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THE DETERMINATION OF HESITATION VALUE FOR SUGENO TYPE INTUITIONISTIC FUZZY GENERATOR VIA FUZZY LIMIT

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Abstract: In intuitionistic fuzzy set (IFS) theory, the membership and non-membership values are needed to model the uncertainties. One of the ways to compute the non-membership is by implementing the Sugeno type intuitionistic fuzzy generator. In this paper, fuzzy limit will be used to determine the value of parameter, namely λ, in the non-membership function. Different values of λ will directly control the level of hesitation, namely π, in the output image. Hence, the output images of Flat EEG (fEEG) during epileptic seizures are enhanced by using the window based enhancement scheme.

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1. Introduction

Medical imaging is one of the applications in image processing that deal with medical images. It is a fast-growing field that has attracted wide attention among researchers to explore new techniques in order to obtain better images for further interpretation. Medical image itself contains a lot of uncertainties. Therefore, much effort has been carried out by various researchers to enhance ambiguous medical images.

One of the ways in handling these uncertainties is by implementing the fuzzy concept which was introduced by Zadeh in 1965. Atanassov introduced the extension of the ordinary fuzzy set which is known as IFS. The advantage of IFS compared to the ordinary fuzzy set is that it considers more uncertainties in terms of membership and non-membership functions [1].

According to [2], there are five main steps that need to be considered in the framework of intuitionistic fuzzy image processing which are fuzzification, intuitionistic fuzzification, modification of intuitionistic fuzzy components, intuitionistic defuzzification, and defuzzification.

2. Basic Terminologies

This section presents some important terminologies that are used in this paper.

2.1. Fuzzy Topographic Topological Mapping (FTTM)

FTTM is a novel non-invasive technique for solving neuromagnetic inverse problem. This technique was introduced by [3] to accommodate static simulated, experimental magnetoencephalography (MEG) and recorded electroencephalography (EEG) signals. MEG works in such a manner to record the magnetic field.

FTTM consists of three algorithms that link four components of the model as shown in Figure 1 [4]

2.2. Electroencephalography (EEG)

EEG (see Figure 2) is a system that measures and records the electrical activity of the brain in graphic form [5]. According to [6], it is a method of visualizing physiology to discover the hidden causes of epilepsy. It reads voltage differences on the head, relative to a given point.

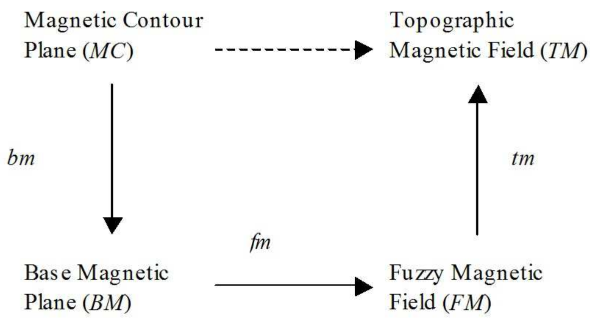

Figure 1: FTTM

Hence, if the electrical activity between two hemispheres is to be ascertained, then one will need to place three electrodes, one on each hemisphere, and another in the centre, connected to both electrodes. This will give an absolute difference between hemispheric brain activities [7].

Figure 2: An EEG System [8]

A sample of EEG signal during seizure is given in Figure 3.

2.3. Flat EEG (fEEG)

fEEG (see Figure 4) is a method for mapping high dimensional signal, namely EEG into a low dimensional space (Magnetic Contour)[9].

Developed by Fuzzy Research Group of UTM in 1999, fEEG method has been used purely for visualization [10]. The main scientific value lies in the ability of flattening method to preserve information recorded during seizure. The jewel of the fEEG method is that EEG signals can be compressed and analyzed.

