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ON ϕ-PSEUDO SYMMETRIES OF GENERALIZED SASAKIAN-SPACE-FORMS

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Abstract: In this paper we study ϕ-pseudo symmetric and ϕ- pseudo Ricci symmetric generalized Sasakian-space-forms with various geometric properties.

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1. Introduction

The notion of generalized Sasakian-space-forms was first studied by Alegre et al. [1]. A generalized Sasakian-space-form is an almost contact metric manifold (M, g) whose curvature tensor is given by

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned} \tag{1.1}$$

where f_1, f_2, f_3 are differentiable functions on M and X, Y, Z are vector fields on M. In such case we will write the manifold as $M(f_1, f_2, f_3)$. This kind of manifolds appears as a natural generalization of the Sasakian-space-forms by taking:

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$f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, where c denotes constant ϕ-sectional curvature. The ϕ-sectional curvature of generalized Sasakian-space-form $M(f_1, f_2, f_3)$ is $f_1 + 3f_2$. Moreover, cosymplectic space-form and Kenmotsu space-form are also considered as particular types of generalized Sasakian-space-form [1, 2, 3, 4, 11, 16, 18, 19, 20].

The geometry of a space mainly depends on the curvature of the space. One of the most important geometric property of a space is symmetry. The study of symmetry of a manifold began with the works of Cartan [7] and then this notion has been weakenend by many authors by giving several defining conditions and some curvature restrictions in different directions.

In 1987, Chaki [8] introduced the notion of pseudo symmetric manifold, which is weaker than the local symmetry. A non-flat Riemannian manifold $(M^n, g)(n > 2)$ is called pseudo symmetric in the sense of Chaki if it satisfies the relation

$$\begin{aligned}(\nabla_W R)(X,Y)Z &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z \\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W \\ &+ g(R(X,Y)Z,W)\rho\end{aligned} \tag{1.2}$$

for any vector fields X, Y, Z, U and W, where R is the Riemannian curvature tensor of the manifold. A is a non-zero 1-form such that $g(X, \rho) = A(X)$ for every vector field X. Such an n-dimensional manifold is denoted by $(PS)_n$.

In 1988, Chaki [9] introduced the notion of pseudo Ricci symmetric manifolds defined as follows:

A non-flat Riemannian manifold (M^n, g) is said to be pseudo Ricci symmetric [9] if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_W S)(X,Y) = 2A(W)S(X,Y) + A(X)S(W,Y) + A(Y)S(X,W) \tag{1.3}$$

for any vector field X, Y and Z, where A is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an n-dimensional manifold is denoted by $(PRS)_n$.
The relation (1.3) can be written as

$$(\nabla_W Q)(X) = 2A(W)Q(X) + A(X)Q(W) + S(X,W)\rho \tag{1.4}$$

where ρ is the vector field associated 1-form A such that $A(X) = g(X, \rho)$ and Q is the Ricci operator i.e., $g(QX, Y) = S(X, Y)$ for all X and Y.

The object of the paper is to study ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric generalized Sasakian-space-forms. The paper is organized as follows: Section 2 is devoted to preliminaries on generalized Sasakian-space-forms. Section 3 deals with a study of ϕ-pseudo symmetric generalized Sasakian-space-forms. In Section 4, we study ϕ-pseudo symmetric generalized Sasakian-space-forms. It is proved that, a ϕ-pseudo symmetric trans-Sasakian generalized Sasakian-space-forms is cosymplectic provided $f_1 \neq f_3$. Also, we prove that a ϕ-pseudo symmetric generalized Sasakian-space-form, which is β-Kenmotsu is an η-Einstein manifold. Section 5 is concerned on ϕ-pseudo Ricci symmetric generalized Sasakian-space-forms.

2. Preliminaries

Let M be an almost contact metric manifold [5] with an almost contact structure (ϕ, ξ, η, g), where ϕ is a (1, 1)-tensor field, ξ is a vector field, η is a 1-form and g is a compatible Riemannian metric such that,

$$(a)\ \phi^2(X) = -X + \eta(X)\xi, \quad (b)\ \ \phi(\xi) = 0, \quad (c)\ \eta(\phi X) = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$(a)\ g(X, \phi Y) = -g(\phi X, Y), \quad (b)\ g(X, \xi) = \eta(X), \tag{2.3}$$

for all vector fields X, Y on M.

