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ON ALMOST WO-CONTINUOUS FUNCTIONS ON ASSOCIATED w-SPACES

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Abstract: We introduce the notion of almost WO-continuity and study some characterizations and properties of such functions. In particular, we study the relation among WK-continuity, weakly WO-continuity and almost WO-continuity.

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1. Introduction

In [19], Siwiec introduced the notions of weak neighborhoods and weak base in a topological space. We introduced the weak neighborhood systems defined by using the notion of weak neighborhoods in [13]. And we also introduced a weak neighborhood space (briefly WNS) which is independent of neighborhood spaces [5] and general topological spaces [2]. We introduced the notion of w-spaces in [14] and investigated some basic properties. In [15], we introduced and studied the notions WK-continuity and WO-continuity on associated w-

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spaces. In this paper, we introduce the notion of almost WO-continuity and study some characterizations and properties of such functions.

2. Preliminaries

Let S be a subset of a topological space X. The closure (resp., interior) of S will be denoted by $Cl(S)$ (resp., $Int(S)$). A subset S of X is called a *preopen* set [11] (resp., α-set, β-open set [15], semi-open [7]) if $S \subset Int(Cl(S))$ (resp., $S \subset Int(Cl(Int(S)))$, $S \subset Cl(Int(Cl(S)))$, $S \subset Cl(Int(S$. The complement of a preopen set (resp., α-set, β-open set, semi-open) is called a *preclosed* set (resp., α-closed set, β-closed set, semi-closed). A subset A of X is said to be *quasi H-closed* relative to X [20] if every collection $\{U_i : i \in J\}$ of open subsets of X such that $A \subset \cup\{U_i : i \in J\}$, there exists a finite subset J_0 of J such that $A \subset \cup\{Cl(U_i) : i \in J_0\}$.

A subset A of a topological space (X, τ) is said to be:

(a) g-closed [6] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X ;
(b) gp-closed [8] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X;
(c) gs-closed [1, 4] if $sCl(A) \subset U$ whenever $A \subset U$ and U is open in X;
(d) $g\alpha$-closed [10] if $\tau^\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is α-open in X where $\tau^\alpha = \alpha(X)$;
(e) $g\alpha^{\square}$-closed [9] if $\tau^\alpha Cl(A) \subset Int(U)$ whenever $A \subset U$ and U is α-open in X;
(f) $g\alpha^{\square\square}$-closed [9] if $\tau^\alpha Cl(A) \subset Int(Cl(U))$ whenever $A \subset U$ and U is α-open in X;
(g) αg-closed [10] if $\tau^\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X;
(h) $\alpha^{\square\square}g$-closed [10] if $\tau^\alpha Cl(A) \subset Int(Cl(U))$ whenever $A \subset U$ and U is open in X;
(i) g-open (resp., gp-open, gs-open, $g\alpha$-open, $g\alpha^{\square}$-open, $g\alpha^{\square\square}$-open, αg-open, $\alpha^{\square\square}g$-open) if the complement A is g-closed (resp., gp-closed, gs-closed, $g\alpha$-closed, $g\alpha^{\square}$-closed, $g\alpha^{\square\square}$-closed, αg-closed, $\alpha^{\square\square}g$-closed).

The family of all g-open (resp., gp-open, gs-open, $g\alpha$-open, $g\alpha^{\square}$-open, $g\alpha^{\square\square}$-open, αg-open, $\alpha^{\square\square}g$-open) sets in X will be denoted by $gO(X)$ (resp., $gpO(X), gsO(X), g\alpha O(X), g\alpha^{\square}O(X), g\alpha^{\square\square}O(X), \alpha gO(X), \alpha^{\square\square}gO(X)$.

Definition 2.1 ([14]). Let X be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a *weak structure* on X if it satisfies the following:
(1) $\emptyset \in w_X$ and $X \in w_X$.

(2) For $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$.

Then the pair (X, w_X) is called a *w-space* on X. Then $V \in w_X$ is called a *w-open* set and the complement of a w-open set is a *w-closed* set.

