SOME FUNCTIONS VIA $\Lambda_b(V_b)$ -SETS

G. Shanmugam^{1 §}, N. Karthikeyan², N. Rajesh³

^{1,2}Department of Mathematics

Jeppiaar Engineering College

Chennai 600119, Tamilnadu, INDIA

³Department of Mathematics

Rajah Serfoji Govt. College

Thanjavur, 613005, Tamilnadu, INDIA

Abstract: In this paper, we introduce and study some new types of functions by the use of Λ_b -sets and V_b -sets.

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1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. The concept of b -open set in a topological space was introduced by Andrijevic in 1996 [1]. But one year later, this notion was also called γ -open sets due to El-Atik [7]. A subset A of a topological space (X, τ) is said to be b -open ($=\gamma$ -open [7]) if $A \subset \text{Int}(Cl(A)) \cup Cl(\text{Int}(A))$, where $Cl(A)$ denotes the closure of A and $\text{Int}(A)$ denotes the interior of A in (X, τ) . The complement A^c of a b -open

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[§]Correspondence author

set A is called b -closed [1] ($=\gamma$ -closed [7]). The family of all b -open (resp. b -closed) sets in (X, τ) is denoted by $BO(X, \tau)$ (resp. $BC(X, \tau)$). The intersection of all b -closed sets containing A is called the b -closure of A [1] and is denoted by $bCl(A)$. Quite recently Caldas et. al. [3] used b -open sets to define and investigate the Λ_b -sets (resp. V_b -sets) which are intersection of b -open (resp. union of b -closed) sets. The purpose of the present paper is introduced and studied some new type of functions by using Λ_b -sets and V_b -sets. The family of all b -open sets of (X, τ) containing a point $x \in X$ is denoted by $BO(X, x)$.

2. Preliminaries

Throughout this paper (X, τ) , (Y, σ) and (Z, ν) (or simply X , Y and Z) will always denote topological spaces on which no separation axioms are assumed, unless explicitly stated.

Definition 2.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be b -continuous [7] ($=\gamma$ -continuous [7]) (resp. b -irresolute [7] ($=\gamma$ -irresolute [7])) if for every $A \in \sigma$ (resp. $A \in BO(Y, \sigma)$) $f^{-1}(A) \in BO(X, \tau)$, or equivalently, f is b -continuous (resp. b -irresolute) if and only if for every closed set A (resp. b -closed set A) of (Y, σ) , $f^{-1}(A) \in BC(X, \tau)$.

Definition 2.2. Let B be a subset of a topological space (X, τ) . B is a Λ_b -set (resp. V_b -set) [3], if $B = B^{\Lambda_b}$ (resp. $B = B^{V_b}$), where: $B^{\Lambda_b} = \cap \{O : O \supseteq B, O \in BO(X, \tau)\}$ and $B^{V_b} = \cup \{F : F \subset B; F^c \in BO(X, \tau)\}$. The family of all Λ_b -sets (resp. V_b) of (X, τ) is denoted by τ^{Λ_b} (resp. τ^{V_b}).

Proposition 2.3. For a space (X, τ) , the following statements hold:

- (i) $\emptyset$ and X are Λ_b -sets and V_b -sets.
- (ii) Every union of Λ_b -sets (resp. V_b -sets) is a Λ_b -set (resp. V_b -set).
- (iii) Every intersection of Λ_b -sets (resp. V_b -sets) is a Λ_b -set (resp. V_b set).

Proposition 2.4. Let A, B for some $\{B : \alpha \in \Omega\}$ be subsets of a topological space (X, τ) . Then the following properties are valid [3]:

- (a) $B \subset B^{\Lambda_b}$;
- (b) If $A \subset B$, then $A^{\Lambda_b} \subset B^{\Lambda_b}$;
- (c) $B^{\Lambda_b \Lambda_b} = B^{\Lambda_b}$;
- (d) $(\bigcup_{\alpha \in \Omega} B)^{\Lambda_b} = \bigcup_{\alpha \in \Omega} B^{\Lambda_b}$;
- (e) If $A \in BO(X, \tau)$, then $A = A^{\Lambda_b}$ (i.e, A is an Λ_b -set);
- (f) $(B^c)^{\Lambda_b} = (B^{V_b})^c$;
- (g) $B^{V_b} \subset B$;