An almost contact structure (ϕ, ξ, η, g), on M is called trans-Sasakian structure [17] if there exist two functions α and β on M such that

$$(\nabla_X \phi)(Y) = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi) - \eta(Y)\phi X) \tag{2.4}$$

for some smooth functions α and β on M. From (2.4), it follows that

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi), \tag{2.5}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y). \tag{2.6}$$

Again in [10] the authors have introduced two subclasses of trans-Sasakian structures, the C_5 and C_6 structures which contain the Kenmotsu and Sasakian structure respectively. The subclass C_5 and C_6 are respectively called α-Sasakian and β-Kenmotsu structure. A trans-Sasakian structures of type $(0, 0)$, $(0, \beta)$ and $(\alpha, 0)$ are cosymplectic [5], β-Kenmotsu [12] and α-Sasakian [12] respectively.

In addition to the relation (1.1), for an $(2n+1)$-dimensional $(n>1)$ generalized Sasakian-space-form $M^{2n+1}(f_1,f_2,f_3)$ the following relation holds [1]

$$\begin{aligned} S(X,Y) &= [(2n)f_1+3f_2-f_3)]g(X,Y)-[3f_2+(2n-1)f_3]\eta(X)\eta(Y) && (2.7)\\ QX &= [(2n)f_1+3f_2-f_3]X-[3f_2+(2n-1)f_3]\eta(X)\xi, && (2.8)\\ S(X,\xi) &= (2n)(f_1-f_3)\eta(X), && (2.9)\\ Q\xi &= 2n(f_1-f_3)\xi, && (2.10)\\ r &= 2n(2n+1)f_1+6nf_2-4nf_3, && (2.11)\\ R(X,Y)\xi &= (f_1-f_3)\{\eta(Y)X-\eta(X)Y\}, && (2.12)\\ R(\xi,X)Y &= (f_1-f_3)\{g(X,Y)\xi-\eta(Y)X\}, && (2.13)\\ S(\phi X,\phi Y) &= S(X,Y)-2n(f_1-f_3)\eta(X)\eta(Y). && (2.14) \end{aligned}$$

Where R, S and Q are the curvature tensor, Ricci tensor and the Ricci operator of the space-form, respectively.

3. ϕ-Pseudo Symmetric Generalized Sasakian-Space-Forms

Definition 1. A generalized Sasakian-space-form $M^{2n+1}(f_1,f_2,f_3)(n>2)$ is said to be a ϕ-pseudo symmetric if the curvature tensor satisfies

$$\begin{aligned} \phi^2((\nabla_W R)(X,Y)Z) &= 2A(W)R(X,Y)Z)+A(X)R(W,Y)Z\\ &+ A(Y)R(X,W)Z+A(Z)R(X,Y)W\\ &+ g(R(X,Y)Z,W)\rho, && (3.1) \end{aligned}$$

for any vector fields X,Y,Z and W, where A is a non zero 1-form. If, in particular, $A=0$, then the manifold is called a ϕ-symmetric [21]. The author, Hui have been studied the above notions to Kenmotsu manifolds [13], $(LCS)_n$-manifolds [15] and para-Sasakian manifolds [14].

We now consider a generalized Sasakian-space-form $M^{2n+1}(f_1,f_2,f_3)(n>1)$ which is ϕ-pseudo symmetric. Then, by virtue of (2.1), it follows from (3.1) that

$$\begin{aligned} &-(\nabla_W R)(X,Y)Z+\eta((\nabla_W R)(X,Y)Z)\xi\\ =\ & 2A(W)R(X,Y)Z+A(X)R(W,Y)Z+A(Y)R(X,W)Z\\ +\ & A(Z)R(X,Y)W+g(R(X,Y)Z,W)\rho. && (3.2) \end{aligned}$$

Taking $Z=\xi$ in (3.2) and then using (2.12), we have Then

$$-(\nabla_W R)(X,Y)\xi+\eta((\nabla_W R)(X,Y)\xi)\xi$$

$$\begin{aligned} &= (f_1 - f_3)[2A(W)\{\eta(Y)X - \eta(X)Y\} + A(X)\{\eta(Y)W - \eta(W)Y\} \\ &+ A(Y)\{\eta(W)X - \eta(X)W\}] + A(\xi)R(X,Y)W \\ &+ (f_1 - f_3)\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\rho. \end{aligned} \tag{3.3}$$

