TRANSLATES OF VAGUE NORMAL GROUPS

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Abstract: In this paper we introduce translate operators on vague cut set, vague normal groups, and studied their properties. These concepts are used in the development of some important results and theorems about translates of vague homomorphism, vague normal groups. Also some of their important properties have been investigated.

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1. Introduction

The concept proposed by L.A. Zadeh [14] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A

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by means of a membership function μ_A defined from X into $[0, 1]$ has revolutionized the theory of Mathematical modeling decision making etc. In handling the imprecise real life situations mathematically. Now several branches of fuzzy mathematics like fuzzy algebra, fuzzy topology, fuzzy control theory, fuzzy measure theory etc. have emerged. But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence against x belonging to A . It is felt by several decision makers and researchers that in proper decision making, the evidence belongs to A and evidence not belongs to A are both necessary and how much X belongs to A or how much x does not belongs to A are necessary.

Several generalizations of Zadeh. L. A's fuzzy set theory have been proposed, such as L-fuzzy sets (see [4]), Interval valued fuzzy sets. Intuitionistic fuzzy sets by K.T. Atanassov [1] and W.L. Gau, D.J. Buehrer Vague sets, see [3], are mathematically equivalent. Any such set A of a given Universe X can be characterized by means of a pair of function (t_A, f_A) where t_A and f_A are function from X in to $[0, 1]$ such that $0 \leq t_A(x) + f_A(x) \leq 1$ for all x in X . The set t_A is called the truth function and the set f_A is called false function or non membership function and $t_A(x)$ gives the evidence of how much $x \in A$ $f_A(x)$ gives the evidence of how much x does not $\in A$. These concepts are being applied in several areas like decisionmaking, fuzzy control, knowledge discovery and fault diagnosis etc. It is believed the vague sets (or equivalently intuitionistic fuzzy sets) will more useful in decision making, and other areas of Mathematical modeling. Through K.T. Atanassov [1] intuitionistic fuzzy sets, W.L. Gau and D.J. Buehrer [3] and some other areas of Mathematical modeling. Since then the theory fuzzy sets developed extensively and embraced almost all subjects like engineering science and Technology. But the membership function μ_A gives only a approximation belong to A . To avoid this and obtain a better estimation and analysis of data decision making.

W.L. Gau and D.J. Bueher in [3] have initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague, than the theory of Fuzzy sets do. Ranjit Biswas [9] initiated the study of vague groups and N. Ramakrishna [8], [10], [11], T. Eswarlal [2], [8], [10] create extended the study of vague algebra. The objective of this paper is to contribute further to the study of vague algebra by introducing concepts of translate operators on vague cut set, translate operators on vague homomorphism and translate operators on vague normal groups.

2. Preliminaries

We give here a review of some definitions and results which one may find in W.L. Gau and D.J. Buehrer [3], Ranjit Biswas [9], N. Ramakrishna [8], [10], [11].

Definition 1.1. A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \rightarrow [0, 1]$, $f_A : U \rightarrow [0, 1]$, are mappings such that $t_A(u) + f_A(u) \leq 1$, for all $u \in U$. The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 1.2. The interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A , and it is denoted by $V_A(u)$, i.e. $V_A(u) = [t_A(u), 1 - f_A(u)]$.

Definition 1.3. Let $(G, *)$ be a group. A vague set A of G is called a vague group of G if, for all x, y in G , $V_A(xy) \geq \min\{V_A(x), V_A(y)\}$ and $V_A(x^{-1}) \geq V_A(x)$, i.e.

$$t_A(xy) \geq \min\{t_A(x), t_A(y)\}, \quad f_A(xy) \leq \max\{f_A(x), f_A(y)\}$$

and

$$t_A(x^{-1}) \geq t_A(x), \quad f_A(x^{-1}) \leq f_A(x).$$

Here the element xy stands for $x * y$.

Definition 1.4. Let A be a vague set of a universe G with true-membership function t_A , and false membership function f_A . For $\alpha, \beta \in [0, 1]$ with $\alpha \leq \beta$, the (α, β) cut or vague cut of a vague set A is the crisp subset of G is given by $A_{(\alpha, \beta)} = \{x | x \in G, V_A(x) \geq [\alpha, \beta]\}$, i.e. $A_{(\alpha, \beta)} = \{x | x \in G, t_A(x) \geq \alpha, \text{ and } 1 - f_A(x) \geq \beta\}$.

Definition 1.5. The α -cut, A_α of the vague set A is the (α, α) cut of A , and hence given by $A_\alpha = \{x | x \in G, t_A(x) \geq \alpha\}$.

Definition 1.6. Let A be a Vague group of a group G Then A is Called Vague normal group if for all $x, y \in G$, $V_A(xy) = V_A(yx)$. Alternatively, we can say that, a vague group A is said to be vague normal group of G if $V_A(x) = V_A(yxy^{-1})$ for all $x, y \in G$.

We now introduce the following definitins.

Definition 1.7. Let A be a Vague group of a group G then the set $N(A) = \{a \in G | V_A(asa^{-1}) = V_A(x) \text{ for all } a, x \in G\}$ is called vague normalizer of A .

Definition 1.8. Let A be a vague group of a group G Then the set

$$\mathbb{C}(A) = \{a \in G | V_A([a, x]) = V_A(e) \text{ for all } x \in G\}$$

is called vague centralizer of A , where $[a, x] = a^{-1}x^{-1}ax$.

Definition 1.9. A function $f : G \rightarrow G^1$ is said to be vague homomorphism if $f(a.b) = f(a).f(b)$ for all $a, b \in G$.