Let (X, τ) be a topological space. Then the family τ, $GO(X)$, $g\alpha O(X)$, $g\alpha^{*}O(X)$, $g\alpha^{**}O(X)$, $\alpha gO(X)$ and $\alpha^{**}gO(X)$ on X are all weak structures on X. But $PO(X)$, $GPO(X)$ and $SO(X)$ are not weak structures on X.

Definition 2.2 ([14]). Let (X, w_X) be a w-space. For a subset A of X, the *w-closure* of A and the *w-interior* of A are defined as the following:

(1) $wCl(A) = \cap\{F : A \subset F, X - F \in w_X\}$.

(2) $wInt(A) = \cup\{U : U \subset A, U \in w_X\}$.

Theorem 2.3 ([14]). *Let (X, w_X) be a w-space and $A \subset X$.*

(1) If A is w-open, then $wInt(A) = A$.

(2) If A is w-closed, then $wCl(A) = A$.

Theorem 2.4 ([14]). *Let (X, w_X) be a w-space and $A, B \subset X$.*

(1) $X = wInt(X)$ and $\emptyset = wCl(\emptyset)$.

(2) $wInt(A) \subset A$ and $A \subset wCl(A)$.

(3) $wInt(A \cap B) = wInt(A) \cap wInt(B)$ and $wCl(A \cup B) = wCl(A) \cup wCl(B)$.

(4) $wInt(wInt(A)) = wInt(A)$ and $wCl(wCl(A)) = wCl(A)$.

(5) $wCl(X - A) = X - wInt(A)$ and $wInt(X - A) = X - wCl(A)$.

Let X be a nonempty set and let (X, τ) be a topological space. A subfamily w_τ of the power set $P(X)$ is called an associated *weak structure* [15] on X if $\tau \subseteq w_\tau$. Then the pair (X, w_τ) is called an associated *w-space* with τ.

Let $f : (X, w_X) \to (Y, \mu)$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then f is said to be

(1) *weakly WO-continuous* [16] if for $x \in X$ and for each open subset V containing $f(x)$, there is a w-open subset U containing x such that $f(U) \subset Cl(V)$,

(2) *WO-continuous* [15] if for $x \in X$ and for each open subset V containing $f(x)$, there is a w-open subset U of X containing x such that $f(U) \subset V$,

(3) *WK-continuous* [15] if for every open set V in Y, $f^{-1}(V)$ is a w-open set in X.

A subset A of a w-space (X, w_X) is called *W-compact* [16] relative to A if every collection $\{U_i : i \in J\}$ of w-open subsets of X such that $A \subset \cup\{U_i : i \in J\}$, there exists a finite subset J_0 of J such that $A \subset \cup\{U_i : i \in J_0\}$. A subset A

of a w-space (X, w_X) is said to be *W-compact* if A is W-compact as a subspace of X.

3. Almost WO-Continuous Functions

Definition 3.1. Let (X, w_X) be an associated w-space and (Y, μ) a topological space. Then $f : X \to Y$ is said to be *almost WO-continuous* if for $x \in X$ and for each open set V containing $f(x)$, there is a w-open set U containing x such that $f(U) \subseteq Int(Cl(V))$.

We get the following implications but the converses are not true from Example 3.2 in [16] and the next example:

continuous $\Rightarrow$ WK-continuous $\Rightarrow$ WO-continuous $\Rightarrow$ almost WO-continuous $\Rightarrow$ weakly WO-continuous

Example 3.2. In Example 3.2 [16], the function f is almost WO-continuous but it is not WO-continuous. Next, let $X = \{a, b, c, d, e\}$ and $Y = \{a, b, c, d\}$. Consider a weak structure $w_X = \{\emptyset, \{a, b, c\}, \{d\}, \{e\}, X\}$ and a topological space $\mu = \{\emptyset, \{d\}, \{a, b\}, \{a, b, d\}, Y\}$. Let $f : (X, w_X) \to (Y, \mu)$ be a function defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ and $f(d) = f(e) = d$. Then f is weakly WO-continuous. But for $a \in X$ and $V = \{a, b\}$ containing $f(a)$, there is no any w-open set U in X such that $f(U) \subseteq Int(Cl(V)) = \{a, b\}$. Hence f is not almost WO-continuous.