- (h) If $B \in BC(X, \tau)$, then $B = B^{V_b}$ (i.e, A is an V_b -set);
 (i) $(\bigcup_{\alpha \in \Omega} B)^{\Lambda_b} \subset \bigcup_{\alpha \in \Omega} B^{\Lambda_b}$;
 (j) $(\bigcup_{\alpha \in \Omega} B)^{V_b} \supseteq \bigcup_{\alpha \in \Omega} B^{V_b}$.

Definition 2.5. A topological space X is said to be:

- (a) R_0 -space [5] if for each open set V of X and each $x \in V$, $Cl(\{x\}) \subset V$
 (b) $b-R_0$ [6] if every b -open set contains the b -closure of each of its singletons.
 (c) $b-T_1$ [3] if to each pair of distinct points x, y of X there corresponds a b -open set A containing x but not y a b -open set B containing y but not x , or equivalently, (X, τ) is a $b-T_1$ -space if and only if every singleton is b -closed.

3. Λ_b -Continuity and Λ_b -Irresoluteness

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

- (i) Λ_b -continuous if $f^{-1}(V)$ is a Λ_b -set in (X, τ) for every open set V of (Y, σ) ,
 (ii) Λ_b -irresolute if $f^{-1}(V)$ is a Λ_b -set in (X, τ) for every Λ_b -set V of (Y, σ) ,
 (iii) pre- Λ_b -open (resp. pre- b -open) if $f(A)$ is a Λ_b -set (resp. b -open set) of (Y, σ) for every Λ_b -set (resp. b -open set) A of (X, τ) .

Proposition 3.2. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties are equivalent:

- (i) f is Λ_b -continuous,
 (ii) $f : (X, \tau^{\Lambda_b}) \rightarrow (Y, \sigma)$ is continuous,
 (iii) $f : (X, \tau^{V_b}) \rightarrow (Y, \sigma)$ is continuous.

Proof. Clear. □

Definition 3.3. (i): A filterbase Λ is said to be Λ_b -convergent to a point x in X if for any $U \in S^{\Lambda_b}$ containing x , there exists $B \in \Lambda$ such that $B \subset U$.
 (ii): A filterbase Λ is said to be convergent to a point x in X if for any open set U of X containing x , there exists $B \in \Lambda$ such that $B \subset U$.

Theorem 3.4. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is Λ_b -continuous, then for each point $x \in X$ and each filterbase $\mathcal{F}$ in X Λ_b -converging to x , the filter base $f(\mathcal{F})$ is convergent to $f(x)$.

Proof. Let $x \in X$ and $\mathcal{F}$ be any filterbase in X Λ_b -converging to x . Since f is Λ_b -continuous, then for any open set V of (Y, σ) containing $f(x)$, there exists

$U \in S^{\Lambda_b}$ containing x such that $f(U) \subset V$. Since Λ_b is Λ_b -converging to x , there exists $B \in \mathcal{F}$ such that $B \subset U$. This means that $f(B) \subset V$ and hence the filter base $f(\mathcal{F})$ is convergent to $f(x)$. $\square$

Definition 3.5. . A sequence (x_n) is said to be Λ_b -convergent to a point x if for every Λ_b set V containing x , there exists an index x_0 such that for $n \geq n_0$, $x_n \in V$.

Theorem 3.6. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is Λ_b -continuous, then for each point $x \in X$ and each net (x_n) which is Λ_b -convergent to x , the net $f(x_n)$ is convergent to $f(x)$.

Proof. The proof is similar to that of Theorem 4.15.

Recall that for a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$. $\square$

Definition 3.7. A graph $G(f)$ of a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strong Λ_b -set if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in S^{\Lambda_b}$ containing x and a closed set V of Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 3.8. A graph $G(f)$ of a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is strong Λ_b -set in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in S^{\Lambda_b}$ containing x and a closed set V of Y containing y such that $f(U) \cap V = \emptyset$.