3. Translate Operators on Vague Cut-Set

Definition 2.1. Let A be a vague set of universal set X then $A_{(\alpha, \beta)}$ cut-set of a vague translation operators can be defined by:

$$(a) T_{\theta+}(A_{(\alpha, \beta)}) = (t_{A_{(\alpha, \beta)T_{\theta+}}}(x), f_{A_{(\alpha, \beta)T_{\theta+}}}(x)), \text{ where}$$

$$t_{A_{(\alpha, \beta)T_{\theta+}}}(x) = \min\{t_{A_{(\alpha, \beta)}}(x) + \theta, 1\} \text{ and } f_{A_{(\alpha, \beta)T_{\theta+}}}(x) = \max\{f_{A_{(\alpha, \beta)}}(x) - \theta, 0\}.$$

$$(b) T_{\theta-}(A_{(\alpha, \beta)}) = (t_{A_{(\alpha, \beta)T_{\theta-}}}(x), f_{A_{(\alpha, \beta)T_{\theta-}}}(x)), \text{ where}$$

$$t_{A_{(\alpha, \beta)T_{\theta-}}}(x) = \max\{t_{A_{(\alpha, \beta)}}(x) - \theta, 0\} \text{ and } f_{A_{(\alpha, \beta)T_{\theta-}}}(x) = \min\{f_{A_{(\alpha, \beta)}}(x) + \theta, 1\}.$$

Here: $A_{(\alpha, \beta)} = \{x \in X, t_A(x) \geq \alpha, f_A(x) \leq \beta\}$, where θ, α , and $\beta \in [0, 1]$; $T_{\theta+}(A_{(\alpha, \beta)})$ and $T_{\theta-}(A_{(\alpha, \beta)})$ are called translation of vague cut-set increasing operator and translation of vague cut-set decreasing operator respectively.

Remark 2.2. Let A be a vague set of a universal set X , $A_{(\alpha, \beta)}$ be a vague cut-set then

$$0 \leq t_{A_{(\alpha, \beta)T_{\theta+}}}(x) + f_{A_{(\alpha, \beta)T_{\theta+}}}(x) \leq 1.$$

Example 2.3. Let $X = \{1, -1\}$, $A = \{(1, 0.6, 0.02), (-1, 0.5, 0.04)\}$ and $\alpha = 0.4$, $\beta = 0.1$ and $\theta = 0.2$.

Let $X = \{1, -1\}$, $A = \{(1, 0.6, 0.02), (-1, 0.5, 0.04)\}$ and $\alpha = 0.4$, $\beta = 0.1$ and $\theta = 0.2$ since $A_{(\alpha, \beta)} = \{x \in X, t_A(x) \geq \alpha, f_A(x) \leq \beta\}$.

Now $T_{\theta+}(A(\alpha, \beta)) = (t_{A(\alpha, \beta)T_{\theta+}}(x), f_{A(\alpha, \beta)T_{\theta+}}(x))$, where $t_{A(\alpha, \beta)T_{\theta+}}(x) = \min\{t_{A(\alpha, \beta)}(x) + \theta, 1\}$ and

$$f_{A(\alpha, \beta)T_{\theta+}}(x) = \max\{f_{A(\alpha, \beta)}(x) - \theta, 0\}$$

$$t_{A(\alpha, \beta)T_{\theta+}}(1) = \min\{t_{A(\alpha, \beta)}(1) + \theta, 1\} = \min\{0.6 + 0.2, 1\} = 0.8$$

$$f_{A(\alpha, \beta)T_{\theta+}}(1) = \max\{f_{A(\alpha, \beta)}(1) - \theta, 0\} = \max\{0.02 - 0.2, 0\} = 0$$

$$\Rightarrow t_{A(\alpha, \beta)T_{\theta+}}(1) + f_{A(\alpha, \beta)T_{\theta+}}(1) = 0.8 + 0 = 0.8 \leq 1.$$

Therefore $t_{A(\alpha, \beta)T_{\theta+}}(1) + f_{A(\alpha, \beta)T_{\theta+}}(1) \in T_{\theta+}(A(\alpha, \beta))$,

$$t_{A(\alpha, \beta)T_{\theta+}}(-1) = \min\{0.5 + 0.2, 1\} = \min\{0.7, 1\} = 0.7,$$

$$f_{A(\alpha, \beta)T_{\theta+}}(-1) = \max\{0.04 - 0.2, 0\} = \max\{-0.16, 0\} = 0,$$

$$\Rightarrow t_{A(\alpha, \beta)T_{\theta+}}(-1) + f_{A(\alpha, \beta)T_{\theta+}}(-1) = 0.7 + 0 = 0.7 \in T_{\theta+}(A(\alpha, \beta)).$$

Therefore $0 \leq t_{A(\alpha, \beta)T_{\theta+}}(-1) + f_{A(\alpha, \beta)T_{\theta+}}(-1) \in T_{\theta+}(A(\alpha, \beta))$. Hence $T_{\theta+}(A(\alpha, \beta))$ is a translate of vague $A(\alpha, \beta)$ cut-set. similarly we have to show that $T_{\theta-}A(\alpha, \beta)$ is a translate of vague $A(\alpha, \beta)$ cut-set.

Example 2.4. Let $X = \{1, w, w^2\}$, $A = \{(1, 0.3, 0.6), (w, 0.1, 0.25), (w^2, 0.5, 0.1)\}$.

It is given: $X = \{1, w, w^2\}$ and $A = \{(1, 0.3, 0.1), (w, 0.1, 0.25), (w^2, 0.5, 0.1)\}$
 $\alpha = 0.3, \beta = 0.2, \theta = 0.4$. Since $A(\alpha, \beta) = \{x \in X : t_A(x) \geq \alpha, 1 - f_A(x) \geq \beta\}$,
 where $t_{A(\alpha, \beta)T_{\theta-}}(x) = \max\{t_{A(\alpha, \beta)}(x) - \theta, 0\}$ and

$$f_{A(\alpha, \beta)T_{\theta-}}(x) = \min\{f_{A(\alpha, \beta)}(x) + \theta, 1\},$$

$$t_{A(\alpha, \beta)T_{\theta-}}(1) = \max\{t_{A(\alpha, \beta)}(1) - \theta, 0\} = \max\{0.3 - 0.4, 1\} = 1,$$

$$f_{A(\alpha, \beta)T_{\theta-}}(1) = \min\{f_{A(\alpha, \beta)}(1) + \theta, 1\} = \min\{0.1 + 0.4, 1\} = 0.5,$$

$$\Rightarrow t_{A(\alpha, \beta)T_{\theta-}}(1) + f_{A(\alpha, \beta)T_{\theta-}}(1) = 1 + 0.5 = 1.5 \leq 1.$$