Theorem 3.3. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then the following statements are equivalent:*

(1) f is almost WO-continuous at $x \in X$.

(2) $x \in wInt(f^{-1}(Int(Cl(V))))$ for every open set V containing $f(x)$.

(3) $x \in wInt(f^{-1}(V))$ for every regular open set V containing $f(x)$.

(4) For each regular open set V containing $f(x)$, there is a w-open set U containing x such that $f(U) \subseteq V$.

Proof. (1) $\Rightarrow$ (2) Let V be any open set in Y containing $f(x)$. By (1), there exists a w-open set U of X containing x such that $f(x) \in f(U) \subseteq Int(Cl(V))$. Since $x \in U \subseteq f^{-1}(Int(Cl(V)))$, we have $x \in wInt(f^{-1}(Int(Cl(V))))$.

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (4) Let V be any regular open set in Y containing $f(x)$. By (3), it is $x \in wInt(f^{-1}(V))$ and so there exists a w-open set U of X containing x such that $x \in U \subseteq f^{-1}(V)$. Thus $f(U) \subseteq V$

(4) $\Rightarrow$ (1) Let V be any open set in Y containing $f(x)$. Then $f(x) \in f(V) \subseteq Int(Cl(V))$. Since $Int(Cl(V))$ is regular open, there exists a w-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Hence f is almost WO-continuous at $x \in X$. □

Theorem 3.4. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then the following statements are equivalent:*

(1) f is almost WO-continuous.

(2) $f^{-1}(V) \subseteq wInt(f^{-1}(Int(Cl(V))))$ for every open set V of Y.

(3) $wCl(f^{-1}(Cl(Int(F)))) \subseteq f^{-1}(F)$ for every closed set F of Y.

(4) $wCl(f^{-1}(Cl(Int(Cl(B))))) \subseteq f^{-1}(Cl(B))$ for every set B of Y.

(5) $f^{-1}(Int(B)) \subseteq wInt(f^{-1}(Int(Cl(Int(B)))))$ for every set B of Y.

(6) $f^{-1}(V) = wInt(f^{-1}(V))$ for every regular open set V of Y.

(7) $f^{-1}(F) = wCl(f^{-1}(F))$ for every regular closed set F of Y.

Proof. (1) $\Rightarrow$ (2) Let V be an open set in Y and $x \in f^{-1}(V)$. There exists a w-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Since $x \in U \subseteq f^{-1}(Int(Cl(V)))$, we have $x \in wInt(f^{-1}(Int(Cl(V))))$. Hence $f^{-1}(V) \subseteq wInt(f^{-1}(Int(Cl(V))))$.

(2) $\Rightarrow$ (3) Let F be a closed set in Y. Then $Y - F$ in open in Y and, by (2) and Theorem 2.4,

$f^{-1}(Y - F) \subseteq wInt(f^{-1}(Int(Cl(Y - F)))) = wInt(f^{-1}(Y - Cl(Int(F)))) = X - wCl(f^{-1}(Cl(Int(F))))$.

Thus $wCl(f^{-1}(Cl(Int(F)))) \subseteq f^{-1}(F)$.

(3) $\Rightarrow$ (4) It is obvious.

(4) $\Rightarrow$ (5) Let B be a set of Y. Then by (4) and Theorem 2.4,

$f^{-1}(Int(B)) = X - f^{-1}(Cl(Y - B)) \subseteq X - wCl(f^{-1}(Cl(Int(Cl(Y - B))))) = wInt(f^{-1}(Int(Cl(Int(B)))))$. Thus we get the result.