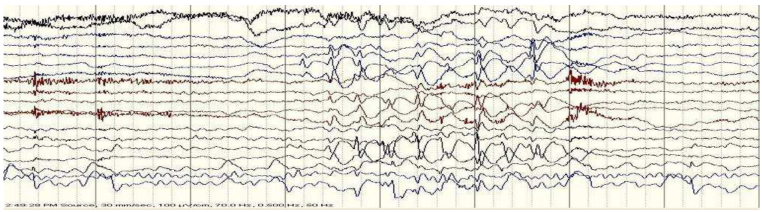

Figure 3: EEG Signal

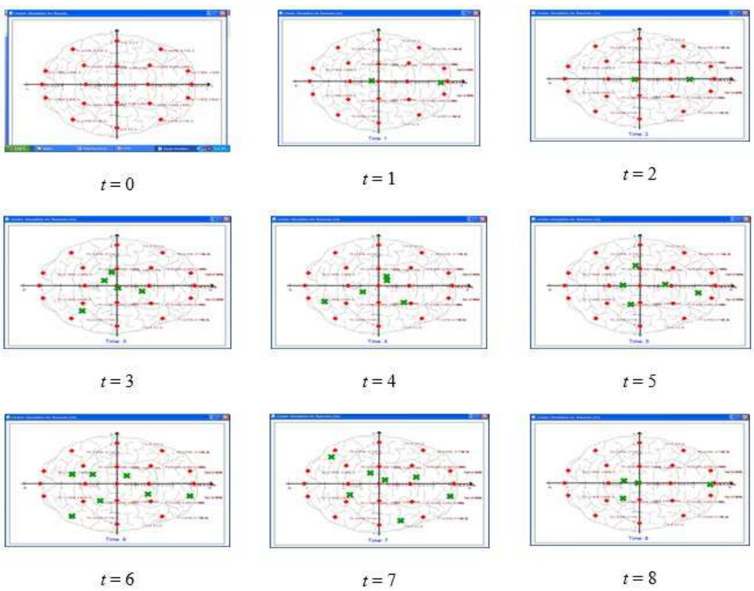

Figure 4: fEEG

Fauziah's fEEG coordinate system [10] (see Figure 5) is defined as

$$C_{EEG} = \{((x, y, z), e_p) : \quad x, y, z, e_p \in \Re \text{ and } x^2 + y^2 + z^2 = r^2\} \tag{1}$$

where r is the radius of a patient's head.

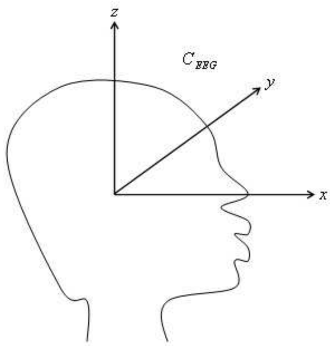

Figure 5: EEG Coordinate System

Furthermore, the mapping of C_{EEG} to a plane is defined as follows. $S_t : C_{EEG} \longrightarrow MC$ (see Figure 6) such that

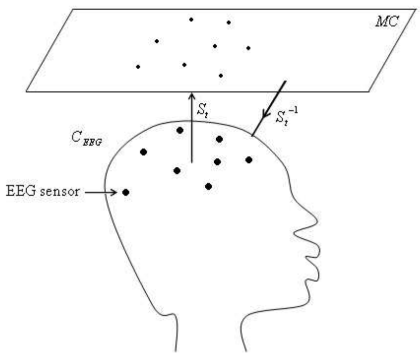

Figure 6: EEG Projection

$$S_t((x, y, z), e_p) = (\frac{rx + iry}{r + z}, e_p) = (\frac{rx}{r + z}, \frac{ry}{r + z})_{e_p(x,y,z)} \tag{2}$$

where $MC = \{((x, y)_0, e_p) : x, y, e_p \in \Re\}$ is the first component of FTTM. Both C_{EEG} and MC were designed and proven as 2-manifolds [10].

Meanwhile S_t is designed to be a one to one function as well as being conformal. Details of the proofs are contained in [10].

The EEG signal during seizure (see Fig. 3) can be compressed to Figure 7 and analyzed second by second as shown in Figure 8.