Theorem 3.9. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Λ_b -continuous function and (Y, σ) is a T_1 -space, then $G(f)$ is strong Λ_b -set.

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since Y is T_1 there exists an open set V in Y such that $f(x) \in V$ and $y \notin V$. Since f is Λ_b -continuous, there exist $U \in S^{\Lambda_b}$ containing x such that $f(U) \subset V$. Therefore, $f(U) \cap (Y \setminus V) = \emptyset$ and $Y \setminus V$ is closed subset of Y containing y . This shows that $G(f)$ is strong Λ_b -set. $\square$

Definition 3.10. A topological space X is said to be Λ_b -connected if there does not exist disjoint Λ_b -set A and B such that $A \cup B = X$.

Theorem 3.11. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Λ_b -continuous surjective function and X is Λ_b -connected, then Y is connected.

Proof. Follows from the definitions. $\square$

Definition 3.12. A collection $\{G_\alpha : \alpha \in \Delta\}$ is said to be Λ_b -cover of a subset A of a topological space (X, τ) if $A \subset \cup\{G_\alpha : X \setminus G_\alpha \in S^{\Lambda_b}, \alpha \in \Delta\}$.

Definition 3.13. A topological space X is said to be

- (i) Λ_b -compact if every Λ_b -open cover of X has a finite subcover;
- (ii) countably Λ_b -compact if every Λ_b -open countable cover of X has a finite subcover;
- (iii) Λ_b -Lindelöf if every cover of X by Λ_b -open set has a countable subcover.

Theorem 3.14. *if $f : (X, \tau) \rightarrow (Y, \sigma)$ is Λ_b -continuous surjection and (X, τ) is Λ_b -compact (resp. countably Λ_b -compact, Λ_b -Lindelöf), then Y is compact (resp. countably compact, Lindelöf).*

Proof. Follows from the definitions. □

Theorem 3.15. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Λ_b -continuous injective function and (Y, σ) is a T_2 -space, then (X, τ) is Λ_b - T_2 -space.*

Proof. For any pair of distinct points x any y in X , there exist distinct open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is Λ_b -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are Λ_b -sets in X containing x any y , respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ because $U \cap V = \emptyset$. This show that (X, τ) is Λ_b - T_2 □

Proposition 3.16. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$ the following properties are equivalent:*

- (i) f is Λ_b -irresolute,
- (ii) $f : (X, \tau^{\Lambda_b}) \rightarrow (Y, \sigma^{\Lambda_b})$, is continuous,
- (iii) $f : (X, \tau^{V_b}) \rightarrow (Y, \sigma^{V_b})$, is continuous.

Proof. (i) $\implies$ (ii) This is obvious. (ii) $\implies$ (iii): Let B be any V_b -sets of (Y, σ) . Then B^c is a Λ_b -sets of (Y, σ) and $f^{-1}(B^c)$ is a Λ_b -sets of (X, τ) (iii) $\implies$ (i): Let B be any Λ_b -sets of (Y, σ) then B^c is a V_b -sets. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is a V_b -set. Thus $f^{-1}(B)$ is Λ_b -sets. □

Theorem 3.17. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties hold.*

- (i) *If f is b -irresolute, then it is Λ_b -irresolute;*
- (ii) *If f is b -open and injective, then it is pre- Λ_b -open*

Proof. (i) Let B be a Λ_b -sets of (Y, σ) . Since f is b -irresolute, we have $f^{-1}(B) \subset (f^{-1}(B))^{\Lambda_b} = \cap \{U | f^{-1}(B) \subset U \in BO(X, \tau)\} \subset \cap \{f^{-1}(V) | B \subset V \in BO(Y, \sigma)\} = f^{-1}(\cap \{V | B \subset V \in BO(Y, \sigma)\}) = (f^{-1}(B))^{\Lambda_b} = f^{-1}(B)$. Therefore, we obtain $f^{-1}(B) = (f^{-1}(B))^{\Lambda_b}$ which show that $f^{-1}(B)$ is a Λ_b -set. Consequently, f is Λ_b -irresolute.