Therefore $t_{A(\alpha, \beta)T_{\theta-}}(1) + f_{A(\alpha, \beta)T_{\theta-}}(1) \in T_{\theta-}(A(\alpha, \beta))$, $t_{A(\alpha, \beta)T_{\theta-}}(w) = \max\{0.1 - 0.4, 1\} = \min\{-0.3, 1\} = -0.3$, $f_{A(\alpha, \beta)T_{\theta-}}(w) = \min\{0.25 + 0.4, 1\} = \min\{0.65, 1\} = 0.65$,
 $\Rightarrow t_{A(\alpha, \beta)T_{\theta-}}(w) + f_{A(\alpha, \beta)T_{\theta-}}(w) = -0.3 + 0.65 = 0.35 \in T_{\theta-}(A(\alpha, \beta))$, $t_{A(\alpha, \beta)T_{\theta-}}(w^2) = \max\{0.5 - 0.4, 1\} = \min\{0.1, 1\} = 0.1$, $f_{A(\alpha, \beta)T_{\theta-}}(w^2) = \min\{0.1 + 0.4, 1\} = \min\{0.5, 1\} = 0.5$,
 $\Rightarrow t_{A(\alpha, \beta)T_{\theta-}}(w^2) + f_{A(\alpha, \beta)T_{\theta-}}(w^2) = 0.1 + 0.5 = 0.6 \in T_{\theta-}(A(\alpha, \beta))$.

Hence $T_{\theta-}(A_{(\alpha,\beta)})$ is a translate of vague $A_{(\alpha,\beta)}$ cut-set. Similarly we have to show that $T_{\theta+A_{(\alpha,\beta)}}$ is a translate of vague $A_{(\alpha,\beta)}$ cut-set.

Theorem 2.5. *If P and Q be two vague sets of a universal set X , then following holds:*

- (1) $A_{(\alpha,\beta)}(P) \subseteq A_{(\gamma,\delta)}(Q)$ if $\alpha \geq \gamma, \beta \leq \delta$,
- (2) $A_{(1-\beta,\beta)}(P) \subseteq A_{(\alpha,\beta)}(P) \subseteq A_{(\alpha, 1-\alpha)}(P)$,
- (3) $P \subseteq Q \Rightarrow A_{(\alpha,\beta)}(P) \subseteq A_{(\alpha,\beta)}(Q)$,
- (4) $A_{(\alpha,\beta)}(P \cap Q) = A_{(\alpha,\beta)}(P) \cap A_{(\alpha,\beta)}(Q)$,
- (5) $A_{(\alpha,\beta)}(P \cup Q) \supseteq A_{(\alpha,\beta)}(P) \cup A_{(\alpha,\beta)}(Q)$ equality holds if $\alpha + \beta = 1$,
- (6) $A_{(\alpha,\beta)}(\bigcap P_i) = \bigcap A_{(\alpha,\beta)}(P_i)$,
- (7) $A_{(1,0)}(P) = X$.

Proof. (1) Let $x \in A_{(\alpha,\beta)}(P) \Rightarrow t_p(x) \geq \alpha$ and $f_p(x) \leq \beta$. Since $\gamma \leq \alpha$ and $\delta \geq \beta, \alpha \geq \gamma$ and $\beta \leq \delta: \Rightarrow t_p(x) \geq \alpha \geq \gamma$ and $f_p(x) \leq \beta \leq \delta \Rightarrow t_p(x) \geq \gamma$ and $f_p(x) \leq \delta \Rightarrow x \in A_{(\gamma,\delta)}$ therefore $Let x \in A_{(\alpha,\beta)}(P) \Rightarrow x \in A_{(\gamma,\delta)} \Rightarrow A_{(\alpha,\beta)}(P) \subseteq A_{(\gamma,\delta)}(Q)$.

(2) Since $\alpha + \beta \leq 1 \Rightarrow \alpha \leq 1 - \beta$ and $\beta \leq 1 - \alpha$. Therefore using part (1), we obtain

$$A_{(1-\beta,\beta)}(P) \subseteq A_{(\alpha,\beta)}(P). \quad (1)$$

Again $\alpha + \beta \leq 1 \Rightarrow \alpha \geq \alpha$ and $\beta \leq 1 - \alpha$. Therefore using part (1) we obtain

$$A_{(\alpha,\beta)}(P) \subseteq A_{(\alpha,1-\alpha)}(P). \quad (2)$$

From (1) and (2) we get $A_{(1-\beta,\beta)}(P) \subseteq A_{(\alpha,\beta)}(P) \subseteq A_{(\alpha,1-\alpha)}(P)$.

(3) Let $x \in A_{(\alpha,\beta)}(P) \Rightarrow t_p(x) \geq \alpha$ and $f_p(x) \leq \beta$, as $Q \supseteq P \Rightarrow t_Q(x) \geq t_P(x) \geq \alpha$ and $f_p(x) \leq \beta \leq \beta, \Rightarrow t_P(x) \geq \alpha$ and $f_p(x) \leq \beta \Rightarrow x \in A_{(\alpha,\beta)}(Q), \Rightarrow x \in A_{(\alpha,\beta)}(P) \Rightarrow x \in A_{(\alpha,\beta)}(Q)$.

Therefore $A_{(\alpha,\beta)}(P) \subseteq A_{(\alpha,\beta)}(Q)$.