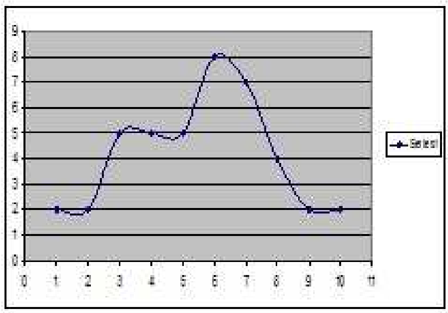

Figure 7: Compressed EEG Signal

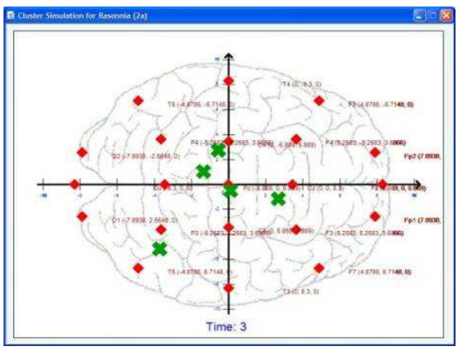

Figure 8: Analyzed EEG Signal

Moreover, in the previous study by [10], the EEG signal was transformed into low dimensional space, known as fEEG, via the flattening method. Then the fEEG was transformed into image by using fuzzy approach [11, 12]. This procedure is summarized in Figure 9 as follows

There are three main steps that are involved in the transformation of fEEG into fEEG image [11, 12].

1. fEEG is divided into pixels as seen in Figure 10

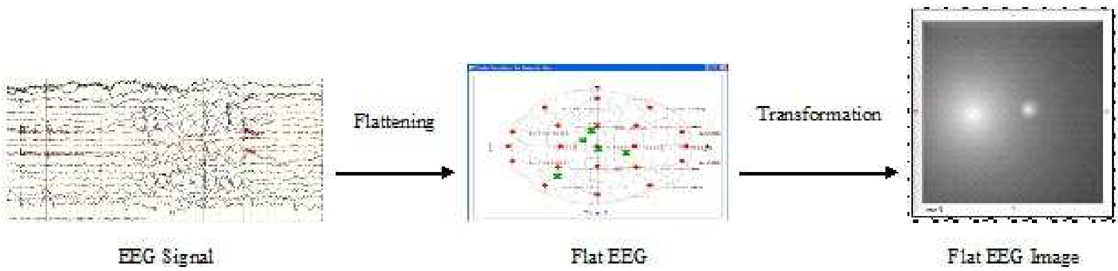

Figure 9: Transformation of EEG Signal

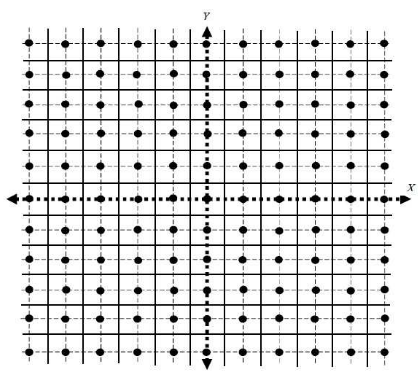

Figure 10: fEEG pixels

2. The membership value for each pixel is determined in a cluster centre and the maximum operator of fuzzy set is implemented (see Figure 11).

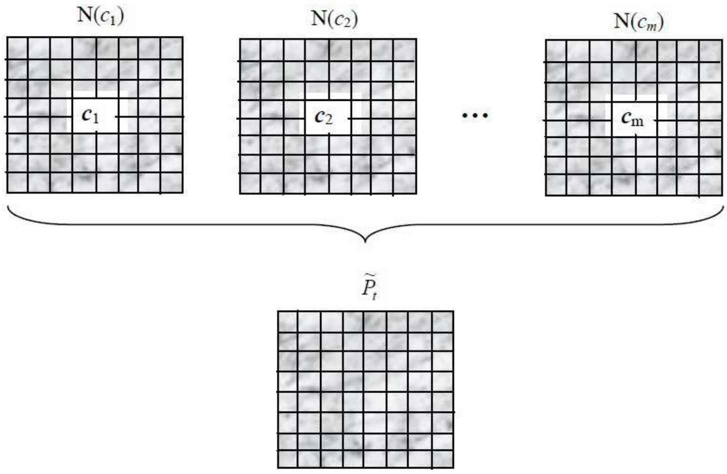

Figure 11: Fuzzy neighborhood of each cluster centre c_j of a fEEG

3. The membership value of pixel is transformed into image data (see Figure 12).

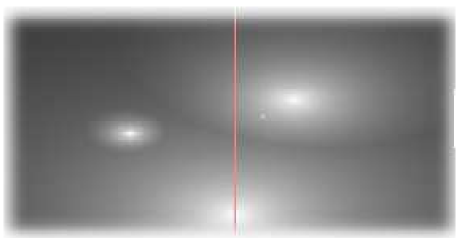

Figure 12: fEEG image

3. Methodology

In the process of imaging and transformations such as fEEG, it is hard to avoid the inheritance of different kinds of noise during recording of the EEG signals.

