window based NAF representation $\sum d_i 2^i$ of d
 $j \leftarrow 0$
while $d \geq 0$ **do**
 if d is odd **then**
 $d_j \leftarrow 2^{w-1} - (d \bmod 2^w)$
 else $\{d_j = 0\}$
 $d \leftarrow d/2, j \leftarrow j + 1$
 end if
end while
return $d_0, d_1, \dots, d_{j-1}$

Algorithm 5: Window based NAF Scalar Multiplication

Input: An integer $d = \sum_{j=0}^{n-1} d_j 2^j$ in w -NAF
 Output: dP
 pre-compute $X_j \leftarrow iX$, i in $[-2^{w-1}, 2^{w-1} - 1]$, j is odd
 $Y \leftarrow X$
for $j = n - 1$ **to** 0 **do**
 $Y = 2Y$
 if $d \neq 0$ **then**
 if $d_j > 0$ **then**
 $Y = Y + X_{d_j}$
 else
 $Y = Y - X_{d_j}$
 end if
 end if
end for
 return Q

Algorithm 6: Scalar Multiplication for Sliding Window

$Y \leftarrow X_\infty$
while $j \geq 0$ **do**
 if $n_j = 0$ **then**
 $Y \leftarrow [2]Y$ and $j \leftarrow i - 1$
 else
 $t \leftarrow \max(j - k + 1, 0)$
 end if
end while
while $n_t = 0$ **do**
 $t \leftarrow t + 1$
end while

```

Input: Integer  $d > 0$ 
Output: The sequence  $(a_i, b_i)$ ,  $d = \sum_i 2^{a_i} 3^{b_i}$ ,  $a_i, b_i \geq 0$ 
Integer  $d > 0$ 
while  $d \neq 0$  do
    Compute  $\mu = 2^a 3^b$ 
    return  $(a, b)$ 
     $d \leftarrow d - 1$ 
end while

```

Example 1. Let $d = 2010$. Binary expansion of this number is $(11111011010)_2$. 1's compliment $c_1 = 00000100101$, $d = 11$. We can express the number by the above equation as $1642 = 2^{11} - 00000100101 - 1$. This can be written as $2010 = 10000000000 - 00000100101 - 1$.

4.1. Experiments using Hamming Weight

• **Double base chains:** Consider this algorithm to compute nP given the point P on Elliptic Curve. Let $n = 314159$. So $314159P$ is given by $2(2(2(2(2(2(2(2(2(2(2(P)) + P)) - P))) + P) + P)) - P))) + P) + P)))) - P$. Starting from the point P results a of $18DBL$ and $8ADD$ of the point P . We consider the subtraction as addition and count.

Type of Algorithm	Double base chains	Double base-2 & 3	Sliding-windows with base-2	Sliding-windows with double-base-2 & 3
Computational Cost	18DBL + 8ADD	17DBL+6ADD	17DBL+ 7ADD	16DBL+ 9ADD

Table 1: Computation Cost

The point to be consider that makes appear an intermediary point with co-ordinate of zero-value in computation of $[k]P$ have been described in [20]. From the leakage trace, the appearance of co-ordinate with zero value that allows an SPA attacker to distinguish the value of k can be traced out and computed. [21] has described this attack elaborately and exploited occurrence of zero-value co-ordinate inside the computations of point addition. This type of attack can be controlled to thwart to randomize the scalar and include dummy operation inside the algorithm [22] [23]. Courrège et al. in [27] introduced recently another type of zero-value SPA attacks. In this type of attack, modular exponentiation is targeted . It transposes directly to an attack against specific scalar multiplication algorithms. In fact, SPA adversary can distinguish, whenever multiplication is performed by zero or a digit with low hamming weight value by hardware multiplier of the target device. Generally a hardware multiplier computes the multiplication over the $\mathbb{F}_q$. The multiplication comprises $(n \times m)$ -bits multiplications. The adversary can get the ideas if the operand of some multiplication over $\mathbb{F}_q$ consisting of n -bit (resp. m -bit) digit to zero or digit with low value hamming weight. This exploits the secret information.

6. Conclusion

In this paper, we have proposed the technique to decrease the computation cost of scalar multiplication by considering the hamming weight of the multiplier in Double base chains, Double base-2 and -3, Sliding-windows base-2 and Sliding-windows double-base-2 and-3. It is low cost and resistant on SCA and zero-value attacks. This can be implemented on low processor constrained devices such as smart phone, PDA, wireless sensor, smart card etc [9] [24].

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