(4) $(P \cap Q) \subseteq P$ and $(P \cap Q) \subseteq Q$. Therefore by (1):

$$A_{(\alpha,\beta)}(P \cap Q) \subseteq A_{(\alpha,\beta)}(P) \text{ and } A_{(\alpha,\beta)}(P \cap Q) \subseteq A_{(\alpha,\beta)}(Q). \quad (3)$$

Also, let $x \in X A_{(\alpha,\beta)}(P) \cap A_{(\alpha,\beta)}(Q), x \in X A_{(\alpha,\beta)}(P)$ and $x \in X A_{(\alpha,\beta)}(Q) \Rightarrow t_P(x) \geq \alpha$ and $f_p(x) \leq \beta$ and $t_Q(x) \geq \alpha$ and $f_Q(x) \leq \beta$

$$\begin{aligned}
&\Rightarrow t_P(x) \geq \alpha \text{ and } t_Q(x) \geq \alpha; f_P(x) \leq \beta \text{ and } f_Q(x) \leq \beta \\
&\Rightarrow t_P(x) \geq \alpha \text{ and } t_Q(x) \geq \alpha; f_P(x) \leq \beta \text{ and } f_Q(x) \leq \beta \\
&\Rightarrow t_P(x) \wedge t_Q(x) \geq \alpha; f_P(x) \wedge f_Q(x) \leq \beta \\
&\Rightarrow (t_P \wedge t_Q)(x) \geq \alpha; (f_P \wedge f_Q)(x) \leq \beta \\
&x \in A_{(\alpha, \beta)}(P \cap Q).
\end{aligned}$$

Thus

$$A_{(\alpha, \beta)}(P) \cap A_{(\alpha, \beta)}(Q) \subseteq A_{(\alpha, \beta)}(P \cap Q). \quad (4)$$

From (3) and (4), we get $A_{(\alpha, \beta)}(P \cap Q) = A_{(\alpha, \beta)}(P) \cap A_{(\alpha, \beta)}(Q)$.

(5) Since $P \subseteq (P \cup Q)$ and $Q \subseteq (P \cup Q)$. Therefore using part (1):

$$\begin{aligned}
A_{(\alpha, \beta)}(P) &\subseteq A_{(\alpha, \beta)}(P \cup Q) \text{ and } A_{(\alpha, \beta)}(Q) \subseteq A_{(\alpha, \beta)}(P \cup Q), \\
&\Rightarrow A_{(\alpha, \beta)}(P) \cup A_{(\alpha, \beta)}(Q) \subseteq A_{(\alpha, \beta)}(P \cup Q).
\end{aligned}$$

Now equality holds if $\alpha + \beta = 1$ we show that

$$A_{(\alpha, \beta)}(P \cup Q) \subseteq A_{(\alpha, \beta)}(P) \cup A_{(\alpha, \beta)}(Q).$$

Let $x \in A_{(\alpha, \beta)}(P \cup Q)$

$$\begin{aligned}
&\Rightarrow (t_P \cup t_Q)(x) \geq \alpha \text{ and } (f_P \cup f_Q)(x) \leq \beta \\
&\Rightarrow (t_P(x) \vee t_Q(x) \geq \alpha \text{ and } f_P(x) \wedge f_Q(x) \leq \beta.
\end{aligned}$$

If $t_P(x) \geq \alpha$, then $f_P(x) \leq 1 - t_P(x) \leq 1 - \alpha = \beta$ implies that $x \in A_{(\alpha, \beta)}(Q) \subseteq A_{(\alpha, \beta)}(P) \cup A_{(\alpha, \beta)}(Q)$.

Also

$$A_{(\alpha, \beta)}(P \cup Q) \subseteq A_{(\alpha, \beta)}(P) \cup A_{(\alpha, \beta)}(Q). \quad (6)$$

From (5) and (6), we get $A_{(\alpha, \beta)}(P \cup Q) = A_{(\alpha, \beta)}(P) \cup A_{(\alpha, \beta)}(Q)$.

(6) let $x \in A_{(\alpha, \beta)}(\bigcap P_i) \Rightarrow t_{P_i}(x) \geq \alpha$ and $\bigcap f_{P_i}(x) \leq \beta$
and $\bigwedge t_{P_i}(x) \geq \alpha$ and $\bigcap f_{P_i} \leq \beta \Rightarrow x \in A_{(\alpha, \beta)}P_i$ for all $i \Rightarrow x \in A_{(\alpha, \beta)} \bigcap (P_i) \Rightarrow x \in A_{(\alpha, \beta)}(P_i)$ for all i , and hence

$$A_{(\alpha, \beta)}(\bigcap P_i) = \bigcap A_{(\alpha, \beta)}(P_i).$$

(7) Since $A_{(\alpha, \beta)}(P) = \{x : t_P(x) \geq \alpha \text{ and } f_P(x) \leq \beta\}$.

Setting $\alpha = 0$ and $\beta = 1$, $\Rightarrow A_{(\alpha, \beta)}(P) = X$.

Theorem 2.6. *If P is vague group of G . Then*

$$T_{\theta+}(A_{(\alpha, \beta)}(P)) \text{ and } T_{\theta-}(A_{(\alpha, \beta)}(P))$$

are a translate vague group of G , where $t_P(e) \geq \alpha$, $f_P(e) \leq \beta$ and e is the identity element of G .

Proof. Let P is vague group of G . $A_{(\alpha,\beta)}(P)$ is vague cut-set. We shall show that $T_{\theta+}(A_{(\alpha,\beta)}(P))$ is a translate of vague group of G .

