SUPER EDGE ANTI-MAGIC VERTEX LABELING AND SUPER EDGE ANTI-MAGIC TOTAL LABELING OF A CYCLE WITH ZIG-ZAG CHORDS

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Abstract: In this paper, we show that super edge anti-magic vertex labeling and super edge anti-magic total labeling of cycles by adding $(n-3)$ zig-zag chords.

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Key Words: graph labeling, edge anti-magic vertex labeling, super edge antimagic vertex labeling, edge anti-magic total labeling, super edge antimagic total labeling, cycle with $(n-3)$ zig-zag chords

1. Introduction

All graphs are finite, simple and undirected. The graph G has vertex-set $V(G)$ and edge-set $E(G)$. Unless otherwise noted, $|V(G)| = p$ and $|E(G)| = q$.

A labeling of a graph is any map that carries some set of graph elements to numbers (usually to the positive or non-negative integers). Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of "constant-sum" property.

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Simanjuntak, Miller and Bertault [8] defined an (a, d) -edge-antimagic vertex $((a, d)$ -EAV) labeling for a graph $G(V, E)$ as an injective mapping f from V onto the set $\{1, 2, \dots, n\}$ with the property that the edge-weights $\{w(xy) : w(xy) = f(x) + f(y), xy \in E\}$, form an arithmetic sequence with the first term a and difference d , where $a > 0$ and $d \geq 0$ are two fixed integers. An (a, d) -EAV labeling is called super (a, d) -edge antimagic vertex $((a, d)$ -SEAV) labeling if $f(v) = \{1, 2, \dots, n\}$.

Acharya and Hegde [1] (see also [6]) introduced the concept of a strongly (a, d) -indexable labeling which is equivalent to (a, d) -EAV labeling. The relationship between the sequential graphs and the graphs having an (a, d) -EAV labeling is shown in [3].

An (a, d) -edge antimagic total $((a, d)$ -EAT) labeling is a bijection f from $V \cup E$ onto $\{1, 2, \dots, v + e\}$ with the property that the sums of the label on the edges and the labels of their end points form an arithmetic sequence starting from a and having a common difference d . This labeling is a natural extension of the notion of edge magic labeling which was originally introduced by Kotzig and Rosa in [7], where edge-magic labeling is called magic valuation. Relationships between (a, d) -EAT labeling and other labelings, namely, (a, d) -EAV labeling are presented in [2].

An (a, d) -EAT labeling is called super (a, d) -edge antimagic total $((a, d)$ -SEAT) labeling if $f(V) = 1, 2, \dots, n$. This labeling is a natural extension of the notion of a super edge-magic labeling defined by Enomoto et al. in [4]. A graph that has an (a, d) -EAV $((a, d)$ -EAT or (a, d) -SEAT) labeling is called an (a, d) -EAV $((a, d)$ -EAT or (a, d) -SEAT) graph. J.A. MacDougall and W.D. Wallis conjecture in [9] that every cycle with a chord is strong edge magic labeling. Also M. Baca and M. Murugan conjecture in [10] that every cycle with a chord is super edge-antimagic labelling.

A cycle with $(n-3)$ chords is a graph obtained from a cycle $C_n (n \geq 4)$ by adding $(n-3)$ zig-zag chords to the non-consecutive vertices of the cycle, that is between each pair of non adjacent vertices $v_i v_{(n-i+2)}$, for $2 \leq i \leq \frac{n}{2}$ and $v_i v_{(n-i+3)}$, for $3 \leq i \leq \frac{n}{2}$ for even cycles and $v_i v_{(n-i+2)}$, for $2 \leq i \leq \frac{n-1}{2}$ and $v_i v_{(n-i+3)}$, for $3 \leq i \leq \frac{n+1}{2}$ for odd cycles.

2. Main Results

Theorem 1. *A cycle C_n for $n \geq 4$ with $(n-3)$ zigzag chords has a $(3,1)$ -super edge anti-magic vertex labeling.*

Proof. Let C_n be a cycle on n vertices. We denote the vertices of C_n as

$v_1, v_2, v_3, \dots, v_n$ in the clockwise direction and denote the edges of C_n with $(n-3)$ zigzag chords as $e_1, e_2, e_3, \dots, e_{(2n-3)}$ such that $e_i = v_i v_{(i+1)}$ for $1 \leq i \leq (n-1)$, $e_n = v_n v_1$ and for chords, when n is even

$$\begin{aligned} e_{(n+i-1)} &= v_i v_{(n-i+2)}, \quad 2 \leq i \leq \frac{n}{2} \\ e_{\frac{(3n+2i-6)}{2}} &= v_i v_{(n-i+3)}, \quad 3 \leq i \leq \frac{n}{2} \end{aligned}$$

when n is odd

$$\begin{aligned} e_{(n+i-1)} &= v_i v_{(n-i+2)}, \quad 2 \leq i \leq \frac{n-2}{2} \\ e_{\frac{(3n+2i-5)}{2}} &= v_i v_{n-i+3}, \quad 3 \leq i \leq \frac{n+1}{2} \end{aligned}$$

