International Journal of Pure and Applied Mathematics
Volume 104 No. 3 2015, 339-352
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v104i3.5

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A MODIFIED FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING TO SOLVE AGGREGATE PRODUCTION PLANNING

Bayda Atiya Kalaf[1,2], Rizam Abu Bakar[1], Lee Lai Soon[1], Mansor Bin Monsi[1], Abdul Jabbar Khudhur Bakheet[3], Iraq Tereq Abbas[1 §]

[1]Department of Mathematics
Faculty of Science
University Putra Malaysia
43300, Serdang, MALAYSIA

[2]Department of Mathematics
College of Ibn AL-Haitham Education
University of Baghdad
Jadriyah, Baghdad, IRAQ

[3]Department of Statistics
College of Administration and Economics
University of Baghdad
Jadriyah, Baghdad, IRAQ

Abstract: This paper develops a fuzzy multi-objective model for solving aggregate production planning problems that contain multiple products and multiple periods in uncertain environments. We seek to minimize total production cost and total labor cost. We adopted a new method that utilizes a Zimmermans approach to determine the tolerance and aspiration levels. The actual performance of an industrial company was used to prove the feasibility of the proposed model. The proposed model shows that the method is useful, generalizable, and can be applied to APP problems with other parameters.

Received: May 26, 2015

[§]Correspondence author

Key Words: aggregate production planning (APP), fuzzy multi-objective linear programming, mathematical model, aspiration level, tolerance level

1. Introduction

Aggregate production planning (APP) is an operational plan for the production process in advance of 3 to 18 months. The objectives of APP are to set overall production levels for each product category in order to meet future fluctuating demand and to evaluate decisions concerning hiring, firing, over time, subcontracting, and carrying inventory level; in this way, the appropriate resources that are needed can be determined (Lai et al.1992)The APP process is conducted at an aggregate level without the need to provide detailed material and capacity resource requirements for individual products and detailed schedules for facilities and personnel. Although APP is considered higher level planning in the process of production management, other forms of disaggregation plans, such as master schedule, capacity plan, and material requirements plan, depend on APP in a hierarchical manner.

Many APP models and solutions with different degrees of sophistication have been introduced since 1955 (Holt et al.,1955), such as linear programming (Kanyallar and Adil,2005), mixed integer linear programming (Gomes et al., 2006; Vassilliadis et al.2000; Christou et al.,2007; Sillekens et al., 2011), and goal programming (Rifai, 1996; Baykasoglu, 2001; Romero,C.2004; Leung and Chang, 2009), Heuristic algorithms such as the genetic algorithm (Ramezanian et al., 2012; Che and Chiang 2010; Liu et al., 2011) and tabu search (Masud and Hwang 1980; Yan et al,2003; Pradenas et al, 2004) have also been applied.

However, there is no clear and specific path for production companies to adopt scientific methods in the aggregate process for the preparation of implementation plans.The planning process requires modern methods to be consistent with the innovation in technology over time and with the complexity of the dynamic movement of the market and competition.Such requirements lead to difficulties when applying mathematical models for classical APP. Because the fuzzy model is more suitable than the other models to give accurate results, therefore, fuzzy programming technique is considered to be more efficient to solve real APP problems. The fuzzy multi-objective linear programming (FMOLP) model is considered particularly efficient because most companies seek to satisfy more than one objective function to develop a response and flexibility production planning system. In this section, we review several studies on the application of FMOLP for APP problems.

Zimmermann (1978) first applied fuzzy set theory to an ordinary multi-objective linear programming problem. Since then, fuzzy mathematical programming has been extended to several fuzzy methods for solving APP problems.