We have:

$$\begin{aligned} t_{A_{(\alpha,\beta)}T_{\theta+}}(xy^{-1}) &\geq \min\{t_{A_{(\alpha,\beta)}}(xy^{-1}) + \theta, 1\} \\ &\geq \min\{t_{A_{(\alpha,\beta)}}(x) + \theta, 1\} \bigwedge \{t_{A_{(\alpha,\beta)}}(y^{-1}) + \theta, 1\} \\ &= \min\{t_{A_{(\alpha,\beta)}}(x) + \theta, 1\}, \{t_{A_{(\alpha,\beta)}}(y) + \theta, 1\} \\ &= \min\{t_{A_{(\alpha,\beta)}}(x), \{t_{A_{(\alpha,\beta)}}(y)\}. \end{aligned}$$

Therefore

$$\begin{aligned} t_{A_{(\alpha,\beta)}T_{\theta+}}(xy^{-1}) &\geq \min\{t_{A_{(\alpha,\beta)}}(x), t_{A_{(\alpha,\beta)}}(y)\}, \\ f_{A_{(\alpha,\beta)}T_{\theta+}}(xy^{-1}) &\leq \max\{f_{A_{(\alpha,\beta)}}(xy^{-1}) + \theta, 0\} \\ \max\{f_{A_{(\alpha,\beta)}}(xy^{-1}) + \theta, 0\} &\bigvee f_{A_{(\alpha,\beta)}}(y^{-1}) + \theta, 0\} \\ &\max\{f_{A_{(\alpha,\beta)}}(x) \bigvee f_{A_{(\alpha,\beta)}}(y)\}. \end{aligned} \tag{1}$$

Therefore

$$f_{A_{(\alpha,\beta)}T_{\theta+}}(xy^{-1}) \leq \max\{f_{A_{(\alpha,\beta)}}(x), f_{A_{(\alpha,\beta)}}(y)\}. \tag{2}$$

Hence from (1) and (2) it follows that $T_{\theta+}(A_{(\alpha,\beta)}(P))$ is a vague group of G .

Similarly, we can show that $T_{\theta-}(A_{(\alpha,\beta)}(P))$ is a vague group of G .

Theorem 2.7. P and Q are two translates of vague groups of G then $P \cap Q$ is translates of vague group of G .

Proof. P and Q are two translates of vague groups of G . Then $T_{\theta+}(P \cap Q)$ is a translates of vague group of G . Since the result holds true.

Now

$$\begin{aligned} t_{(P \cap Q)T_{\alpha+}}(xy) &\geq \min\{t_{(P \cap Q)}(xy) + \alpha, 1\}, \\ \min\{t_{(P \cap Q)}(x) + \alpha, 1\}, \{t_{(P \cap Q)}(y) + \alpha, 1\} &= \min\{t_{(P \cap Q)}(x), t_{(P \cap Q)}(y)\}, \end{aligned}$$

Therefore

$$t_{(P \cap Q)T_{\alpha+}}(xy^{-1}) \geq \min\{t_{(P \cap Q)}(x), t_{(P \cap Q)}(y)\}. \tag{1}$$

$$\begin{aligned} f_{(P \cap Q)T_{\alpha+}}(xy^{-1}) &\leq \max\{f_{(P \cap Q)}(xy^{-1}) - \alpha, 0\} \\ &= \max\{f_{(P \cap Q)}(x) - \alpha, 0\}, \{f_{(P \cap Q)}(y^{-1}) - \alpha, 0\} \end{aligned}$$

$$\max\{f_{(P \cap Q)}(x) - \alpha, 0\}, \{f_{(P \cap Q)}(y) - \alpha, 0\} = \max\{f_{(P \cap Q)}(x), f_{(P \cap Q)}(y)\}.$$

Hence

$$f_{(P \cap Q)T_{\alpha+}}(xy^{-1}) \geq \max\{f_{(P \cap Q)}(x), f_{(P \cap Q)}(y)\}. \quad (2)$$

From (1) and (2): $T_{\theta+(P \cap Q)}$ is a translates of vague group of G .

Theorem 2.8. *Let A be a translates of vague group of G . Then α -cut A_α is a translates of vague group of G .*

Proof. Let $A_\alpha = \{x \in G : t_A(x) \geq \alpha\}$ and let $x, y \in A_\alpha$

$$\begin{aligned} \Rightarrow t_{AT_{\alpha+}}(xy^{-1}) &= \min\{t_A(xy^{-1}) + \alpha, 1\} \geq \min\{t_A(x) + \alpha, 1\} \min\{t_A(y^{-1}) + \alpha, 1\} \\ &= \min\{t_A(x), t_A(y)\} = \{\alpha, \alpha\} \end{aligned}$$

$$\Rightarrow t_{AT_{\alpha+}}(xy^{-1}) \geq \min\{\alpha, \alpha\} = \alpha.$$

So $xy^{-1} \in G$. Therefore

$$t_{AT_{\alpha+}}(xy^{-1}) \geq \min\{t_A(x), t_A(y)\}. \quad (1)$$

$$\begin{aligned} f_{AT_{\alpha+}}(xy^{-1}) &= \max\{f_A(xy^{-1}) - \alpha, 0\} \leq \max\{f_A(x) - \alpha, 0\} \max\{f_A(y^{-1}) - \alpha, 0\} \\ &= \max\{f_A(x), f_A(y)\} = \{\alpha, \alpha\}, \end{aligned}$$

$$\Rightarrow f_{AT_{\alpha+}}(xy^{-1}) \leq \max\{\alpha, \alpha\} = \alpha.$$

So $xy^{-1} \in G$,

$$f_{AT_{\alpha+}}(xy^{-1}) \leq \max\{f_A(x), f_A(y)\}. \quad (2)$$

From (1) and (2) A_α cut set is a translates of vague group of G .

4. Translate Operators on Vague Homomorphism Groups and Vague Normal Groups

Definition 3.1. Let G be a vague normal group of be a vague set P then the translate operators of Vague normal group can be defined by

$$T_{\alpha+}(V_P(xy)) = T_{\alpha+}(V_P(yx) \Rightarrow (t_{P\alpha+}(xy), f_{P\alpha+}(xy)) = (t_{P\alpha+}(yx), f_{P\alpha+}(yx))$$

and

$$T_{\alpha-}(V_P(xy)) = T_{\alpha-}(V_P(yx) \Rightarrow (t_{P\alpha-}(xy), f_{P\alpha-}(xy)) = (t_{P\alpha-}(yx), f_{P\alpha-}(yx)).$$

Theorem 3.2. If P is a translate of vague normal group of G . Then $T_{\theta+}(A_{(\alpha,\beta)}(P))$ is a translate of vague normal group of G . where $t_P(e) \geq \alpha, f_P(e) \leq \beta$ and e is the identity element of G .