(ii) Let A be a Λ_b -set of (X, τ) . Since f is pre- b -open and injective. We have

$f(A) = f(\cap\{U|A \subset U \in BO(X, \tau)\}) = \cap\{f(\{U|A \subset U \in BO(X, \tau)\})\} = \cap\{f(U)|A \subset U \in BO(X, \tau)\} \supset \cap\{V|f(A) \subset V \in BO(Y, \sigma)\} = (f(A))^{\Lambda_b} \supset f(A)$. Therefore we obtain $f(A) = (f(A))^{\Lambda_b}$. Which show that $f(A)$ is a Λ_b -set. Consequently, f is pre- Λ_b -open. $\square$

Corollary 3.18. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, b -irresolute and pre- b -closed, then*

- (i) *for every V_b -set B of (Y, σ) , then $f^{-1}(B)$ is a V_b -set of (X, τ)*
- (ii) *for every V_b -set B of (X, τ) , then $f(B)$ is a V_b -set of (Y, σ)*

Proposition 3.19. (i) *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Λ_b -irresolute function and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is a Λ_b -continuous function, then the composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is Λ_b -continuous.*

(ii) *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ are both Λ_b -irresolute, then the composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is Λ_b -irresolute*

Proof. It follows directly from the definitions. $\square$

Definition 3.20. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be V_b -closed if for each closed set F of X , $f(F)$ is a V_b -set of (Y, σ)

Theorem 3.21. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is V_b -closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$, there is a Λ_b -set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.*

Proof. . Let S be a subset of Y and U be an open subset of X such that $f^{-1}(S) \subset U$. Then, $Y \setminus f(X \setminus U) = V$ (say), is a Λ_b -set containing S such that $f^{-1}(V) \subset U$. Conversely, let F be an arbitrary closed set of X . Then $f^{-1}(Y \setminus f(F)) \subset X \setminus F$ and $X \setminus F$ is open in X . By hypothesis, there is a Λ_b -set V of Y such that $(Y \setminus f(F)) \subset V$ and $f^{-1}(V) \subset X \setminus F$ hence $Y \setminus V \subset f(F) \subset f(X \setminus f^{-1}(V)) \subset Y \setminus V$, which implies $f(F) = Y \setminus V$. Since $Y \setminus V$ is a V_b -set, $f(F)$ is a V_b -set; hence f is a V_b -closed function. $\square$

We consider now some composition properties in terms of V_b -sets.

Theorem 3.22. . *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two functions such that $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is V_b -closed. Then, (i) if f is continuous and surjective, then g is V_b -closed, (ii) if g is b -irresolute, pre- b -closed and bijective, then f is V_b -closed.*

Proof. (i) Let F be an arbitrary closed set in (X, τ) . Then $f(F)$ is a V_b -set in (Y, σ) . Since g is bijective, b -irresolute and pre- b -closed, $(g \circ f)(F) = g(f(F))$ is a

V_b -set by Corollary 3.21(ii). (ii) The proof follows immediately from definitions. Regarding the restriction $f|_A$ of a function $f : (X, \tau) \rightarrow (Y, \sigma)$ to a subset A of X , we have the following: $\square$

Theorem 3.23. (i) If $f : (X, \tau) \rightarrow (Y, \sigma)$ is V_b -closed and A is a closed set in (X, τ) , then its restriction $f|_A : (A, \tau|_A) \rightarrow (Y, \sigma)$ is V_b -closed.
(ii) Let B be a V_b -set of (Y, σ) . If $f : (X, \tau) \rightarrow (Y, \sigma)$ is V_b -closed, then $f|_A : (A, \tau|_A) \rightarrow (Y, \sigma)$ is V_b -closed, where $A = f^{-1}(B)$.