Kavehand Ayda (2014) established a fuzzy goal programming approach to solve multi-objective mixed-integer mathematical programming. Three objective functions, including , minimizing total cost, maximizing customer service level, and maximizing the quality of the end product, were simultaneously considered. Najmeh and Kuan (2014) introduced a multi-objective fuzzy APP model and assumed the performance and availability of production lines. Comparison of results showed the significant importance of these two factors in developing a real and practical aggregate production plan. Navid et al(2013) expressed multi-objective APP with imprecise parameters, and used the method suggested by Okada et al (1991) to convert a fuzzy APP model into a crisp model. Since then, crisp APP models have been solved using a genetic algorithm. Phruksaphanrat (2011) suggested a preemptive possibilistic linear programming model to solve multi-objective APP problems with two objective functions: maximizing profit and minimizing labor changes. Each objective was transformed to three crisp objective functions. Then, the problem was solved according to objective priorities. Baykasoglu and Gocken (2010) used fuzzy ranking methods and the tabu search algorithm to solve multi-product fuzzy APP directly, and tested four different vague ranking methods. They observed that fuzzy decision-making problems can be solved effectively by using meta-heuristics, incorporating fuzzy ranking methods without converting them into crisp equivalent models. Liang (2007) introduced possibilistic programming to solve multi-product and multi-time period APP problems, with multiple fuzzy goals and fuzzy cost coefficients having triangular possibility distributions. Wang and Liang (2005) introduced a novel interactive FMOLP model to solve APP decision problems in an uncertain environment, They adopted Zimmermanns maxmin approach to transform FMOLP into crisp equivalent. Wang and Liang (2004) developed a fuzzy multi-objective linear programming model to solve APP problem in uncertainty environment, Three objectives used to address the problem were to minimize total cost, minimize inventory and back-ordered, and minimize the rates of change in the workforce level with reference to inventory level, labor level,capacity, warehouse space, and the time value of money. Wang and Fang (2001) established a fuzzy linear programming model to solve multi-production APP problems with multiple goals, where product price, unit cost to subcontract, work force level, production capacity,and market demand were uncertain over the production-planning horizon. Tang Et al.(2000)

formulated a multi-product APP problem as a fuzzy quadratic programming with both fuzzy demand and fuzzy constraint appeared in the same model.

The present study introduces FMOLP to solve APP problems in a fuzzy environment. In addition, we present a new method for determining the tolerance level and aspiration level based on Zimmermans approach.The model aims to minimize total production costs and labor costs. WINQSB computer software is used to run this fuzzy linear programming model.

This paper is organized as follows. Section 2 describes the FMOLP, the mathematical model of APP problem, and introduces the procedure to solve the model. A case study and its results presented in Section 3.Finally, conclusions are given in Section 4.

2. The Fuzzy Multiobjective Linear Programming Model

We proposed mathematical model for aggregate production planning problem. Assumed that an industrial company manufacturing produces n types of products to fulfill market demand over planning time horizon T. We considered two objective function in this paper: to minimize production costs and minimize workforce costs.

Notations:

- $n=$ number of products.
- $t=$ number of periods in the planning horizon.
- cnt=production cost per ton of product n per period t, (dolar/ton).
- int=inventory carrying cost per ton of product n per period t, (dolar/ton).
- ht=hiring cost per worker in period t, (dolar,worker).
- ft=firing cost per worker in period t, (dolar,worker).
- ot=cost per man- hour of overtime labor per period t.
- wt=cost of regular labor per period t.
- Dnt=forecasted demand for product n per period t, (tons).
- Pnt=production of product n per period t, (tons).
- Int=inventory level of product n per period t, (tons).

- Ot=man-hours of overtime labor per period t.
- Wt=workforce level per period t, (workers).
- Ht=hired workers per period t, (workers).
- Ft=fired workers per period t, (workers).
- Mn=hours required to produce one ton of product n.
- AR=working regular hours per period t.
- AO=working overtime hours, which allowed during per period t.

Objective function:

- Minimize production costs.

$$\min Z_1 = \sum_{n=1}^{10} \sum_{t=1}^{6} C_{nt} P_{nt} + i_{nt} I_{nt} \tag{1}$$

- Minimize workforce costs.

$$\min Z_2 = \sum_{t=1}^{6} w_t W_t + h_t H_t + f_t F_t + o_t O_t \tag{2}$$

Constraint

- inventory level constraint

$$P_{nt} + I_{n(t-1)} - I_{nt} = D_{nt}, \quad \forall n, \forall t \tag{3}$$

- Workforce level constraint

$$F_t - H_t + W_t - W_{t-1} = 0 \quad \forall t \tag{4}$$

- Overtime constraint

$$O_t - AO \times W_t \leq 0 \quad \forall t \tag{5}$$

- Production constraint

$$\sum_{n=1}^{10} M_n P_{nt} - AR \times W_t - O_t \leq o \quad \forall t. \tag{6}$$