Proof. Given that P is a vague normal group of G ie $V_P(xy) = V_P(yx)$ for all $x, y \in G$ and $t_P(xy) = t_P(yx)$ and $f_P(xy) = f_P(yx)$ for all $x, y \in G$.

Now we shall that $T_{\theta+}(A_{(\alpha,\beta)}(P))$ is Translate of vague normal group of G , i.e.

$$T_{\theta+}(A_{(\alpha,\beta)}(xy)) = T_{\theta+}(A_{(\alpha,\beta)}(yx)) = (t_{A_{(\alpha,\beta)}T_{\theta+}}, f_{A_{(\alpha,\beta)}T_{\theta+}}).$$

Now

$$\begin{aligned} t_{A_{(\alpha,\beta)}T_{\theta+}}(xy) &\geq \min\{t_{A_{(\alpha,\beta)}}(xy) + \theta, 1\} = \min\{t_{A_{(\alpha,\beta)}}(yx) + \theta, 1\} \\ &\Rightarrow t_{A_{(\alpha,\beta)}T_{\theta+}}(xy) = t_{A_{(\alpha,\beta)}T_{\theta+}}(yx), \quad (1) \end{aligned}$$

$$\begin{aligned} f_{A_{(\alpha,\beta)}T_{\theta+}}(xy) &\leq \max\{f_{A_{(\alpha,\beta)}}(xy) - \theta, 0\} = \min\{f_{A_{(\alpha,\beta)}}(yx) - \theta, 0\} \\ &f_{A_{(\alpha,\beta)}T_{\theta+}}(xy) = f_{A_{(\alpha,\beta)}T_{\theta+}}(yx). \quad (2) \end{aligned}$$

From (1) and (2):

$$T_{\theta+}(A_{(\alpha,\beta)}(xy)) = T_{\theta+}(A_{(\alpha,\beta)}(yx)).$$

Therefore $T_{\theta+}(A_{(\alpha,\beta)}(P))$ is translate of vague normal group of G .

Theorem 3.3. Let G and G^1 be any two vague groups then the homomorphic image of translates of vague group A of G is a vague group of G^1 .

Proof. Let G and G^1 be any two groups and $\Phi : G \rightarrow G^1$ be a homomorphism.

Therefore $\Phi(x, y) = \Phi(x) \cdot \Phi(y)$ for all $x, y \in G$. Let $V = \Phi(T_{\alpha+})$ be a translates of a vague group of G . We shall show that $V = \Phi(T_{\alpha+})$ is a vague group of G^1 .

For $\Phi(x), \Phi(y) \in G^1$, we have

$$\begin{aligned} t_{PT_{\alpha+}}(\Phi(x)\Phi(y^{-1})) &\leq \min\{t_P\Phi(xy^{-1}) + \alpha, 1\} = \min\{t_P\{\Phi(x)(y^{-1}) + \alpha, 1\} \\ &= \min\{\{t_P\Phi(x) + \alpha, 1\}, \{t_P\Phi(y) + \alpha, 1\}\}. \end{aligned}$$

Therefore

$$t_{PT_{\alpha+}}(\Phi(x)\Phi(y^{-1})) \leq \min\{t_P\Phi(x), t_P\Phi(y)\}. \quad (1)$$

$$\begin{aligned} f_{PT_{\alpha+}}(\Phi(x)\Phi(y^{-1})) &\leq \max\{t_P\Phi(xy^{-1}) - \alpha, 0\} = \max\{f_P\{\Phi(x)(y^{-1}) - \alpha, 0\} \\ &= \max\{\{t_P\Phi(x) - \alpha, 0\}, \{t_P\Phi(y) - \alpha, 0\}\}. \end{aligned}$$

Therefore

$$f_{PT_{\alpha+}}(\Phi(x)\Phi(y^{-1})) \leq \max\{f_P\Phi(x), f_P\Phi(y)\}. \quad (2)$$

From (1) and (2): $V = \Phi(T_{\alpha+})$ is a translate of vague group of G^1 .

Theorem 3.4. *Let G and G^1 be any two vague groups. Then the homomorphic image of an translates of vague normal group A of G is an vague normal group of G^1 .*

Proof. Let G and G^1 be any two groups and $\Phi : G \rightarrow G^1$ be a homomorphism.

Let $V = \Phi(T_{\alpha+})$ where $T_{\alpha+}$ is a translation of vague normal group of A of G .

We shall show that $V = \Phi(T_{\alpha+})$ is a translates of vague normal group G^1 .

For any $\Phi(x), \Phi(y) \in G^1$, we have, for any $\Phi(x), \Phi(y)$ in G^1 ,

$$T_{\alpha+}\Phi(xy) = (t_PT_{\alpha+}\Phi(xy), f_PT_{\alpha+}\Phi(xy)).$$

Consider

$$\begin{aligned} t_PT_{\alpha+}\Phi(xy) &= \min\{t_{PT_{\alpha+}}\Phi(xy) + \alpha, 1\} = \min\{t_{PT_{\alpha+}}\Phi(yx) + \alpha, 1\} \\ \Rightarrow t_PT_{\alpha+}\Phi(xy) &= t_{PT_{\alpha+}}(\Phi(yx)T_{\alpha+}\Phi(xy)) = (t_PT_{\alpha+}\Phi(xy), f_PT_{\alpha+}\Phi(xy)). \end{aligned}$$

Moreover

$$f_P T\alpha + \Phi(xy) = \max\{f_{PT_{\alpha+}}\Phi(xy) - \alpha, 0\} = \max\{f_{PT_{\alpha+}}\Phi(yx) - \alpha, 0\}$$

$$\text{Hence } f_P T\alpha + \Phi(xy) = f_{PT_{\alpha+}}(\Phi(yx)).$$

Theorem 3.5. *Let G and G^1 be any two vague groups. Then the homomorphic pre-image of an translates of vague group V of G^1 is a vague group of G .*

Proof. Let G and G^1 be any two groups and $\Phi : G \rightarrow G^1$ be homomorphism.