Proof. (i) Let F be a closed set of $(A, \tau|_A)$. Since A is closed in (X, τ) , F is closed in (X, τ) and $f_A(F) = f(F)$ is a V_b -closed set of (Y, σ) . Therefore, $f|_A$ is V_b -closed. (ii) Let F be a closed set of A . Then $F = A \cap H$ for some closed set H of (X, τ) . By Proposition 2.3, we have $f(H) \cap B$ is a V_b -set in (Y, σ) since B is a V_b -set. Using $f|_A(F) = f(A \cap H) = f(H) \cap B$, $f|_A$ is V_b -closed. $\square$

Definition 3.24. A topological space (X, τ) is said to be T_b -space if $\tau^{\Lambda_b} = \tau^{V_b}$.

Theorem 3.25. For a topological space (X, τ) , the following properties are equivalent:

- (i) (X, τ) is a b - R_o space;
- (ii) (X, τ^{V_b}) is discrete;
- (iii) (X, τ^{Λ_b}) is discrete;
- (iv) For each $x \in X$, $\{x\}$ is a Λ_b -set of (X, τ) ;
- (v) $P = (P)_b^V$ for each $P \in BO(X, \tau)$;
- (vi) (X, τ) is a T_b -space;
- (vii) (X, τ^{Λ_b}) is a R_o space;

Proof. (i) $\Rightarrow$ (ii): It is shown in Theorem 3.11 of [4] that (X, τ) is b - R_0 if and only if it is b - T_1 . In [3], it is shown that if (X, τ) is b - T_1 , then (X, τ_b^Λ) is discrete. (ii) $\Rightarrow$ (iii): This is obvious. (iii) $\Rightarrow$ (iv): For each $x \in X$, $\{x\}$ is τ^{Λ_b} -open and $\{x\}$ is a Λ_b -set of (X, τ) . (iv) $\Rightarrow$ (v): Let P be a b -open set of X . Let $y \in P^c$ then $(\{y\})_b^\Lambda \supset P^c$ by the assumption. By using Proposition 2.4, we have $P^c \supset \cup \{\Lambda_b(\{y\}) : y \in P^c\} = (P^c)_b^\Lambda$ and hence $P^c = (P^c)_b^\Lambda$. Then it follows from Proposition 4 that $P = (P^c)_b^V$ (v) $\Rightarrow$ (vi): By (v), we have $BO(X, \tau) \subset \tau^{\Lambda_b}$. First we show that $\tau^{\Lambda_b} \subset \tau^{V_b}$. Let A be any Λ_b of (X, τ) . Then $A = \cap \{V \setminus A \subset V \in BO(X, \tau)\}$. Since $BO(X, \tau) \subset \tau^{V_b}$, By Proposition 4 we have $A \in \tau^{V_b}$ and $\tau^{\Lambda_b} \subset \tau^{V_b}$. Next let $A \in \tau^{V_b}$. Then $X \setminus A \in \tau^{\Lambda_b}$. Therefore $A \in \tau^{\Lambda_b}$ and $\tau^{V_b} \subset \tau^{\Lambda_b}$. Consequently, we obtain $\tau^{V_b} = \tau^{\Lambda_b}$ and (X, τ) is a T_b -space. (vi) $\Rightarrow$ (vii) Suppose that $V \in \tau^{\Lambda_b}$ and $x \in V$, Since (X, τ) is a T_b -space, $V \in \tau^{V_b}$ and $V^c \in \tau^{\Lambda_b}$. Since $\{x\} \cap V^c = \emptyset$. $\text{Cl}_{\tau^{\Lambda_b}}(\{x\}) \cap V^c = \emptyset$

and $\text{Cl}_{\tau^{\Lambda_b}}(\{x\}) \subset V$ where $\text{Cl}_{\tau^{\Lambda_b}}(\{x\})$ denotes the closure of $\{x\}$ in (X, τ^{Λ_b}) .
 (vii) $\Rightarrow$ (i): Let $V \in \text{BO}(X, \tau)$ and $x \in V$. Since $\text{BO}(X, \tau) \subset \tau^{\Lambda_b}$, by (vii), $\text{Cl}_{\tau^{\Lambda_b}}(\{x\}) \subset V$. Since $\text{Cl}_{\tau^{\Lambda_b}}(\{x\}) \subset \tau^{V_b}$, we have $\text{Cl}_{\tau^{\Lambda_b}}(\{x\}) = \cup\{F : F \in \text{BC}(X, \tau), F \subset \text{Cl}_{\tau^{\Lambda_b}}(\{x\})\}$ and $x \in \text{Cl}_{\tau^{\Lambda_b}}(\{x\})$. There exist $F \in \text{BC}(X, \tau)$ such that $x \in X$ and hence we have $b\text{Cl}(\{x\}) \subset F \subset \text{Cl}_{\tau^{\Lambda_b}}(\{x\}) \subset V$. This shows that (X, τ) is a $b\text{-}R_0$ -space. $\square$