- Integer constraint

$$H_t, F_t, W_t \quad are\ integer$$

- non-negativity constraint

$$P_{nt}, I_{nt}, H_t F_t, W_t, O_t \geq 0$$

2.1. Model Development

We used the a new method in fuzzy multi objective linear programming in aggregate production planning and we shall describe the procedure of how to apply this method as a general procedure as follows:

Step 1
This method adopted on Zimmermans approach as a follows formulation:

$$\min CX \tilde{\leq} Z^{\square}$$

Subject to :

$$AX \tilde{\leq} T, \quad X \geq 0$$

Here $\leq$ is fuzzy inequalities, $Z^{\square}$ is an aspiration level of the decision maker, and T is the tolerance level. The linear membership function is written as :

$$\mu_k(Z_k) = \begin{cases} 1 & Z_k \leq Z_k^{\square} \\ 1 - \frac{Z_k - Z_k^*}{T_k} & Z_k^{\square} \leq Z_k \leq Z_k^{\square} + T_k \\ 0 & Z_k > Z_k^{\square} + T_k \end{cases} \tag{7}$$

where$k = 1, 2, , N$. which N is number of objectives.

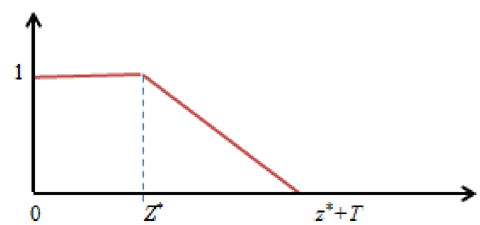

Figure 1. Linear membership function

Step 2

Using winqsb programming or any other program to solve each fuzzy objective function individual to find optimum solution that means $Z_k^{\square}$ (aspiration level). After that, we find Tk from accruing costs or profits from the program by taking smaller two variable decision values from it and subtract the minimum number from supreme. This way is general it can any decision maker use it and get the same result. Because, in the past they adopted on decision maker's guessing and estimating to get the tolerance level value (Tk), that leads to becoming more than one solution to same problem.

Step 3

Write membership function for each objective, according to Zimmermans approach .

$$\mu_1(Z_1) = \begin{cases} 1 & Z_1 \leq Z_1^{\square} \\ 1 - \frac{Z_1 - Z_1^*}{T_1} & Z_1^{\square} \leq Z_1 \leq Z_1^{\square} + T_1 \\ 0 & Z_1 > Z_1^{\square} + T_1 \end{cases} \tag{8}$$

$$\mu_2(Z_2) = \begin{cases} 1 & Z_2 \leq Z_2^{\square} \\ 1 - \frac{Z_2 - Z_2^*}{T_2} & Z_2^{\square} \leq Z_2 \leq Z_2^{\square} + T_2 \\ 0 & Z_2 > Z_2^{\square} + T_2 \end{cases} \tag{9}$$

After we solve the general equation (7), we can get to

$$\alpha \leq 1 - ((Z_k - Z_k^{\square})/T_k) \tag{10}$$

where μ represent membership function $= \mu k(Z_k)$ for each k.

$$\alpha \leq 1 - ((Z_1 - Z_1^{\square})/T_1) \tag{11}$$

$$\alpha \leq 1 - ((Z_2 - Z_2^{\square})/T_2) \tag{12}$$

After simplifying the equations we get

$$\alpha + Z_1^{\square}/T_1 \tag{13}$$

$$\alpha + Z_2^{\square}/T_2 \tag{14}$$

Step 4

	A	B	C	D	E	F	G	H	J	K
c_{nt}	3285	385.082	450.7	1005.776	800.834	486.999	449.419	1007.323	494.509	738.755
i_{nt}	38	47	53.6	35.511	35.415	24.6	37.75	37.666	58.666	37