Let $V = \Phi(T_{\alpha+})$ be translates of vague group of G^1 . We shall show that P is a vague group V of G .

Let $x, y \in G$. Then

$$\begin{aligned} V &= \Phi^{-1}(T_{\alpha+}) = (t_{AT_{\alpha+}}\phi^{-1}(x), f_{AT_{\alpha+}}\phi^{-1}(x)) \\ t_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) &= t_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) = t_{AT_{\alpha+}}(\Phi^{-1}(x)\Phi^{-1}(y^{-1})) \\ &\geq \min\{t_P[(\Phi^{-1}(x)\Phi^{-1}(y^{-1}))]+\alpha, 1\} = \min\{(t_P(\Phi^{-1}(x)+\alpha) \bigwedge (\Phi^{-1}(y^{-1})+\alpha, 1)\} \\ &\min\{(t_P(\Phi^{-1}(x) + \alpha) \bigwedge (\Phi^{-1}(y^{-1})) + \alpha, 1)\} \\ &= \min\{t_P\Phi^{-1}(x) + \alpha, 1\}, \{t_P\Phi^{-1}(y) + \alpha, 1\}\} \\ &\min\{t_P\Phi^{-1}(x) \bigwedge t_P\Phi^{-1}(y)\} = \min\{t_P(x)t_P(y)\}. \end{aligned}$$

Therefore

$$t_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) \geq \min\{t_P(x)t_P(y)\}. \quad (1)$$

$$\begin{aligned} f_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) &= f_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) = f_{AT_{\alpha+}}(\Phi^{-1}(x)\Phi^{-1}(y^{-1})) \\ &\leq \max\{f_P[(\Phi^{-1}(x)\Phi^{-1}(y^{-1}))]-\alpha, 0\} = \max\{(f_P(\Phi^{-1}(x)-\alpha) \bigwedge (\Phi^{-1}(y^{-1})-\alpha, 0)\} \\ &\max\{(f_P(\Phi^{-1}(x) - \alpha) \bigwedge (\Phi^{-1}(y^{-1})) - \alpha, 0)\} = \max\{\{f_P\Phi^{-1}(x) - \alpha, 0\}, \\ &\{f_P\Phi^{-1}(y) - \alpha, 0\}\} = \max\{f_P\Phi^{-1}(x) \bigwedge f_P\Phi^{-1}(y)\} = \max\{f_P(x), f_P(y)\}. \end{aligned}$$

Hence

$$f_{AT_{\alpha+}}\Phi^{-1}(xy^{-1}) \geq \max\{f_P(x), f_P(y)\}. \quad (2)$$

From (1) and (2) the homomorphic pre-image of an translates of vague group V of G^1 is a vague group of G .

Theorem 3.6. *Let G and G^1 be any two vague groups . Then the homomorphic pre-image of an translates of vague group of a vague normal group V of G^1 is translates of vague normal group of G .*

Proof. Let G and G^1 be any two groups and $\Phi : G \rightarrow G^1$ be a homomorphism.

Let $V = \Phi^{-1}(T_{\alpha+})$ where $T_{\alpha+}$ is a translates of vague operators of vague normal group of G . We shall show that P is vague normal group of G . Now, Let $x, y \in G \Rightarrow V = \Phi^{-1}(T_{\alpha+}) = (t_{PT_{\alpha+}}\phi^{-1}(x), f_{PT_{\alpha+}}\phi^{-1}(x))$.

Consider

$$\begin{aligned} t_{PT_{\alpha+}}\phi^{-1}(xy) &\geq \min\{t_P\phi^{-1}(xy) + \alpha, 1\} = \min\{t_P\phi^{-1}(yx) + \alpha, 1\} \\ &= t_{PT_{\alpha+}}\phi^{-1}(yx)t_{PT_{\alpha+}}\phi^{-1}(xy) = t_{PT_{\alpha+}}\phi^{-1}(yx) \end{aligned} \quad (1)$$

for all $x, y \in G$. Also

$$\begin{aligned} f_{PT_{\alpha+}}\phi^{-1}(xy) &\leq \max\{f_P^{-1}(xy) - \alpha, 0\} = \max\{f_P^{-1}(yx) - \alpha, 0\} = f_{PT_{\alpha+}}\phi^{-1}(yx) \\ f_{PT_{\alpha+}}\phi^{-1}(xy) &= f_{PT_{\alpha+}}\phi^{-1}(yx), \end{aligned} \quad (2)$$

for all $x, y \in G$.

Therefore from (1) and (2) the homomorphic pre-image of an translates of vague group of a vague normal group V of G^1 is translates of vague normal group of G .

Theorem 3.7. *If P is translate of vague normal group of G , then $K = \{x \in G : GV_P = GV_P(e)\}$ is translate of crisp normal group of G .*

Proof. Let P is a translates of vague normal group of G , let $x, y \in K \Rightarrow V_P(x) = V_P(e)$ and $V_P(y) = V_P(e)$. Now

$$\begin{aligned} t_{V_PT_{\alpha+}}(xy^{-1}) &\geq \min\{t_{V_P}(xy^{-1}) + \alpha, 1\} \\ &= \min\{\{t_{V_P}(x) + \alpha, 1\}, \{t_{V_P}(y^{-1}) + \alpha, 1\}\} = \min\{\{t_{V_P}(e) + \alpha, 1\}, \{t_{V_P}(e) + \alpha, 1\}\}, \\ t_{V_PT_{\alpha+}}(xy^{-1}) &\geq t_{V_P}(e). \end{aligned} \quad (1)$$

$$\begin{aligned} f_{V_PT_{\alpha+}}(xy^{-1}) &\leq \max\{f_{V_P}(xy^{-1}) - \alpha, 0\} \\ &= \max\{\{f_{V_P}(x) - \alpha, 0\}, \{f_{V_P}(y^{-1}) - \alpha, 0\}\} = \max\{\{f_{V_P}(e) - \alpha, 0\}, \{f_{V_P}(e) - \alpha, 0\}\}. \end{aligned}$$

Therefore

$$f_{V_PT_{\alpha+}}(xy^{-1}) \leq f_{V_P}(e). \quad (2)$$

Hence from (1) and (2) $V_{PT_{\alpha+}}$ is a vague crisp group of G .