Corollary 3.26. *If (X, τ) is a $b\text{-}R_0$ -space, then (X, τ^{Λ_b}) is a R_0 -space.*

Definition 3.27. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a Λ_b -homeomorphism if it is Λ_b -irresolute, pre- Λ_b -open and bijective.

Theorem 3.28. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties hold:*

- (i) *If f is Λ_b -irresolute injection and (Y, σ) is a T_b -space, then (X, τ) is a T_b -space.*
- (ii) *If f is pre- Λ_b -open surjection and (X, τ) is a T_b -space, then (Y, σ) is a T_b -space.*
- (iii) *Let f be Λ_b -homeomorphism. Then (X, τ) is a T_b -space, if and only if (Y, σ) is a T_b -space.*

Proof. (i) This follows from Theorem 4.17. (ii) This is analogous to the proof of (i). (iii) This is an immediate consequence of (i) and (ii) $\square$

4. Weakly Pre- Λ_b -Open Functions

Definition 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly pre- Λ_b -open if the image of every Λ_b -set in X is open in Y .

Clearly, every pre- Λ_b -open function is weakly pre- Λ_b -open. But the converse is not true in general.

Example 4.2. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, X\}$. Then the identity function f on X is weakly pre- Λ_b -open but not pre- Λ_b -open.

Remark 4.3. It is evident that, the concepts weakly pre- Λ_b -openness and Λ_b -continuity are coincide if the function is a bijective

Theorem 4.4. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly pre- Λ_b -open if and only if for every subset U of X , $f(\text{Int}_{\tau^{\Lambda_b}}(U)) \subset \text{Int}(f(U))$.*

Proof. Let f be weakly pre- Λ_b -open map. Now we have $\text{Int } f(U) \subset U$ and $\text{Int}_{\tau\Lambda_b}(U)$ is a Λ_b -set. Hence we obtain that $f(\text{Int}_{\tau\Lambda_b}(U)) \subset f(U)$. As $f(\text{Int}_{\tau\Lambda_b}(U))$ is open, then $f(\text{Int}_{\tau\Lambda_b}(U)) \subset \text{Int}(f(U))$. Conversely, assume that U be a Λ_b -set in X . Then $f(U) = f(\text{Int}_{\tau\Lambda_b}(U)) \subset \text{Int}(f(U))$. But usually $\text{Int}(f(U)) \subset f(U)$. Consequently $f(U) = \text{Int } f(U)$ and hence f is weakly pre- Λ_b -open. $\square$

Lemma 4.5. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly pre- Λ_b -open then $\text{Int}_{\tau\Lambda_b}(f^{-1}(G)) \subset f^{-1}(\text{Int}(G))$ for every subset G of Y*

Proof. Let G be any arbitrary subset of Y . Then $\text{Int}_{\tau\Lambda_b}(f^{-1}(G))$ is a Λ_b -set in X and f is weakly pre- Λ_b -open, then $f(\text{Int}_{\tau\Lambda_b}(f^{-1}(G))) \subset \text{Int}(f(f^{-1}(G))) \subset \text{Int}(G)$. Thus $\text{Int}_{\tau\Lambda_b}(f^{-1}(G)) \subset f^{-1}(\text{Int}(G))$. $\square$

Definition 4.6. A subset S is called a Λ_b -neighbourhood of a point x of X if there exist a Λ_b -set U such that $x \in U \subset S$.

