**A COMMON FIXED POINT THEOREM FOR FOUR SELF
MAPS IN CONE RECTANGULAR METRIC SPACE
UNDER KANNAN TYPE CONTRACTIONS**

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Abstract: The purpose of this paper is to establish a common fixed point theorem for four self mappings which satisfy Kannan type contractive conditions in cone rectangular metric space.

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1. Introduction and Preliminaries

Recently, Huang and Zhang [5] introduced the notion of cone metric spaces. They have replaced real number system by an ordered Banach space and established some fixed point theorems for contractive type mappings in a normal cone metric space. The study of fixed point theorems in such spaces is followed by some other mathematicians, see [1], [4], [9], [11], [14]. In 2009, Azam, Ar-

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shad and Beg [2] introduced the notion of cone rectangular metric spaces. They have replaced triangular inequality by a rectangular inequality and proved fixed point theorem for Banach contraction in cone rectangular metric space. Several authors proved some fixed point theorems in such spaces, see [5], [8], [10], [12]. In 2009, Jleli, Samet [5] extended the Kannan's fixed point theorem in a complete normal cone rectangular metric space. In 2012, R. A. Rashwan and S. M. Saleh [12] have proved common fixed point theorems for two self mappings in cone rectangular metric space. In this paper, we prove a common fixed point theorem for four self mappings which satisfy Kannan type contractive conditions in cone rectangular metric space. Our result generalizes and extends the result of Jleli et al. [5] for four self mappings in a cone rectangular metric space.

First, we recall some standard definitions and other results that will be needed in the sequel.

Definition 1.1. (see [4]) A subset P of a real Banach space E is called a *cone* if it has following properties:

- (1) P is nonempty, closed and $P \neq \{\theta\}$;
- (2) $a, b \in R, a, b \geq 0, x, y \in P$ implies $ax + by \in P$;
- (3) $P \cap (-P) = \{\theta\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int}(P)$, where $\text{int}(P)$ denotes the interior of P .

Definition 1.2. (see [4]) The cone P is called *normal* if there is a number $k > 1$ such that for all $x, y \in E, 0 \leq x \leq y$ implies $\|x\| \leq k \|y\|$.

The least positive number k satisfying the above condition is called the *normal constant* of P .

Proposition 1.3. (see [14]) Let (X, d) be a cone metric space with cone P is not necessary to be normal. Then for $a, c, u, v, w \in E$, we have

- (1) If $a \leq \lambda a$ and $\lambda \in [0, 1)$, then $a = \theta$.
- (2) If $\theta \leq u \ll c$, for each $\theta \ll c$, then $u = \theta$.
- (3) If $u \leq v$ and $v \ll w$, then $u \ll w$.

Definition 1.4. (see [4]) Let X is a non empty set, E is a real Banach space and P is a cone in E with $\text{int } P \neq \Phi$ and $\leq$ is a partial ordering with respect to P . Suppose the mapping $d : X \times X \rightarrow E$ satisfies:

- (1) $\theta \leq d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
- (2) $d(x, y) = d(y, x)$, for all $x, y \in X$;
- (3) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a *cone metric* on X and (X, d) is called a *cone metric space*.

Definition 1.5. (see [2]) Let X is a non empty set, E is a real Banach space and P is a cone in E with $\text{int } P \neq \Phi$ and $\leq$ is a partial ordering with respect to P . Suppose the mapping $d : X \times X \rightarrow E$ satisfies:

- (1) $\theta \leq d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
- (2) $d(x, y) = d(y, x)$, for all $x, y \in X$;
- (3) $d(x, y) \leq d(x, w) + d(w, z) + d(z, y)$, for all $x, y \in X$ and for all distinct points $w, z \in X - \{x, y\}$ [rectangular inequality].

Then d is called a *cone rectangular metric* on X and (X, d) is called a *cone rectangular metric space*.

Definition 1.6. (see [2]) Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d) and $x \in X$. If for every $c \in E$, with $\theta \ll c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit of $\{x_n\}$. We denote this by $\lim_{n \rightarrow \infty} x_n = x$ or $x_n \rightarrow x$, as $n \rightarrow \infty$.