For $X = \xi$ in (3.3), we have

$$\begin{aligned} & -(\nabla_W R)(\xi,Y)\xi + \eta((\nabla_W R)(\xi,Y)\xi)\xi \\ = \; & (f_1 - f_3)[2A(W)\{\eta(Y)\xi - Y\} + A(\xi)\{\eta(Y)W - \eta(W)Y\} \\ & + A(Y)\{\eta(W)\xi - W\}] + A(\xi)(f_1 - f_3)\{g(Y,W)\xi - \eta(W)Y\} \\ & + (f_1 - f_3)\{\eta(Y)\eta(W)\rho - g(Y,W)\rho\}. \end{aligned} \tag{3.4}$$

By definition of the covariant derivative, we have

$$\begin{aligned} (\nabla_W R)(\xi,Y)\xi = \nabla_W R(\xi,Y)\xi - R(\nabla_W \xi, Y)\xi \\ - R(\xi, \nabla_W Y)\xi - R(\xi,Y)\nabla_W \xi. \end{aligned} \tag{3.5}$$

Using (2.12) (2.13) and (2.5) in (3.5), we have

$$\begin{aligned} (\nabla_W R)(\xi,Y)\xi \;=\; & \nabla_W[(f_1 - f_3)(\eta(Y)\xi - Y)] - (f_1 - f_3)[\eta(\nabla_W Y)\xi - \nabla_W Y] \\ & -(f_1 - f_3)[-\alpha\eta(Y)\phi W + \beta\eta(Y)W - \beta\eta(W)\eta(Y)\xi] \\ & -(f_1 - f_3)[-\alpha g(Y,\phi W)\xi \\ & +\beta(g(Y,W)\xi) - \eta(W)\eta(Y)\xi)]. \end{aligned} \tag{3.6}$$

Using (2.5) and (2.6), equation (3.6) reduces to

$$(\nabla_W R)(\xi,Y)\xi = d(f_1 - f_3)(W)(\eta(Y)\xi - Y) + (f_1 - f_3)\nabla_W Y. \tag{3.7}$$

From (3.4) and (3.7), we have

$$\begin{aligned} & d(f_1 - f_3)(W)(Y - \eta(Y)\xi) - (f_1 - f_3)\nabla_W Y \\ = \; & (f_1 - f_3)[2A(W)\{\eta(Y)\xi - Y\} \\ & + A(\xi)\{\eta(Y)W - \eta(W)Y\} + A(Y)\{\eta(W)\xi - W\}] \\ & + (f_1 - f_3)A(\xi)\{g(Y,W)\xi - \eta(W)Y\} \\ & + (f_1 - f_3)\{\eta(Y)\eta(W)\rho - g(Y,W)\rho\}. \end{aligned} \tag{3.8}$$

Taking $Y = \xi$ in (3.8), we obtain

$$(f_1 - f_3)\nabla_W \xi = 0. \tag{3.9}$$

From (3.9), we get that M is co-symplectic provided $f_1 \neq f_3$. Thus, we have

Theorem 1. *A trans-Sasakian generalized Sasakian-space-form which is ϕ-pseudosymmetric, is cosymplectic provided $f_1 \neq f_3$.*

Taking inner product of (3.3) with U and then taking contraction over X and U, we get

$$\begin{aligned} & -(\nabla_W S)(Y,\xi) + g((\nabla_W R)(\xi,Y)\xi,\xi) \\ = \; & (f_1 - f_3)[2(2n+1)A(W)\eta(Y) + (2n-1)A(Y)\eta(W)] \\ & -(f_1 - f_3)A(\xi)g(Y,W) + A(\xi)S(Y,W). \end{aligned} \tag{3.10}$$

Using (2.12), we have

$$g((\nabla_W R)(\xi,Y)\xi,\xi) = 0. \tag{3.11}$$

By definition of covariant derivative, we have

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi). \tag{3.12}$$

Using (2.5) and (2.9) in (3.12), we get

$$\begin{aligned} (\nabla_W)(Y,\xi) \;=\; & 2n[d(f_1 - f_3)(W)\eta(Y) - (f_1 - f_3)\{\alpha g(\phi W,Y) - \beta g(W,Y)\}] \\ + \; & \alpha S(Y,\phi W) - \beta g(Y,W). \end{aligned} \tag{3.13}$$