Since the regions of clusters in fEEG are not always defined, uncertainty can arise within every transformation.

Thus, in the presence of noise, pre-processing steps such as image enhancement are needed. The objectives of image enhancement are to remove noise, to smooth non impulsive noise, and to enhance the edges or other salient structures on fEEG. Thus, in this paper, the image of fEEG has been enhanced by using intuitionistic fuzzy sets approach.

Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an IFS in a finite set $X = \{x_1, x_2, ..., x_n\}$ whereby $\mu_A(x), \nu_A(x) : X \longrightarrow [0, 1]$ represent the membership and non-membership respectively. The necessary conditions that must be fulfilled are as follows

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

such that $0 \leq \pi_A(x) \leq 1$ [1, 13].

In IFS, the sum of $\mu_A(x)$ and $\nu_A(x)$ will not always equal to one as in the ordinary fuzzy set. Therefore, there exist parameter π that makes the summation equal to 1.

The occurrence of π is due to the lack of knowledge about the membership degree or personal error while calculating the distances between two fuzzy sets. Therefore, the membership value will lie in the interval $[\mu_A(x) - \pi_A(x), \mu_A(x) + \pi_A(x)]$ because of the hesitation, π, that occurs in the membership function [1, 13].

Furthermore, in order to enhance the image of fEEG, a window based enhancement scheme is applied during epileptic seizures at time 1 of size 201x201. However, there is a slight difference in the initial step with the proposed step by [13]. In [13], the image is initially divided into four windows and fuzzification is carried out for each window. On the other hand, in this work the fEEG image is initially undergoing the fuzzification process based on [11] which is applied to the entire image. After that, the image is divided into four windows.

The details of the algorithm is given in the following steps [13]

1. The input image (see Figure 13) is fuzzified by using the maximum operator of fuzzy set.

2. The image is divided into four windows and enhancement is carried out for each window.

3. The non-membership function is computed by using the Sugeno type intuitionistic fuzzy generator as follows:
$$\nu_A(g_{ij}) = \frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})} \tag{5}$$

4. The hesitation degree is calculated as
$$\pi_A(g_{ij}) = 1-\mu_A(g_{ij}) - \frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})} \tag{6}$$

5. The mean of each window is calculated

6. The modified membership value is given by
$$\mu_A{}^{mod}(g_{ij}) = \mu_A(g_{ij}) - mean\ window \times \pi_A(g_{ij}) \tag{7}$$

7. The contrast enhancement is applied to each window by using the intensifier operator
$$\mu_A{}^{enh}(g_{ij}) = \begin{cases} 2\left[\mu_A{}^{mod}(g_{ij})\right]^2 & \text{if} \quad \mu_A{}^{mod}(g_{ij}) \le 0.5 \\ 1-2\left[1-\mu_A{}^{mod}(g_{ij})\right]^2 & \text{if} \quad 0.5 < \mu_A{}^{mod}(g_{ij}) \le 1 \end{cases} \tag{8}$$

Here, g_{ij} refers to the $(i,j)^{th}$ gray level of the image.

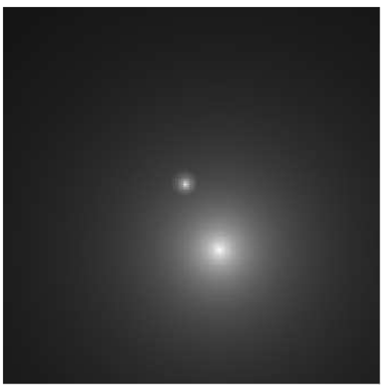

Figure 13: Input Image [12]

In this paper, the deterioration of the fEEG images has been demonstrated as λ increased. Moreover, the determination of parameter λ for Sugeno type intuitionistic fuzzy generator will be discussed further in the later section of this paper.