Definition 1.7. (see [2]) Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d) . If for every $c \in E$, with $\theta \ll c$ there is $n_0 \in N$ such that for all $m, n > n_0$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in (X, d) .

Definition 1.8. (see [2]) Let (X, d) be a cone rectangular metric space. If every Cauchy sequence is convergent in (X, d) , then (X, d) is called a complete cone rectangular metric space.

Lemma 1.9. (see [2]) Let (X, d) be a cone rectangular metric space and P be a normal cone with a normal constant k . Let $\{x_n\}$ be a sequence in X . Then $\{x_n\}$ converges to x if and only if $\|d(x_n, x)\| \rightarrow 0$, as $n \rightarrow \infty$.

Lemma 1.10 (see [2]) *Let (X, d) be a cone rectangular metric space, P be a normal cone with a normal constant k . Let $\{x_n\}$ be a sequence in X . Then $\{x_n\}$ is a Cauchy sequence if and only if $\|d(x_n, x_m)\| \rightarrow 0$, as $n, m \rightarrow \infty$.*

Definition 1.11. (see [7]) A self map $T: X \rightarrow X$ on a metric space (X, d) is called *Kannan contraction* if for all $x, y \in X$, there exists $\lambda \in [0, 1/2)$ such that

$$d(Tx, Ty) \leq \lambda[d(x, Tx) + d(y, Ty)].$$

Definition 1.12. (see [12]) Let T and S are self maps of a nonempty set X . If $w = Tx = Sx$, for some $x \in X$, then x is called a *coincidence point* of T and S and w is called a *point of coincidence* of T and S .

Definition 1.13. (see [14]) Two self mappings T and S are said to be *weakly compatible* if they commute at their coincidence points, that is, $Tx = Sx$ implies that $TSx = STx$.

2. Main Results

In this section we establish a common fixed point theorem for four self mappings in cone rectangular metric space under Kannan type contractive conditions.

Theorem 2.1. *Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant k and let the mappings S, T, U and $V: X \rightarrow X$ satisfy the following:*

- (1) $d(Sx, Ty) \leq \lambda[d(Sx, Ux) + d(Ty, Vy)],$
- (2) $d(Sx, Sy) \leq \lambda[d(Sx, Ux) + d(Sy, Uy)],$
- (3) $d(Tx, Ty) \leq \lambda[d(Tx, Vx) + d(Ty, Vy)],$

for all $x, y \in X$, where $\lambda \in [0, 1/2)$. If $S(X) \subseteq V(X)$, $T(X) \subseteq U(X)$ and one of $S(X)$, $T(X)$, $U(X)$ or $V(X)$ is a complete subspace of X , then the pairs (S, U) , and (T, V) have a unique point of coincidence in X . Moreover, if (S, U) and (T, V) are weakly compatible pairs then S, T, U and V have a unique common fixed point in X .

Proof. Let x_0 be any arbitrary point of X . Since, $S(X) \subseteq V(X)$ and $T(X) \subseteq U(X)$, starting with x_0 we define the sequence $\{y_n\}$ such that

$$y_{2n} = Sx_{2n} = Vx_{2n+1},$$

and $y_{2n+1} = Tx_{2n+1} = Ux_{2n+2}$, for all $n = 0, 1, 2, \dots$.

Suppose that $y_m = y_{m+1}$, for some $m \in N$.

If $m = 2k$, then $y_{2k} = y_{2k+1}$, for some $k \in N$, then from (1) we get

$$\begin{aligned} d(y_{2k+2}, y_{2k+1}) &= d(Sx_{2k+2}, Tx_{2k+1}) \\ &\leq \lambda [d(Sx_{2k+2}, Ux_{2k+2}) + d(Tx_{2k+1}, Vx_{2k+1})] \\ &= \lambda [d(y_{2k+2}, y_{2k+1}) + d(y_{2k+1}, y_{2k})], \end{aligned}$$

i.e.

$$d(y_{2k+2}, y_{2k+1}) \leq \lambda d(y_{2k+2}, y_{2k+1})$$

which implies that, $y_{2k+2} = y_{2k+1}$.