Plugging (3.11) and (3.13) in (3.10), we have

$$\begin{aligned} & -2n[d(f_1 - f_3)(W)\eta(Y) - (f_1 - f_3)\{\alpha g(\phi W,Y) - \beta g(W,Y)\}] \\ & -\alpha S(Y,\phi W) + \beta g(Y,W) \\ = \; & (f_1 - f_3)[2(2n+1)A(W)\eta(Y) + (2n-1)A(Y)\eta(W)] \\ & -(f_1 - f_3)A(\xi)g(Y,W) + A(\xi)S(Y,W). \end{aligned} \tag{3.14}$$

Taking $Y = \xi$ in (3.14) and then using (2.1) and (2.9), we get

$$d(f_1 - f_3)(W) = -\frac{(f_1 - f_3)}{2n}[2(2n+1)A(W) + 2A(\xi)\eta(W)]. \tag{3.15}$$

Using (3.15) in (3.14), we have

$$\begin{aligned} & [\beta - A(\xi)]S(Y,W) - \alpha[S(Y,\phi W) - 2n(f_1 - f_3)g(Y,\phi W)] \\ = \; & (f_1 - f_3)[2n\beta - A(\xi)]g(Y,W) + (2n-1)(f_1 - f_3)[A(Y)\eta(W) \\ & -2A(\xi)\eta(Y)\eta(W)]. \end{aligned} \tag{3.16}$$

For $W = \xi$, the equation (3.16) reduces to

$$A(Y) = A(\xi)\eta(Y). \tag{3.17}$$

By taking account of (3.17) in (3.16), one can get

$$\begin{aligned} & [\beta - A(\xi)]S(Y,W) \\ = \ & \alpha[S(Y,\phi W) - 2n(f_1 - f_3)g(Y,\phi W)] \\ & +(f_1 - f_3)[(2n\beta - A(\xi))g(Y,W) - (2n-1)A(\xi)\eta(Y)\eta(W)]. \end{aligned} \tag{3.18}$$

Changing W by ϕW in (3.18), and using (2.1) and (2.9), we obtain

$$S(Y,\phi W) = \frac{\alpha}{\beta - A(\xi)}\{-S(Y,W) + 2n(f_1 - f_3)g(Y,W)\} + \frac{2n\beta - A(\xi)}{\beta - A(\xi)}g(Y,\phi W). \tag{3.19}$$

Using (3.19), (3.18) gives

$$\begin{aligned} [\alpha^2 + (\beta - A(\xi)^2]S(Y,W) &= [2n\alpha^2 + (2n\beta - A(\xi))(\beta - A(\xi))](f_1 - f_3)g(Y,W) \\ &- (2n-1)(f_1 - f_3)[\beta - A(\xi)]A(\xi)\eta(Y)\eta(W) \\ &+ (2n-1)(f_1 - f_3)\alpha A(\xi)g(Y,\phi W). \end{aligned} \tag{3.20}$$

Thus, we can able to state the following:

Theorem 2. *In a ϕ-pseudosymmetric trans-Sasakian generalized Sasakian-space-form, the Ricci tensor S is of the form (3.20).*

If we taking $\alpha = 0$ in (3.20), then

$$S(Y,W) = \frac{(2n\beta - A(\xi))}{\beta - A(\xi)}(f_1 - f_3)g(Y,W) - \frac{(2n-1)A(\xi)}{\beta - A(\xi)}\eta(Y)\eta(W). \tag{3.21}$$

That is, $M^{2n+1}(f_1, f_2, f_3)$ is an η-Einstein manifold.Hence, we state.

Corollary 1. *A ϕ-pseudosymmetric generalized Sasakian-space-form*

$$M^{2n+1}(f_1, f_2, f_3) \quad (n > 1),$$

which is β- Kenmotsu is an η-Einstein manifold.

4. ϕ-Pseudo Ricci Symmetric Generalized Sasakian-Space-Forms

Definition 2. A generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is said to be a ϕ-pseudo Ricci symmetric if the Ricci operator Q satisfies

$$\phi^2((\nabla_W Q)(X)) = 2A(W)QX + A(X)QW + S(X,W)\rho, \tag{4.1}$$

for any vector field X and W, where A is a non-zero 1-form.
If, in particular, $A = 0$, then (4.1) turns into the notion of ϕ-Ricci symmetric generalized Sasakian-space-form.