Table 1: Production and inventory costs in dolar

The equivalent linear programming problem is formulated as:
$max\, \alpha$
Subject to

$$\begin{aligned}
\alpha + Z_1/T_1 &\leq 1 + Z_1^{\square}/T_1 \\
\alpha + Z_2/T_2 &\leq 1 + Z_2^{\square}/T_2 \\
P_{nt} + I_{n(t-1)} - I_{nt} &= D_{nt} \\
F_t - H_t + W_t - W_{(t-1)} &= 0 \\
O_t - AO \times W_t &\leq 0 \\
\sum_{n=1}^{10} M_n P_{nt} - AR \times W_t - O_t &\leq 0
\end{aligned}$$

H_t, F_t, W_t, O_t are integer.

$$P_{nt}, I_{nt}, H_{nt}, W_{nt}, O_{nt} \geq 0 \tag{15}$$

Step 5
Solve the model to find the optimal solution for α, after that we can find Z_1 and Z_2 from (13), (14) respectively.

3. Implement FMOLP

3.1. Case Study

In this section, General Company for Vegetable Oils is used as a case study to demonstrate the proposed model. This Company produces ten types of products. We represented for each product by letter, A=solid detergent, B= liquid detergent, C= fat solid, D= liquid oil, E= toilet soap, F=liquid soap, G=detergent bleach, H=shaving cream, J=shampoo, and K= toothpaste. The time horizon of APP decision six months.

The relevant data are as follows:

Product	A	B	C	D	E	F	G	H	J	K
Hours	92	525	69	64	50	121	42	607	172	692

Table 2: Hours required to produce one ton of product

Period	A	B	C	D	E	F	G	H	J	K
1	3049.1	539	340.6	100	606.4	23.1	1.7	1.2	3.1	0/74
2	1664.1	509	708.1	152	482.7	265	3.3	2	1.8	1.1
3	1236.4	35.4	700	138	496.8	14.8	7.4	1.7	2.3	0.47
4	782.5	40.8	650	77	429.9	25	8.7	2.5	2.9	0.76
5	914.4	275	439	56	324.7	15	215	2.4	2.1	2.3
6	652.9	379	619.1	50	652.9	12.4	29.1	1.3	2.7	0.71

Table 3: Forecast demand for all products

- The initial inventory for solid detergent is 105 tons, toilet soap is 333 tons, shaving cream is 0.25 tons and shampoo is 1.8 tons
- The initial worker level is 3313 workers ($W_0 = 3313$).
- The costs of regular worker per month is500 dollar/man, the hours worked in the in one month is 140 hours
- The costs associated with hiring and firing are 774.910 dollar and 581.182 dollar per worker, respectively .
- Hours of overtime, which allowed during the period is60 hours per period$t$.
- The overtime costs 5.357 dollar per worker hour.
- Hours of regular worker per period is140hours.

3.2. Computational Results

We determined the aspiration level and the tolerance level for each objective function using a proposed approach.

The aspiration and the tolerance levels for the first objective function are $Z_1^{\square}$ =7162577 dollar,$T_1 = 177$ respectively. Either $Z_2^{\square}$=5635496 dollar and T_2=207 dollar 207 represented the aspiration level and the tolerance level for second objective.

Product	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
A	2944.1000	1664.1000	1236.4000	782.5000	914.4000	652.9000
B	53.9000	50.9000	35.4000	40.8000	27.5000	37.9000
C	340.6000	708.1000	700.0000	650.0000	439.0000	619.1000
D	100	152	138	77	56	50
E	273.4000	482.7000	496.8000	429.9000	324.7000	652.9000
F	23.1000	26.5000	14.8000	25.0000	17.1244	10.2756
G	1.7000	3.3000	7.4000	8.7000	21.5000	29.1000
H	0.9500	2.0000	1.7000	2.5000	3.7000	0
J	1.3000	1.8000	2.3000	2.9000	2.1000	2.7000
K	0.7441	1.1560	0.5081	0.6618	3.0100	0

Table 4: Production yield

Product	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
A	0	0	0	0	0	0
B	0	0	0	0	0	0
C	0	0	0	60	0	60
D	0	0	0	0	0	0
E	0	0	0	0	0	0
F	0	0	0	0	2.1.244	0
G	0	0	0	0	0	0
H	0	0	0	0	1.3000	0
J	0	0	0	0	0	0
K	0.0041	0.0601	0.0982	0	0.7100	0

Table 5: Inventory level

The overall degree of satisfaction of fuzzy multi-objective was 0.2($\alpha = 0.2$). Due to is small, we can increase α by repeating the solution to increase the value of the tolerance level, but we must stop when the value of α becomes fixed (stable) or decrease from the previous step, despite the increase in the value of the tolerance level. We increase T_1 to 1234 to get $\alpha = 0.8962$. Consequently, the optimum solutions for each objective function are

$$z_1 = 7162709.8538 \text{ dollar}, \quad z_2 = 5635803.2318 \text{ dollar}.$$

The following tables presented solution for each decision variable.