Similarly, we can deduce that $y_{2k+2} = y_{2k+3} = y_{2k+4} \dots$.

Hence $y_n = y_m$, for all $n \geq m$.

Thus $\{y_n\}$ is a Cauchy sequence in X in this case.

Assume that, $y_n \neq y_{n+1}$, for all $n \in N$.

Then from (1) it follows that,

$$\begin{aligned} d(y_{2k}, y_{2k+1}) &= d(Sx_{2k}, Tx_{2k+1}) \\ &\leq \lambda [d(Sx_{2k}, Ux_{2k}) + d(Tx_{2k+1}, Vx_{2k+1})] \\ &= \lambda [d(y_{2k}, y_{2k-1}) + d(y_{2k+1}, y_{2k})] \end{aligned}$$

which implies that

$$d(y_{2k}, y_{2k+1}) \leq \mu d(y_{2k-1}, y_{2k}), \quad (4)$$

for all $k \geq 1$, where $\mu = \frac{\lambda}{1-\lambda} < 1$.

Also

$$\begin{aligned} d(y_{2k+1}, y_{2k+2}) &= d(Tx_{2k+1}, Sx_{2k+2}) \\ &\leq \lambda [d(Sx_{2k+2}, Ux_{2k+2}) + d(Tx_{2k+1}, Vx_{2k+1})] \\ &= \lambda [d(y_{2k+2}, y_{2k+1}) + d(y_{2k+1}, y_{2k})] \end{aligned}$$

which implies that

$$d(y_{2k+1}, y_{2k+2}) \leq \mu d(y_{2k}, y_{2k+1}), \quad (5)$$

for all $k \geq 1$, where $\mu = \frac{\lambda}{1-\lambda} < 1$.

Using (4) and (5) we obtain

$$\begin{aligned} d(y_{2k}, y_{2k+1}) &\leq \mu d(y_{2k-1}, y_{2k}) \\ &\leq \mu^2 d(y_{2k-2}, y_{2k-1}) \end{aligned}$$

$$\begin{aligned} & \vdots \\ & \leq \mu^{2k} d(y_0, y_1), \end{aligned} \quad (6)$$

for all $k \geq 1$ and

$$\begin{aligned} d(y_{2k+1}, y_{2k+2}) & \leq \mu d(y_{2k}, y_{2k+1}) \\ & \leq \mu^2 d(y_{2k-1}, y_{2k}) \\ & \vdots \\ & \leq \mu^{2k+1} d(y_0, y_1), \end{aligned} \quad (7)$$

for all $k \geq 1$.

From (6) and (7) we have

$$d(y_n, y_{n+1}) \leq \mu^n d(y_0, y_1), \quad (8)$$

for all $n \geq 1$.

Using (2), (8), the facts that $\lambda \leq \mu$, and $\mu < 1$, we obtain

$$\begin{aligned} d(y_{2k}, y_{2k+2}) & = d(Sx_{2k}, Sx_{2k+2}) \\ & \leq \lambda [d(Sx_{2k}, Ux_{2k}) + d(Sx_{2k+2}, Ux_{2k+2})] \\ & \leq \lambda [d(y_{2k}, y_{2k-1}) + d(y_{2k+2}, y_{2k+1})] \\ & \leq \lambda [\mu^{2k-1} d(y_0, y_1) + \mu^{2k+1} d(y_0, y_1)] \\ & \leq \mu^{2k} d(y_0, y_1) + \mu^{2k+2} d(y_0, y_1) \\ & = (1 + \mu^2) \mu^{2k} d(y_0, y_1) \\ & \leq (1 + \mu) \mu^{2k} d(y_0, y_1), \end{aligned} \quad (9)$$

for all $k \geq 1$.