The labeling for the vertices of C_n are given as follows. Define

$$\begin{aligned} f(v_1) &= 2n - 2 \\ f(v_2) &= 2n - 1 \\ f(v_i) &= f(v_{i-1}) + 2, \quad 3 \leq i \leq \frac{n+2}{2}, \text{ if } n \text{ even} \\ f(v_i) &= f(v_{i-1}) - 1, \quad i = \frac{n+4}{2}, \text{ if } n \text{ even} \\ f(v_i) &= f(v_{i-1}) - 2, \quad \frac{n+6}{2} \leq i \leq n, \text{ if } n \text{ even} \\ f(v_i) &= f(v_{i-1}) + 2, \quad 3 \leq i \leq \frac{n+1}{2}, \text{ if } n \text{ odd} \\ f(v_i) &= f(v_{i-1}) + 1, \quad i = \frac{n+3}{2}, \text{ if } n \text{ odd} \\ f(v_i) &= f(v_{i-1}) - 2, \quad \frac{n+5}{2} \leq i \leq n, \text{ if } n \text{ odd} \end{aligned}$$

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct. Now the edge-weights can be labelled as,

$$\begin{aligned} f(e_i) &= f(v_i) + f(v_{i+1}), \quad 1 \leq i \leq n-1 \\ f(e_n) &= f(v_n) + f(v_1) \end{aligned}$$

when n is even, the edge labeling of chords are

$$f(e_{(n+i-1)}) = f(v_i) + f(v_{n-i+2}), \quad 2 \leq i \leq \frac{n}{2}$$

$$f(e_{\frac{3n+2i-6}{2}}) = f(v_i) + f(v_{n-i+3}), \quad 3 \leq i \leq \frac{n}{2}$$

when n is odd, the edge labeling of chords are,

$$\begin{aligned} f(e_{n+i-1}) &= f(v_i) + f(v_{n-i+2}), \quad 2 \leq i \leq \frac{n-1}{2} \\ f(e_{\frac{3n+2i-7}{2}}) &= f(v_i) + f(v_{n-i+3}), \quad 3 \leq i \leq \frac{n+1}{2} \end{aligned}$$

Hence the cycle C_n with $(n-3)$ zigzag chords has a $(3,1)$ -super edge anti-magic vertex labeling. $\square$

Theorem 2. *A cycle C_n for $n \geq 4$ with $(n-3)$ zigzag chords has a $(n+4,2)$ -super edge anti-magic total labelling.*

Proof. Let C_n be a cycle on n vertices. We denote the vertices of C_n as $v_1, v_2, v_3, \dots, v_n$ in the clockwise direction and denote the edges of C_n with $n-3$ zigzag chords as $e_1, e_2, e_3, \dots, e_{2n-3}$ such that $e_i = v_i v_{(i+1)}$ for $1 \leq i \leq n-1$, $e_n = v_n v_1$ and for chords, when n is even

$$\begin{aligned} e_{(n+i-1)} &= v_i v_{(n-i+2)}, \quad 2 \leq i \leq \frac{n}{2} \\ e_{\frac{3n+2i-6}{2}} &= v_i v_{(n-i+3)}, \quad 3 \leq i \leq \frac{n}{2} \end{aligned}$$

when n is odd

$$\begin{aligned} e_{(n+i-1)} &= v_i v_{(n-i+2)}, \quad 2 \leq i \leq \frac{n-1}{2} \\ e_{\frac{3n+2i-7}{2}} &= v_i v_{n-i+3}, \quad 3 \leq i \leq \frac{n+1}{2} \end{aligned}$$

The labeling for the vertices of C_n are given as follows. Define

$$\begin{aligned} f(v_1) &= 2n-2 \\ f(v_2) &= 2n-1 \\ f(v_i) &= f(v_{i-1}) + 2, \quad 3 \leq i \leq \frac{n+2}{2} \\ f(v_i) &= f(v_{i-1}) - 1, \quad i = \frac{n+4}{2}, \text{ if } n \text{ even} \\ f(v_i) &= f(v_{i-1}) - 2, \quad \frac{n+6}{2} \leq i \leq n \end{aligned}$$

$$\begin{aligned}
f(v_i) &= f(v_{i-1}) + 2, \quad 3 \leq i \leq \frac{n+1}{2} \\
f(v_i) &= f(v_{i-1}) + 1, \quad i = \frac{n+3}{2}, \text{ if } n \text{ odd} \\
f(v_i) &= f(v_{i-1}) - 2, \quad \frac{n+5}{2} \leq i \leq n
\end{aligned}$$

From the above definition it is observed that the vertices of C_n are labeled from 1 to n and are distinct.

Now the edge-weights can be labelled as,

$$\begin{aligned}
f(e_i) &= f(v_i) + f(v_{i+1}) + (n-2), \quad 1 \leq i \leq n-1 \\
f(e_n) &= f(v_n) + f(v_1) + (n-2)
\end{aligned}$$

when n is even, the edge labeling of chords are,

$$\begin{aligned}
f(e_{n+(i-1)}) &= f(v_i) + f(v_{n-i+2}) + (n-2), \quad 2 \leq i \leq \frac{n}{2} \\
f(e_{\frac{3n+2i-6}{2}}) &= f(v_i) + f(v_{n-i+3}) + (n-2), \quad 3 \leq i \leq \frac{n}{2}
\end{aligned}$$

when n is odd, the edge labeling of chords are,

$$\begin{aligned}
f(e_{n+i-1}) &= f(v_i) + f(v_{n-i+2}) + (n-2), \quad 2 \leq i \leq \frac{n-1}{2} \\
f(e_{\frac{3n+2i-7}{2}}) &= f(v_i) + f(v_{n-i+3}) + (n-2), \quad 3 \leq i \leq \frac{n+1}{2}
\end{aligned}$$

Hence the cycle C_n with $(n-3)$ zigzag chords has a $(n+4,2)$ -super edge anti-magic total labeling. $\square$

Figure 1: $(11,2)$ -Super edge anti-magic total labeling of C_7

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