Let $x \in G, y \in K$. Then

$$t_{PT_{\alpha+}}(xy) \geq \min\{t_P(xy) + \alpha, 1\} = \min\{t_P(yx) + \alpha, 1\}.$$

Therefore

$$t_{PT_{\alpha+}}(xy) = t_{PT_{\alpha+}}(yx).$$

Let $x \in G, y \in K$. Then

$$f_{PT_{\alpha+}}(xy) \leq \max\{f_P(xy) - \alpha, 0\} = \max\{f_P(yx) - \alpha, 0\}.$$

Therefore $f_{PT_{\alpha+}}(xy) = f_{PT_{\alpha+}}(yx)$.

Hence k is translates of crisp normal group of G .

Theorem 3.8. *Let P and Q be two translates vague normal group of a group G . Then $(P \cap Q)$ is translates of vague normal group of G .*

Proof. Since P and Q are a translates vague normal group of a group G and Now we shall so that $(P \cap Q)$ is a translates vague normal group of G

$$\begin{aligned} t_{(P \cap Q)T_{\alpha+}}(xy) &\geq \min\{t_{(P \cap Q)}(xy) + \alpha, 1\} \\ &= \min\{t_{(P \cap Q)}(yx) + \alpha, 1\} = t_{(P \cap Q)T_{\alpha+}}(yx), \end{aligned}$$

$$\begin{aligned} f_{(P \cap Q)T_{\alpha+}}(xy) &\leq \max\{f_{(P \cap Q)}(xy) + \alpha, 1\} \\ &= \max\{f_{(P \cap Q)}(yx) + \alpha, 1\} = f_{(P \cap Q)T_{\alpha+}}(yx). \end{aligned}$$

Therefore $(P \cap Q)$ is translates of vague normal group of G .

Theorem 3.9. *Let P be a translates of vague group of G . Then $K = \{x \in G : t_P(x) = t_P(e) \text{ and } f_P(x) = f_P(e)\}$ is a translates of vague normal group of $N(P)$.*

Proof. Let $x \in K$ and $y \in K \Rightarrow t_P(x) = t_P(e) \text{ and } f_P(x) = f_P(e)$. consider

$$\begin{aligned} t_{(P \cap Q)T_{\alpha+}}(xyx^{-1}) &\geq \min\{\{t_{(P \cap Q)}(xx^{-1}) + \alpha, 1\}t_{(P \cap Q)}(y) + \alpha, 1\} \\ &= \min\{\{t_{(P \cap Q)}(e) + \alpha, 1\}, \end{aligned}$$

$$t_{(P \cap Q)}(xyx^{-1}) + \alpha, 1\} = t_{(P \cap Q)}(e) = t_{(P \cap Q)}(xyx^{-1}).$$

Therefore $t_{(P \cap Q)T_{\alpha+}}(xyx^{-1}) = N(P)$ and

$$\begin{aligned} f_{(P \cap Q)T_{\alpha+}}(xyx^{-1}) &\leq \max\{\{f_{(P \cap Q)}(xx^{-1}) - \alpha, 0\} \\ &\quad f_{(P \cap Q)}(y) - \alpha, 0\}\} = \max\{\{t_{(P \cap Q)}(e) - \alpha, 0\} \\ &\quad f_{(P \cap Q)}(xyx^{-1}) - \alpha, 0\} = f_{(P \cap Q)}(e) = f_{(P \cap Q)}(xyx^{-1}) \end{aligned}$$

Therefore $f_{(P \cap Q)T_{\alpha+}}(xyx^{-1}) = N(P)$, $K = \{x \in G : t_P(x) = t_P(e), \text{ and } f_P(x) = f_P(e)\}$ is a translates of vague normal group of $N(P)$.

Theorem 3.10. *Let P be a translates vague group of a group G , and $K = \{x \in G | V_P(x) = V_P(e)\}$ is translates of vague normal group of G . Then $K \subseteq C(P)$.*

Proof. Let $x \in K$. Then

$$V_P(x) = V_P(e) \Rightarrow t_P(x) = t_P(e)$$

and

$$f_P(x) = f_P(e) \text{ for all } x, y \in G.$$

$$\begin{aligned} t_{PT_{\theta+}}[x, y] &= \min\{t_P[x, y] + \theta, 1\} = \min\{t_P(x^{-1}y^{-1}xy) + \theta, 1\} \\ &\geq \min\{\{t_P(x^{-1}x) + \theta, 1\}, \end{aligned}$$

$$\begin{aligned} &\quad t_P(yy^{-1}) + \theta, 1\}\} = \min\{\{t_P(e) + \theta, 1\} \\ &\quad t_P(e) + \theta, 1\}\} = \min\{t_P(e), t_P(e)\} = t_P(e). \end{aligned}$$

Therefore

$$t_{PT_{\theta+}}[x, y] = t_P(e), \tag{1}$$

$$\begin{aligned} f_{PT_{\theta+}}[x, y] &= \max\{f_P[x, y] - \theta, 0\} = \max\{f_P(x^{-1}y^{-1}xy) - \theta, 0\} \\ &\leq \max\{\{f_P(x^{-1}x) - \theta, 0\}, \end{aligned}$$

$$\begin{aligned} &\quad f_P(yy^{-1}) - \theta, 0\}\} = \max\{\{f_P(e) - \theta, 0\} \\ &\quad f_P(e) - \theta, 0\}\} = \max\{f_P(e), f_P(e)\} = f_P(e). \end{aligned}$$

Therefore

$$t_{PT_{\theta+}}[x, y] = t_P(e). \tag{2}$$

Hence from (1) and (2) it follows $K \subseteq C(P)$.

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