Theorem 4.7. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:*

- (i) f is weakly pre- Λ_b -open ;
- (ii) For each subset U of X , $f(\text{Int}_{\tau\Lambda_b}(U)) \subset \text{Int}(f(U))$;
- (iii) For each $x \in X$ and each Λ_b -neighbourhood U of x in X , there exists a neighbourhood V of $f(x)$ in Y such that $V \subset f(U)$.

Proof. (i) $\Rightarrow$ (ii) : It follows from Theorem 4.4. (ii) $\Rightarrow$ (iii): Let $x \in X$ and U be an arbitrary Λ_b -neighbourhood of x in X . Then there exists a Λ_b -set V in X such that $x \in V \subset U$. Then by (ii), we have $f(V) = f(\text{Int}_{\tau\Lambda_b}(V)) \subset \text{Int}(f(U))$ and hence $f(V) = \text{Int}(f(V))$. Therefore, it follows that $f(V)$ is open in Y such that $f(x) \in f(V) \subset f(U)$. (iii) $\Rightarrow$ (i): Let U be an arbitrary Λ_b -set in X , Then for each $y \in f(U)$, by (iii) there exists a neighbourhood V_y of y in Y such that $V_y \subset f(U)$. As V_y is a neighbourhood of y , there exists an open set W_y in Y such that $y \in W_y \subset V_y$. Thus, $f(U) = \cup\{W_y : y \in f(U)\}$ which is an open set in Y . This implies that f is weakly pre- Λ_b -open function. $\square$

Theorem 4.8. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly pre- Λ_b -open if and only if for any subset B of Y and for any V_b -set F of X containing $f^{-1}(B)$, there exists a closed set G of Y containing B such that $f^{-1}(G) \subset F$.*

Proof. Similar to the proof of Theorem 3.24. $\square$

Theorem 4.9. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly pre- Λ_b -open if and only if $f(\text{Cl}(B)) \subset \text{Cl}_{\tau\Lambda_b}(f(B))$ for every subset B of Y .*

Proof. Suppose that f is weakly pre- Λ_b -open. For any subset B of Y , $f^{-1}(B) \subset \text{Cl}_{\tau\Lambda_b}(f^{-1}(B))$. Therefore by Theorem 4.8, there exists a closed set F in Y such that $B \subset F$ and $f^{-1}(F) \subset \text{Cl}_{\tau\Lambda_b}(f^{-1}(B))$. Therefore, we obtain $f^{-1}(\text{Cl}(B)) \subset f^{-1}(F) \subset \text{Cl}_{\tau\Lambda_b}(f^{-1}(B))$. Conversely, let $B \subset Y$ and F be a Λ_b -set of X containing $f^{-1}(B)$. Put $W = \text{Cl}(B)$, then we have $B \subset W$ and W is closed and $f^{-1}(W) \subset \text{Cl}_{\tau\Lambda_b}(f^{-1}(B)) \subset F$. Then by Theorem 44, f is weakly pre- Λ_b -open. $\square$

Lemma 4.10. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two functions and $g \circ f : X \rightarrow Z$ is weakly pre- Λ_b -open. If g is continuous injective, then f is weakly pre- Λ_b -open.*

Proof. Let U be a Λ_b -set in X , then $(g \circ f)(U)$ is open in Z , since $g \circ f$ is weakly pre- Λ_b -open. Again g is an injective continuous function, $f(U) = g^{-1}(g \circ f(U))$ is open in Y . This show that f is weakly pre- Λ_b -open. $\square$

Theorem 4.11. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are two weakly pre- V_b -closed functions, then $g \circ f : X \rightarrow Z$ is a weakly pre- V_b -closed function.*

Proof. Obvious. $\square$

Furthermore, we have the following.