The number production quantities and carrying inventory are shown in table 4 and 5. The total number of worker and overtime hours in each period

Product	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
A	2944.1000	1664.1000	1236.4000	782.5000	914.4000	652.9000
B	53.9000	50.9000	35.4000	40.8000	27.5000	37.9000
C	340.6000	708.1000	700.0000	650.0000	439.0000	619.1000
D	100	152	138	77	56	50
E	273.4000	482.7000	496.8000	429.9000	324.7000	652.9000
F	23.1000	26.5000	14.8000	25.0000	17.1244	10.2756
G	1.7000	3.3000	7.4000	8.7000	21.5000	29.1000
H	0.9500	2.0000	1.7000	2.5000	3.7000	0
J	1.3000	1.8000	2.3000	2.9000	2.1000	2.7000
K	0.7441	1.1560	0.5081	0.6618	3.0100	0

Table 6: Production yield

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
W_t	2296	1744	1439	1081	1025	1025
H_t	0	0	0	0	0	0
F_t	1019	552	305	359	55	0
O_t	0	0	0	0	0	0.8926

Table 7: The rate of work force level

is shown in table 6.

4. Conclusion

Recently, fuzzy APP problems have fascinated many researchers. This study introduced a new formulation of the FMOLP model for solving APP problems with multiple products and multiple time periods in a fuzzy environment. This method is based on Zimmermans approach, which was used as a general way of determining the tolerance and aspiration levels. This approach can be used by any decision maker and obtain the same result. In the past, decision makers have guessed to determine the aspiration and tolerance levels, which leads to existence more than one solution to the same problem. The proposed model attempts to minimize total production costs and labor costs simultaneously.This model was applied to solve the APP problem of the General Company for Vegetable Oils. The proposed method yielded an efficient solution and it can be applied to APP problems with other parameters. Future studies may use one of the artificial intelligence algorithms to solve multi-objective APP.

References

[1] Baykasoglu A and Gocken, T. Multi-objective aggregate production planning with fuzzy parameters. Journal of advances in engineering software. 41(9), 1124-1131. 2010.

[2] Baykasoglu, A. MOAPPS 1.0: Aggregate production planning using the multiple-objective tabu search. International Journal of Production Research, 39(16), 36853702., 2001.

[3] Che, Z. H. and Chiang, C. J.A modified Pareto genetic algorithm for multi-objective build-to order supply chain planning with product assembly. Advances in Engineering Software, 41(7-8), 10111022. 2010.

[4] Krajewski L. J. and Ritzman L.P. and Malhotra M.K. Operations Management Processes and supply chaius Tenth Edition, New York, 2013.

[5] Christou, I. T., Lagodimos, A.G., and Lycopoulou, D.Hierarchical production planning for multi-product lines in the beverage industry.Production Planning and Control, 18(5), 367376. 2007.

[6] Fung, R.Y.K., J. Tang and D. Wang Multiproduct aggregate production planning with fuzzy demands and fuzzy capacities. IEEE Trans.Syst., Man, Cybern. A, Syst., Humans, 33(3), 302-313. 2003.

[7] Farris P. and Rubinstein D. Marketing Matrices Second Edition, New York, 2010.

[8] Davenport T. and Harris J. Analytics at work smarter Results Book New Jersey, 2010.

[9] Gomes da Silva, C., Figueira,J., Lisboa, J. and Barman, S.An interactive decision support system for an aggregate production planning model based on multiple criteria mixed integer linear programming. Omega, 34 (2), 167177. 2006.

[10] Holt, C. C., Modigliani, F., and Simon, H. A.A linear decision rule for production and employment scheduling. Management Science, 2, 130 1995.