4. Results

Consider the non-membership function as given in (5). It follows that

Proposition 1. *If* $\nu_A(g_{ij}) = \frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})}$, *then* $\lim\limits_{\lambda\to\infty} \nu_A(g_{ij}) = 0$

Proof

$$\begin{aligned}
\lim_{\lambda\to\infty} \nu_A(g_{ij}) &= \lim_{\lambda\to\infty} \frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})} \\
&= \frac{\lim\limits_{\lambda\to\infty} 1-\mu_A(g_{ij})}{\lim\limits_{\lambda\to\infty} 1+\lambda\mu_A(g_{ij})} \\
&= \lim_{\lambda\to\infty} (1-\mu_A(g_{ij})) \cdot \frac{1}{\lim\limits_{\lambda\to\infty} (1+\lambda\mu_A(g_{ij}))} \\
&= (1-\mu_A(g_{ij})) \cdot 0 \\
&= 0
\end{aligned}$$

This proposition indicates that as λ increases, the non-membership value will decrease. This will influence the hesitation value and the results will be demonstrated in the form of images. Different values of λ are tested for $\lambda = 0.0001, \lambda = 0.1, \lambda = 1, \lambda = 3$, and $\lambda = 10$ as displayed in Figure 14 [14].

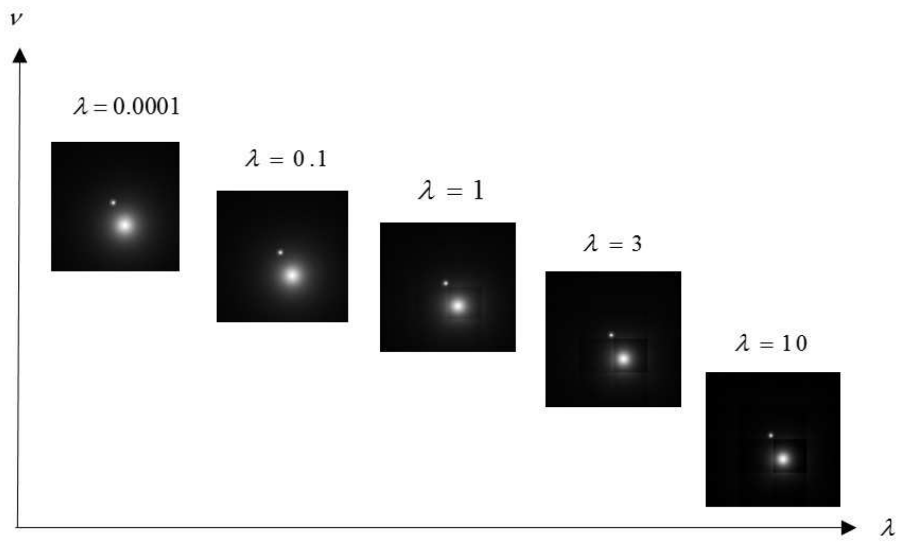

Figure 14: The output images with different values of λ

Hence, from (6), as λ increases, the hesitation value π will increase which shows that unwanted elements that look like squares occur in the images. Thus the images start to deteriorate and are in agreement with the result in [13]. The difference between the input image as seen in Figure 13 and the enhanced images is that the border of the electrical potential has been suppressed in the enhanced images resulting in a darker background area compared to the input image.

Therefore, from (6), the following corollary can be applied

Corollary 1.

$$\begin{aligned}\lim_{\lambda\to\infty} \pi_A(g_{ij}) &= \lim_{\lambda\to\infty} [1 - \mu_A(g_{ij}) - \nu_A(g_{ij})] \\ &= \lim_{\lambda\to\infty} [1] - \lim_{\lambda\to\infty} \mu_A(g_{ij}) - \lim_{\lambda\to\infty} \nu_A(g_{ij}) \\ &= \lim_{\lambda\to\infty} [1] - \lim_{\lambda\to\infty} \mu_A(g_{ij}) - 0 \\ &= 1 - \lim_{\lambda\to\infty} \mu_A(g_{ij})\end{aligned}$$

Moreover, the deterioration of the image can be represented or modeled as a limit of $\nu_A(g_{ij})$. Consider the definition of sequence of fuzzy number and its convergence as given in Definition 1 and 2 as follows [15].