Theorem 4.12. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then*

- (i). *If f is V_b -closed and g is weakly pre- V_b -closed, then $g \circ f$ is closed;*
- (ii) *If f is weakly pre- V_b -closed and g is V_b -closed, then $g \circ f$ is pre- V_b -closed;*
- (iii) *If f is pre V_b -closed and g is weakly pre- V_b -closed, then $g \circ f$ is weakly pre- V_b -closed;*

Proof. Obvious. $\square$

Theorem 4.13. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions such that $g \circ f : X \rightarrow Z$ is weakly pre- Λ_b -closed.*

- (i) *If f is V_b -irresolute surjective, then g is closed.*
- (ii) *If g is V_b -continuous injective, then f is pre- V_b -closed.*

Proof. (i) Suppose F is an arbitrary V_b -closed set in Y . As f is V_b -irresolute, $f^{-1}(F)$ is V_b -set in X . Since $g \circ f$ is weakly pre- V_b -closed and f is surjective, $(g \circ f(f^{-1}(F))) = g(F)$, which is closed in Z . This implies that g is a closed function. (ii) Suppose F is any V_b -closed set in X . Since $g \circ f$

is weakly pre- V_b -closed, $(g \circ f)(F)$ is closed in Z . Again g is a V_b -continuous injective function, $g^{-1}(g \circ f(F)) = f(F)$, which is V_b -closed in Y . This shows that f is pre- V_b -closed. $\square$

Theorem 4.14. *Let (X, τ) and (Y, σ) be topological spaces. Then the function $g : (X, \tau) \rightarrow (Y, \sigma)$ is a weakly pre- V_b -closed if and only if $g(X)$ is closed in Y and $g(V) - g(X - V)$ is open in $g(X)$ whenever V is Λ_b -set in X .*

Proof. Necessity: Suppose $g : (X, \tau) \rightarrow (Y, \sigma)$ is a weakly pre- V_b -closed function. Since X is Λ_b -set, $g(X)$ is closed in Y and $g(V) - g(X - V) = g(X) - g(X - V)$ is open in $g(X)$ when V is Λ_b -set in X . Sufficiency: Suppose $g(X)$ is closed in Y , $g(V) - g(X - V)$ is open in $g(X)$ when V is Λ_b -set in X , and let C be closed in X . Then $g(C) = g(X) - (g(X - C) - g(C))$ is closed in $g(X)$ and hence, closed in Y . $\square$

Corollary 4.15. *Let (X, τ) and (Y, σ) be topological spaces. Then a surjective $g : (X, \tau) \rightarrow (Y, \sigma)$ is a weakly pre- Λ_b -closed if and only if $g(V) - g(X - V)$ is open in Y whenever U is Λ_b -set in X .*

Corollary 4.16. *Let (X, τ) and (Y, σ) be topological spaces and let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a Λ_b -continuous weakly pre- Λ_b -closed surjective function. Then the topology on Y is $\{g(V) - g(X - V) : V \text{ is } V_b\text{-set in } X\}$.*

Proof. Let W be open in Y . Then $g^{-1}(W)$ is Λ_b -set in X , and $g(g^{-1}(W)) - g(X - g^{-1}(W)) = W$. Hence, all open sets in Y are of the form $g(V) - g(X - V)$, V is Λ_b -set in X . On the other hand, all sets of the form $g(V) - g(X - V)$, V is Λ_b -set in X , are open in Y from Corollary 4.21. $\square$

Definition 4.17. A topological space $(X; \tau)$ is said to be V_b -normal if for any pair of disjoint V_b -sets $F1$ and $F2$ of X , there exist disjoint open sets U and V such that $F1 \subset U$ and $F2 \subset V$.

Theorem 4.18. *Let (X, τ) and (Y, σ) be topological spaces with X is V_b normal and let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a V_b -continuous weakly pre- V_b -closed surjective function. Then Y is normal.*

Proof. Let K and M be disjoint closed subsets of Y . Then $g^{-1}(K)$, $g^{-1}(M)$ are disjoint V_b -sets of X . Since X is V_b -normal, there exist disjoint open sets V and W such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Then $K \subset g(V) - g(X - V)$ and $M \subset g(W) - g(X - W)$. Further by Corollary 4.21, $g(V) - g(X - V)$ and $g(W) - g(X - W)$ are open sets in Y and clearly $(g(V) - g(X - V)) \cap (g(W) - g(X - W)) = \emptyset$. This shows that Y is normal. $\square$

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