[11] Kanyalkar, A. P. and Adil, G. K. An integrated aggregate and detailed planning in a multi-site production environment using linear programming. International Journal of Production Research, 43(20), 44314454. 2005.

[12] Kaveh K.D. and Ayda S Solving a New Multi-Period Multi-Objective Multi-Product Aggregate Production Planning Problem Using Fuzzy Goal Programming. Industrial Engineering and Management Systems. 13(4),369-382 ISSN 1598-7248EISSN 2234-6473 2014.

[13] Liang, T. F. Application of interactive possibilistic linear programming to aggregate production planning with multiple imprecise objectives. Production Planning and Control, 18(7), 548-560. 2007.

[14] Leung, S. C. H and Chan, S. S.W. A goal programming model for aggregate production planning with resource utilization constraint. Computers and Industrial Engineering, 56(3), 10531064. 2009.

[15] Liu, Z., Chua, D. K. H., and K. Yeoh Aggregate production planning for shipbuilding with variation-inventory trade-offs. International Journal of Production Research, 49(20), 62496272 2011.

[16] Masud, A. S. M. and Hwang, C. L. An aggregate production planning model and application of three multiple objective decision methods. International Journal of Production Research, 18(6), 741752. 1980.

[17] Navid Mortezaei, Norzima Zulkifli, Tang Sai Hong, Rosnah Mohd Yusuff Multi-objective aggregate production planning model with fuzzy parameters and its solving methods. Life Science Journal.10(4), 2406-2414. 2013.

[18] Najmeh Madadi and Kuan Yew Wong A Multi objective Fuzzy Aggregate Production Planning Model Considering Real Capacity and Quality of Products. Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2014, Article ID 313829, 15 pages http://dx.doi.org/10.1155/2014/313829 2014.

[19] Pradenas, L., F. Penailillo, and Ferland, J.Aggregate production planning problem. A new algorithm. Electronic Notes in Discrete Mathematics. 18, 193199. 2014.

[20] Phruksaphanrat, B.preemptive possibilistic linear programming: application to aggregate production planning world academy of science, engineering and technology. Vol:5, 08-24. 2011.

[21] Ramezanian R., Rahmani, D., and Barzinpour, F.An aggregate production planning model for two phase production systems: solving with genetic algorithmand tabu search. Expert Systems with Applications, 39(1),12561263.. 2012.

[22] Rifai, A. K. A note on the structure of the goal-programming model: assessment and evaluation. International Journal of Operations and Production Management. 16(1), 4049. 1996.

[23] Romero, C. A general structure of achievement function for a goal programming model, European Journal of Operational Research. 153(3), 675686. 2004.

[24] Sillekens, T., Koberstein, A., and Suhl, L. Aggregate production planning in the automotive industry with special consideration of workforce flexibility. International Journal of Production Research, 49(17),50555078 2011.

[25] Tang, J., Wang, D. and R.Y.K. Fung Fuzzy formulation for multi-product aggregate production planning. Production Planning and Control: The Management of Operations, 11(7):670-676 2000.

[26] Vassiliadis, C G., Vassiliadou,M G., Papageorgiou, L G., and Pistikopoulos E.N. Simultaneous maintenance considerations and production planning in multi-purpose plants. in Proceedings of the Annual Reliability and Maintainability Symposium, pp. 228233, IEEE. 2000.

[27] Wang, R.C. and Fang, H.H. Aggregate production planning with multiple objectives in a fuzzy environment. European Journal of Operational Research, 133(3),521-536. 2001

[28] Wang, R.C. and Liang, TF Aggregate production planning with multiple fuzzy goals. Int J Adv Manuf Technol. 25(56):589597 2005

[29] Wang, R.C. and Liang, T.F. Application of fuzzy multi-objective linear programming to aggregate production planning. Computer and formulation Industrial Engineering, 46(1): 17-41. 2004

[30] Yan, H. S., Xia, Q., Zhu, M., Liu X., and Guo, Z. Integrated production planning and scheduling on automobile assembly lines. IIE Transactions, 35(8), 711725. 2003

[31] Zimmermann, H. J. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems, 1(1), pp. 45-56. 1978