(5) $\Rightarrow$ (6) Let V be any regular open set of Y. Since $Int(Cl(Int(V))) = V$, from (5), it follows $f^{-1}(V) \subseteq wInt(f^{-1}(V))$ and so $f^{-1}(V) = wInt(f^{-1}(V))$.

(6) $\Rightarrow$ (7) Let F be any regular closed set of Y. Then $Y - F$ is any regular open and by (6), we have

$X - f^{-1}(F) = f^{-1}(Y - F) = wInt(f^{-1}(Y - F)) = X - wCl(f^{-1}(F))$.

(7) $\Rightarrow$ (1) Let V be any regular open set of Y containing $f(x)$. By (7), $X - f^{-1}(V) = f^{-1}(Y - V) = wCl(f^{-1}(Y - V)) = X - wInt(f^{-1}(V))$. Since $x \in f^{-1}(V) = wInt(f^{-1}(V))$, there exists a w-open set U containing x such that $U \subseteq f^{-1}(V)$. Hence by Theorem 3.3(4), f is almost WO-continuous. □

Theorem 3.5. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then the following statements are equivalent:*

(1) f is almost WO-continuous.

(2) $wCl(f^{-1}(G))) \subseteq f^{-1}(Cl(G))$ for every β-open set G of Y.

(3) $wCl(f^{-1}(G))) \subseteq f^{-1}(Cl(G))$ for every semiopen set G of Y.

(4) $f^{-1}(G) \subseteq wInt(f^{-1}(Int(Cl(G))))$ for every preopen set G of Y.

Proof. (1) $\Rightarrow$ (2) Let G be any β-open set. Since every β-open set is regular closed, by Theorem 3.4(7), it is obtained $f^{-1}(Cl(G)) = wCl(f^{-1}(Cl(G)))$. Thus $wCl(f^{-1}(G))) \subseteq wCl(f^{-1}(Cl(G))) = f^{-1}(Cl(G))$.

(2) $\Rightarrow$ (3) It is obvious since every semiopen set is β-open.

(3) $\Rightarrow$ (1) Let F be any regular closed set of Y; then since F is semiopen, we have $wCl(f^{-1}(F))) \subseteq f^{-1}(Cl(F)) = f^{-1}(F)$. Thus from Theorem 3.4(7), it follows f is almost WO-continuous.

(1) $\Rightarrow$ (4) Let V be any preopen set of Y; then $V \subseteq Int(Cl(V))$ and $Int(Cl(V))$ is regular open, by Theorem 3.4(6),

$$f^{-1}(Int(Cl(V))) = wCl(f^{-1}(Int(Cl(V)))).$$

Thus we have $f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = wCl(f^{-1}(Int(Cl(V))))$.

(4) $\Rightarrow$ (1) Let V be any regular open set of Y; then V is preopen and $f^{-1}(V) \subseteq wInt(f^{-1}(Int(Cl(V)))) = wInt(f^{-1}(V))$.

Hence by Theorem 3.4(6), f is almost WO-continuous. □

We recall that a point x of a topological space X is said to be δ-*cluster point of A if $A \cap Int(Cl(V)) \neq \emptyset$ for every open set V containing x. The set of all δ-cluster points of A is called δ-closure of A [20] and is denoted by $Cl_\delta(A)$. If $A = Cl_\delta(A)$, then A is called δ-closed. The complement of a δ-closed set is said to be δ-open. It is shown in [20] that $Cl(A) = Cl_\delta(A)$ for every open set A and $Cl_\delta(B)$ is closed for every subset B of X.*

Theorem 3.6. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then the following statements are equivalent:*

(1) f is almost WO-continuous.
(2) $wCl(f^{-1}(Cl(Int(Cl_\delta(B)))) \subseteq f^{-1}(Cl_\delta(B))$ for every set B of Y.
(3) $wCl(f^{-1}(Cl(Int(Cl(B)))) \subseteq f^{-1}(Cl_\delta(B))$ for every set B of Y.
(4) $wCl(f^{-1}(Cl(Int(Cl(G)))) \subseteq f^{-1}(Cl(G))$ for every open subset G of Y.
(5) $wCl(f^{-1}(Cl(Int(Cl(G)))) \subseteq f^{-1}(Cl(G))$ for every preopen subset G of Y.