Now, consider a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ which is ϕ-pseudo Ricci symmetric. Then by, virtue of (2.1), it follows from (4.1) that -$(\nabla_W Q)(X) + \eta((\nabla_W Q)(X))\xi = 2A(W)QX + A(X)QW + S(W, X)\rho$.
Then

$$\begin{aligned} & -g(\nabla_W Q(X), Y) + S(\nabla_W X, Y) + \eta(\nabla_W Q)(X)\eta(Y) \\ = \; & 2A(W)S(X, Y) + A(X)S(W, Y) + A(Y)S(X, W). \end{aligned} \tag{4.2}$$

Putting $Y = \xi$ in (4.2) and using (2.9), we have

$$\begin{aligned} & -g(\nabla_W Q\xi, Y) + S(\nabla_W \xi, Y) + \eta(\nabla_W Q)(\xi)\eta(Y) \\ = \; & 2n(f_1 - f_3)[2A(W)\eta(Y) + A(Y)\eta(W)] + A(\xi)S(W, Y). \end{aligned} \tag{4.3}$$

Using (2.5) and (2.9) in (4.3), we have

$$\begin{aligned} & 2n \{-d(f_1 - f_3)(W)\eta(Y) + (f_1 - f_3)(\alpha g(\phi W, Y) - \beta g(W, Y))\} \\ = \; & 2n(f_1 - f_3)[2A(W)\eta(Y) + A(Y)\eta(W)] + A(\xi)S(W, Y) \\ & +\alpha S(\phi W, Y) - \beta S(W, Y). \end{aligned} \tag{4.4}$$

Taking $Y = \xi$ in (4.4) and using (2.1) and (2.9), we obtain

$$d(f_1 - f_3)(W) = (f_1 - f_3)[(\beta - 2A(\xi))\eta(W) - 2A(W)]. \tag{4.5}$$

Using (4.5) in (4.4), gives

$$\begin{aligned} & (n - 1)(f_1 - f_3)A(Y)\eta(W) + A(\xi)S(W, Y) \\ = \; & 2n(f_1 - f_3)[-(\beta - 2A(\xi))\eta(W)\eta(Y) + \alpha g(\phi W, Y) - \beta g(W, Y)] \\ & -\alpha S(\phi W, Y) + \beta S(W, Y). \end{aligned} \tag{4.6}$$

Putting $W = \xi$ in (4.6), we get

$$A(Y) = -(\beta - A(\xi))\eta(Y). \tag{4.7}$$

By taking account of (4.7) in (4.6), one can get

$$\begin{aligned} (\beta - A(\xi))S(W, Y) \; = \; & 2n(f_1 - f_3)[\beta g(W, Y) - A(\xi)\eta(W)\eta(Y)] \\ + \; & \alpha[S(\phi W, Y) - 2n(f_1 - f_3)g(\phi W, Y)]. \end{aligned} \tag{4.8}$$

Changing W by ϕW in (4.8), and using (2.1) and (2.9), we have

$$S(Y,\phi W) = \frac{\alpha}{(\beta - A(\xi))}[-S(Y,W) + 2n(f_1 - f_3)g(Y,W)] + \frac{\beta}{(\beta - A(\xi))}2n(f_1 - f_3)g(Y,\phi W). \quad (4.9)$$

Using (4.9), (4.8) reduces to

$$\begin{aligned}[\alpha^2 + (\beta - A(\xi)^2)]S(Y,W) &= 2n(f_1 - f_3)[\alpha^2 + \beta(\beta - A(\xi)]g(Y,W) \\ &+ 2n(f_1 - f_3)[(\beta - A(\xi))A(\xi)]\eta(Y)\eta(W) \\ &+ 2n(f_1 - f_3)\alpha A(\xi)g(Y,\phi W). \quad (4.10)\end{aligned}$$

Thus, we state the following;

Theorem 3. *In a ϕ- pseudosymmetric trans-Sasakian generalized Sasakian-space-form, rhe Ricci tensor S is of the form (4.11).*

If we taking $\alpha = 0$ in (4.10), then

$$S(Y,W) = \frac{\beta}{(\beta - A(\xi))}2n(f_1 - f_3)g(Y,W) - \frac{A(\xi)}{(\beta - A(\xi))}2n(f_1 - f_3)\eta(Y)\eta(W). \quad (4.11)$$

That is, $M^{2n+1}(f_1, f_2, f_3)$ is an η-Einstein manifold. Hence. we state

Corollary 2. *A ϕ-pseudu Ricci symmetric generalized Sasakian-space-form, which is β-Kenmotsu is an η-Einstein manifold.*

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