Using (3), (8), $\lambda \leq \mu$, and $\mu < 1$, we get

$$\begin{aligned} d(y_{2k+1}, y_{2k+3}) & = d(Tx_{2k+1}, Tx_{2k+3}) \\ & \leq \lambda [d(Tx_{2k+1}, Vx_{2k+1}) + d(Tx_{2k+3}, Vx_{2k+3})] \\ & \leq \lambda [d(y_{2k+1}, y_{2k}) + d(y_{2k+3}, y_{2k+2})] \\ & \leq \lambda [\mu^{2k} d(y_0, y_1) + \mu^{2k+2} d(y_0, y_1)] \\ & \leq \mu^{2k+1} d(y_0, y_1) + \mu^{2k+3} d(y_0, y_1) \\ & = (1 + \mu^2) \mu^{2k+1} d(y_0, y_1) \end{aligned}$$

$$\leq (1 + \mu)\mu^{2k}d(y_0, y_1), \quad (10)$$

for all $k \geq 1$.

Therefore from (9) and (10) we obtain

$$d(y_n, y_{n+2}) \leq (1 + \mu)\mu^n d(y_0, y_1), \quad (11)$$

for all $n \geq 1$.

For the sequence $\{y_n\}$, we consider $d(y_n, y_{n+p})$ in two cases.

If p is odd say $2m + 1$, for $m \geq 1$, then by the use of rectangular inequality and (8) we get

$$\begin{aligned} d(y_n, y_{n+2m+1}) &\leq d(y_{n+2m+1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_n) \\ &\leq d(y_{n+2m}, y_{n+2m+1}) + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-1}, y_{n+2m-2}) \\ &\quad + d(y_{n+2m-2}, y_{n+2m-3}) + \dots + d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n) \\ &= d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{n+2m-2}, y_{n+2m-1}) \\ &\quad + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m+1}) \\ &\leq \mu^n d(y_0, y_1) + \mu^{n+1} d(y_0, y_1) + \mu^{n+2} d(y_0, y_1) + \dots \\ &\quad + \mu^{n+2m-1} d(y_0, y_1) + \mu^{n+2m} d(y_0, y_1) \\ &= [1 + \mu + \mu^2 + \mu^3 + \dots + \mu^{2m-1} + \mu^{2m}] \mu^n d(y_0, y_1) \\ &\leq \frac{\mu^n}{1 - \mu} d(y_0, y_1), \quad \text{for all } n \geq 1, m \geq 1. \end{aligned}$$

If p is even say $2m$, for $m \geq 1$, then by the use of rectangular inequality, (8) and (11) we get

$$\begin{aligned} d(y_n, y_{n+2m}) &\leq d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_{n+2m-2}) + d(y_{n+2m-2}, y_n) \\ &\leq d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-2}, y_{n+2m-1}) + \dots + d(y_{n+4}, y_{n+3}) \\ &\quad + d(y_{n+3}, y_{n+2}) + d(y_{n+2}, y_n) \\ &= d(y_n, y_{n+2}) + d(y_{n+2}, y_{n+3}) + \dots + d(y_{n+2m-2}, y_{n+2m-1}) \\ &\quad + d(y_{n+2m-1}, y_{n+2m}) \\ &\leq (1 + \mu)\mu^n d(y_0, y_1) + [\mu^{n+2} d(y_0, y_1) + \mu^{n+3} d(y_0, y_1) \\ &\quad + \mu^{n+2m-2} d(y_0, y_1) + \mu^{n+2m-1} d(y_0, y_1)] \\ &= [1 + \mu + \mu^2 + \mu^3 + \dots + \mu^{2m-2} + \mu^{2m-1}] \mu^n d(y_0, y_1) \\ &\leq \frac{\mu^n}{1 - \mu} d(y_0, y_1), \quad \text{for all } n \geq 1, m \geq 1. \end{aligned}$$

Thus combining the above two cases we have

$$d(y_n, y_{n+p}) \leq \frac{\mu^n}{1-\mu} d(y_0, y_1), \quad \text{for all } n \in N, p \in N.$$

Since, P is a normal cone with a normal constant k and $0 \leq \mu < 1$, we have

$$\|d(y_n, y_{n+p})\| \leq \frac{k\mu^n}{1-\mu} \|d(y_0, y_1)\| \rightarrow 0, \text{ as } n \rightarrow \infty,$$

i.e.

$$\|d(y_n, y_{n+p})\| \rightarrow 0, \text{ as } n \rightarrow \infty, \quad \forall p \in N.$$

Hence, $\{y_n\}$ is a Cauchy sequence in X.