Proof. (1) $\Rightarrow$ (2) Let B be any subset in Y; then $Cl_\delta(B)$ is closed, by Theorem 3.4 (3), we get the result.

(2) $\Rightarrow$ (3) It is obvious since $Cl(B) \subseteq Cl_\delta(B)$ for every subset B of Y.

(3) $\Rightarrow$ (4) It is obvious since $Cl(G) = Cl_\delta(G)$ for every open subset G of Y.

(4) $\Rightarrow$ (5) Let G be preopen in Y; then $Cl(G) = Cl(Int(Cl(G)))$. By (4),
$wCl(f^{-1}(Cl(Int(Cl(G))))) = wCl(f^{-1}(Cl(Int(Cl(Int(Cl(G))))))$
$\subseteq f^{-1}(Cl(Int(Cl(G)))) = f^{-1}(Cl(G))$.

(5) $\Rightarrow$ (1) Let A be any regular closed set of Y; then since $Int(A)$ is a preopen set, by (5), we have the following:
$wCl(f^{-1}(A)) = wCl(f^{-1}(Cl(Int(A)))) = wCl(f^{-1}(Cl(Int(Cl(Int(A)))))) \subseteq f^{-1}(Cl(Int(A))) = f^{-1}(A)$. Then $wCl(f^{-1}(A)) = f^{-1}(A)$ and so by Theorem 3.4(6), f is almost WO-continuous. □

Theorem 3.7. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). Then the following statements are equivalent:*
(1) f is almost WO-continuous.
(2) $f(wCl(A)) \subseteq Cl_\delta(f(A))$ for every set A of X.
(3) $f^{-1}(F) = wCl(f^{-1}(F))$ for every δ-closed set F of Y.
(4) $f^{-1}(G) = wInt(f^{-1}(G))$ for every δ-open set G of Y.
(5) $f^{-1}(Int_\delta(B)) \subseteq wInt(f^{-1}(B))$ for every set B of Y.
(6) $wCl(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$ for every set B of Y.

Proof. (1) $\Rightarrow$ (2) Let $x \in wCl(A)$ for $A \subseteq X$ and let V be any open set of Y containing $f(x)$. There exists a w-open set U containing x such that $f(U) \subseteq Int(Cl(V))$. Since $x \in wCl(A)$, $U \cap A \neq \emptyset$ and so $\emptyset \neq f(U) \cap f(A) \subseteq Int(Cl(V)) \cap f(A)$. Thus $f(x) \in Cl_\delta(f(A))$.

(2) $\Rightarrow$ (3) Let F be any δ-closed set of Y. Then
$f(wCl(f^{-1}(F))) \subseteq Cl_\delta(f(f^{-1}(F))) \subseteq Cl_\delta(F) = F$. Hence $wCl(f^{-1}(F)) \subseteq f^{-1}(F)$.

(3) $\Rightarrow$ (4) Let G be any δ-open set of Y. Then $Y - G$ is a δ-closed set and by (3) and Theorem 2.4,

$X - f^{-1}(G) = f^{-1}(Y - G) = wCl(f^{-1}(Y - G)) = X - wInt(f^{-1}(G))$. Hence $f^{-1}(G) = wInt(f^{-1}(G))$.

(4) ⇒ (5) Let B be any set in Y. Then $Int_\delta(B)$ is a δ-open set of Y and so $f^{-1}(Int_\delta(B)) = wInt(f^{-1}(Int_\delta(B))) \subseteq wInt(f^{-1}(B))$.

(5) ⇒ (6) Let B be a set of Y; then from (5) and Theorem 2.4, it follows that

$f^{-1}(Cl_\delta(B)) = X - f^{-1}(Int_\delta(Y - B)) \supseteq X - (wInt(f^{-1}(Y - B))) = wCl(f^{-1}(B))$.