Definition 1. A sequence $X = \{X_n\}$ of fuzzy numbers is a function X from the set N of all positive integers into $L(R)$. The fuzzy number X_n denotes the value of the function at $n \in N$ and is called the n -th term of the sequence.

Definition 2. A sequence $X = \{X_n\}$ of fuzzy numbers is said to be convergent to the fuzzy number X_0, written as $\lim_n X_n = X_0$, if $\forall\varepsilon > 0, \exists n_0 > 0 \ni \bar{d}(X_n, X_0) < \varepsilon$ for $n > n_0$.

Definition 2 which is applicable for the limit of fuzzy number can be implemented in the limit of non-membership value as in Proposition 1 mentioned earlier. This can be shown as follows

$$\lim_{\lambda\to\infty} \nu_A(g_{ij}) = 0 \quad \text{iff} \quad \forall\varepsilon > 0, \exists N(N(\varepsilon)) \ni \lambda > N \Rightarrow |\nu_A - 0| < \varepsilon$$

Notice,

$$\begin{aligned}\left|\frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})} - 0\right| &= \left|\frac{1-\mu_A(g_{ij})}{1+\lambda\mu_A(g_{ij})}\right| \\ &\leq \frac{1-\mu_A(g_{ij})}{1+\lambda} \\ &< \frac{1-\mu_A(g_{ij})}{1+N} < \varepsilon\end{aligned}$$

Hence,

$$1 - \mu_A(g_{ij}) < (1+N)\varepsilon$$
$$\frac{1 - \mu_A(g_{ij})}{\varepsilon} < 1+N$$
$$\frac{1 - \mu_A(g_{ij})}{\varepsilon} - 1 < N$$

Now, by taking $N > \frac{1-\mu_A(g_{ij})}{\varepsilon} - 1$
Therefore $\lim\limits_{\lambda \to \infty} \nu_A(g_{ij}) = 0$ since

$$\forall \varepsilon > 0, \exists N \left(N > \frac{1 - \mu_A(g_{ij})}{\varepsilon} - 1 \right) \ni \lambda > N \Rightarrow |\nu_A(g_{ij}) - 0| < \varepsilon$$

.

Furthermore, different values of ε will be tested in order to obtain the values of N and λ. Here, some values of ε are 0.01, 0.02, 0.1, and 0.5. For each particular ε, the maximum value of N will be chosen to determine the value of λ as given in Table 1.

Table 1: Values of Parameters

ε	N	λ
0.01	$N \geq 91$	λ>91
0.02	$N \geq 45$	λ>45
0.1	$N \geq 9$	λ>9
0.5	$N \geq 1$	λ>1

Table 2 shows the values of hesitation, π, for particular values of λ. Here, the maximum value of π for each λ should be chosen.

Table 2: Values of π for particular λ

λ	π
2	0.267949
10	0.536675
46	0.745406
92	0.812095

A sample of relationship between hesitation and membership is given in Figure 15 for $\lambda = 10$.

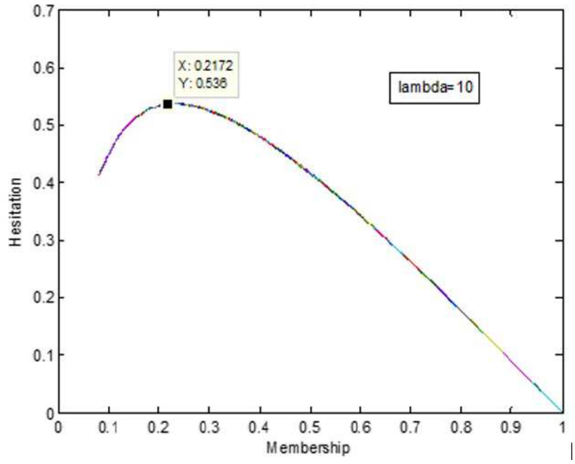

Figure 15: Hesitation vs membership value for $\lambda = 10$

5. Conclusion

The fuzzy limit defined in Definition 2 can assist in obtaining the parameter λ in the window based enhancement scheme. Unsmooth images were produced as λ increased which also increased the hesitation value. This can be seen when $\lambda = 1, 3, 10$ as in Figure 14.

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