Suppose, $U(X)$ is a complete subspace of X, there exists $z \in U(X)$ such that $\lim_{n \rightarrow \infty} y_{2n+1} = \lim_{n \rightarrow \infty} Ux_{2n+2} = z$. Consequently we find $x \in X$ such that $z = Ux$. Then by using rectangular property and (2), we have

$$\begin{aligned} d(Sx, z) &\leq d(Sx, y_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\ &= d(Sx, Sx_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\ &\leq \lambda [d(Sx, Ux) + d(Sx_{2n}, Ux_{2n})] + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\ &= \lambda d(Sx, z) + \lambda d(y_{2n}, y_{2n-1}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z), \end{aligned}$$

which implies

$$d(Sx, z) \leq \frac{\lambda}{1-\lambda} d(y_{2n}, y_{2n-1}) + \frac{1}{1-\lambda} [d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)],$$

for all $n \geq 1$.

Since, P is a normal cone with a normal constant k we have,

$$\begin{aligned} \|d(Sx, z)\| &\leq \frac{k\lambda}{1-\lambda} \|d(y_{2n}, y_{2n-1})\| \\ &\quad + \frac{k}{1-\lambda} [\|d(y_{2n}, y_{2n+1})\| + \|d(y_{2n+1}, z)\|] \rightarrow 0, \text{ as } n \rightarrow \infty. \end{aligned}$$

i.e. $\|d(Sx, z)\| = 0$ or $Sx = z$.

Therefore

$$Sx = Ux = z. \tag{12}$$

Since $z = Sx \in S(X)$ and $S(X) \subseteq V(X)$, there exists $y \in X$ such that $z = Vy$.

Now,

$$\begin{aligned}
 d(Ty, z) &\leq d(Ty, y_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\
 &= d(Ty, Sx_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\
 &\leq \lambda [d(Sx_{2n}, Ux_{2n}) + d(Ty, Vy)] + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z) \\
 &= \lambda [d(y_{2n}, y_{2n-1}) + d(Ty, z)] + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z),
 \end{aligned}$$

which implies

$$\begin{aligned}
 d(Ty, z) &\leq \frac{\lambda}{1-\lambda} d(y_{2n-1}, y_{2n}) \\
 &\quad + \frac{1}{1-\lambda} [d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)], \text{ for all } n \geq 1.
 \end{aligned}$$

Since, P is a normal cone with a normal constant k , we have

$$\begin{aligned}
 \|d(Ty, z)\| &\leq \frac{k\lambda}{1-\lambda} \|d(y_{2n-1}, y_{2n})\| \\
 &\quad + \frac{k}{1-\lambda} [\|d(y_{2n}, y_{2n+1})\| + \|d(y_{2n+1}, z)\|] \rightarrow 0, \text{ as } n \rightarrow \infty,
 \end{aligned}$$

i.e. $\|d(Ty, z)\| = 0$ or $Ty = z$.

Therefore

$$Ty = Vy = z. \quad (13)$$

From (12) and (13), we say that the pairs (S, U), and (T, V) have a common point of coincidence z in X.

Suppose the pairs (S, U), and (T, V) are weakly compatible mappings. Then by using (12) and (13) we get

$$Sz = SUx = USx = Uz = z_1 \quad (14)$$

and

$$Tz = TVy = VTy = Vz = z_2. \quad (15)$$

Using (1), (14) and (15), we obtain

$$\begin{aligned}
 d(z_1, z_2) &= d(Sz, Tz) \\
 &\leq \lambda [d(Sz, Uz) + d(Tz, Vz)] \\
 &= \lambda [d(z_1, z_1) + d(z_2, z_2)] = \theta,
 \end{aligned}$$

or $d(z_1, z_2) = \theta$.

That is $z_1 = z_2$.

Therefore

$$Sz = Tz = Uz = Vz. \quad (16)$$

Now we have to prove that $Tz = z$.