(6) ⇒ (1) Let B be a set of Y. From (6) and $Cl_\delta(B)$ is closed in Y, we have

$wCl(f^{-1}(Int(Cl_\delta(B)))) \subseteq f^{-1}(Cl_\delta(Int(Cl_\delta(B)))) = f^{-1}(Cl(Int(Cl_\delta(B)))) \subseteq f^{-1}(Cl_\delta(B))$. Hence by Theorem 3.6(2), f is almost WO-continuous. □

Definition 3.8. Let (X, w_X) be an associated w-space and (Y, μ) a topological space. Then $f : X \to Y$ is said to be *almost WO-open* if $f(U) \subseteq Int(Cl(f(U)))$ for every w-open set U in X.

Theorem 3.9. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). If f is an almost WO-open and weakly WO-continuous function, then f is almost WO-continuous.*

Proof. For $x \in X$, let V be an open set containing $f(x)$ in Y. Since f is weakly WO-continuous, there exists a w-open set U containing x such that $f(U) \subseteq Cl(V)$. Since f is almost WO-open, $f(U) \subseteq Int(Cl(f(U)))$ and so $f(U) \subseteq Int(Cl(f(U))) \subseteq Int(Cl(V))$. Hence f is almost WO-continuous. □

A topological space X is said to be

(1) almost-regular [18] if for each regular closed set F of X and each point $x \in X - F$ there exist disjoint open sets U and V of X such that $x \in U$ and $F \subseteq V$,

(2) semi-regular [3] if for each open set U of X and each point $x \in U$ there exists a regular open set V of X such that $x \in V \subseteq U$.

Theorem 3.10. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). If f is almost WO-continuous and Y is semi-regular, then f is WO-continuous.*

Proof. For $x \in X$, let V be an open set containing $f(x)$ in Y. Since Y is semi-regular, there exists a regular open set G in Y such that $f(x) \in G \subseteq V$.

Since f is almost WO-continuous, there exists an w-open set U containing x such that $f(U) \subseteq Int(Cl(G)) \subseteq V$. Hence f is WO-continuous. □

Theorem 3.11. *Let $f : X \to Y$ be a function on an associated w-space (X, w_X) and a topological space (Y, μ). If f is weakly WO-continuous and Y is almost-regular, then f is almost WO-continuous.*

Proof. For $x \in X$, let V be an open set containing $f(x)$ in Y. Since Y is almost-regular, there exists a regular open set G in Y such that $f(x) \in G \subseteq Cl(G) \subseteq Int(Cl(V))$. Since f is weakly WO-continuous, there exists an w-open set U containing x such that $f(U) \subseteq Cl(G) \subseteq Int(Cl(V))$. Hence f is almost w-continuous. □

Theorem 3.12. *Let (X, w_X) be W-compact and (Y, μ) Urysohn. If $f : X \to Y$ is a weakly WO-continuous surjection, then f is almost WO-continuous.*

Proof. Let $\{V_i : i \in J\}$ be a cover of Y by open subsets of Y. For each $x \in X$, there exists $i(x) \in J$ such that $f(x) = y \in V_{i(x)}$. Since f is weakly WO-continuous, there exists a w-open set $U(x)$ containing x such that $f(U(x)) \subseteq Cl(V_{i(x)})$. The family $\{U(x) : x \in X\}$ is a cover of X by w-open sets in X. Since X is w-compact, there is a finite subcover $\{U(x_1), U(x_2), \cdots, U(x_n) : x_j \in X, j = 1, 2, \cdots, n\}$ such that $X \subseteq \cup U(x_j)$. Then

$$Y \subseteq f(\cup U(x_j)) \subseteq \cup f(U(x_j)) \subseteq \cup Cl(V_{i(x_j)}),$$

$1 \leq j \leq n$.

Thus Y is quasi H-closed. Since every quasi H-closed and Urysohn space is almost-regular, by Theorem 3.10, f is almost WO-continuous. □

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