Using (1), (12) and (16) we get

$$\begin{aligned} d(z, Tz) &= d(Sx, Tz) \\ &\leq \lambda [d(Sx, Uz) + d(Tz, Vz)] \\ &= \lambda [d(z, Tz) + d(Tz, Tz)]. \end{aligned}$$

That is $d(z, Tz) \leq \lambda d(z, Tz)$, which implies that $d(z, Tz) = \theta$.

Hence $z = Tz$, or $Sz = Tz = Uz = Vz = z$.

Thus z is a common fixed point of S , T , U and V .

For the uniqueness suppose that z^* is another common fixed point of S , T , U , and V . That is, $Sz^* = Tz^* = Uz^* = Vz^* = z^*$. Then:

$$\begin{aligned} d(z, z^*) &= d(Sz, Tz^*) \\ &\leq \lambda [d(Sz, Uz) + d(Tz^*, Vz^*)] \\ &= \lambda [d(Sz, Sz) + d(Tz^*, Tz^*)] = \theta \end{aligned}$$

which implies that $d(z, z^*) = \theta$.

Hence, $z = z^*$ and moreover S , T , U and V have a unique common fixed point in X . Similarly, if $S(X)$, $T(X)$ or $V(X)$ is a complete subspace of X , then we can easily prove that S , T , U and V have a unique common fixed point in X . $\square$

To illustrate Theorem 2.1 we give the following example.

Example 2.2. Let $X = \{1, 2, 3, 4\}$, $E = \mathbb{R}^2$ and $P = \{(x, y) : x, y \geq 0\}$ is a normal cone in E , where $\mathbb{R}$ denotes set of all Real numbers. Define $d : X \times X \rightarrow E$ as follows:

$$\begin{cases} d(x, x) = \theta, & \text{for all } x \in X, \\ d(1, 2) = d(2, 1) = (3, 9), \\ d(1, 3) = d(3, 1) = d(2, 3) = d(3, 2) = (1, 3), \\ d(1, 4) = d(4, 1) = d(2, 4) = d(4, 2) = d(3, 4) = d(4, 3) = (4, 12), \end{cases}$$

Then (X, d) is a complete cone rectangular metric space.

Now, we can define the mappings S , T , U and $V : X \rightarrow X$ as follows:

$$S(x) = 3, \quad \text{for all } x \in X,$$

$$T(x) = \begin{cases} 3 & \text{if } x \neq 4, \\ 1 & \text{if } x = 4, \end{cases}$$

$$U(x) = \begin{cases} 2 & \text{if } x = 1, \\ 1 & \text{if } x = 2, \\ 3 & \text{if } x = 3, \\ 4 & \text{if } x = 4, \end{cases}$$

$$\text{and } V(x) = x, \quad \text{for all } x \in X.$$

It is clear that, $S(X) \subseteq V(X)$, $T(X) \subseteq U(X)$, S and U are weakly compatible mappings. Note that, the Kannan type contractive conditions (1), (2) and (3) of Theorem 2.1 are satisfied if $\lambda = 1/4$.

Hence, all the conditions of Theorem 2.1 are satisfied and 3 is a unique common fixed point of S , T , U and V .

Corollary 2.3. *Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant k and let the mappings S , T and U : $X \rightarrow X$ satisfy the following:*

- (1) $d(Sx, Ty) \leq \lambda[d(Sx, Ux) + d(Ty, Uy)]$,
- (2) $d(Sx, Sy) \leq \lambda[d(Sx, Ux) + d(Sy, Uy)]$,
- (3) $d(Tx, Ty) \leq \lambda[d(Tx, Ux) + d(Ty, Uy)]$,

for all $x, y \in X$, where $\lambda \in [0, 1/2)$. If $S(X) \cup T(X) \subseteq U(X)$ and if $U(X)$ or $S(X) \cup T(X)$ is a complete subspace of X , then the pairs (S, U) and (T, U) have a unique point of coincidence in X . Moreover, if (S, U) and (T, U) are weakly compatible pairs then S , T and U have a unique common fixed point in X .

Proof. If we put $V = U$, in Theorem 2.1, then we obtain the corollary. $\square$

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