International Journal of Pure and Applied Mathematics
Volume 105 No. 3 2015, 415-425
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v105i3.9

ijpam.eu

QUASI CONFORMALLY SYMMETRIC WEYL MANIFOLDS

Fusun Nurcan
Department of Mathematics
Marmara University
Göztepe Campus 34722, İstanbul, TURKEY

Abstract: The present paper deals with a type of non-quasi flat Weyl manifold which is called *quasi conformally symmetric*. Some necessary and sufficient conditions between concircularly symmetric, conformally symmetric and quasi conformally symmetric Weyl manifolds are obtained and a special condition on the Weyl manifold admitting a semi-symmetric non-metric connection is studied.

AMS Subject Classification: 53C05, 53C25, 53C35
Key Words: Weyl manifold, quasi conformal curvature tensor, conformally symmetric manifold, concircularly symmetric manifold, quasi conformally symmetric manifold

1. Introduction

The physical concepts in Weyl geometry has been widely reviewed in the review paper (see [9]). Herman Weyl first introduced a gauge invariant theory to unify gravity and electromagnetic theories in 1918. This theory is not acceptible as a unified theory since the electromagnetic potential does not couple to spinor being essential for the electromagnetic theory. This does not mean that Weyl geometry has a physical meaning as well in different theories and in the second part of the 20th century, the Weyl geometry has been studied

Received: September 2, 2015

in some research filds of physics such as *quantum mechanics, particle physics, gravity* and *cosmology*. Despite the unified theory of Weyl not being acceptable as physical theory, it introduced a useful theory in *differential geometry*. The mathematics of the theory is a generalization of the Riemannian geometry and the connection is an instructive example of non-metric connections.

A *Weylian metric* on a differentiable manifold M can be given by pairs (g, φ) of a non-degenerate symmetric differential 2-form g and a differential 1-form φ. The Weylian metric consists of the equivalance class of such pairs, with $\left(\widetilde{g}, \widetilde{\varphi}\right) \sim (g, \varphi)$ if and only if

$$(i) \quad \widetilde{g} = \lambda^2 g \quad , \quad (ii) \quad \widetilde{\varphi} = \varphi - d \log \lambda \tag{1}$$

for a strictly positive real function $\lambda > 0$ on M. Choosing a representative means to *gauge* the Weylian metric; g is then the *Riemannian component* and φ the *scale connection* of the gauge. A change of representative (1) is called a *Weyl(scale) transformation;* it consists of a rescaling (i) and a *scale gauge transformation* (ii).

A manifold with a Weylian metric (M, g, φ) will be called a *Weyl manifold*.

In the recent mathematical literature, a *Weyl structure* on a differentiable manifold M is spesified by a pair (g, φ) consisting of a conformal metric g and an affine, which is torsion-free, connection Γ, respectively its covariant derivative ∇. For any conformal metric g, there is a differential 1-form φ_g such that

$$\nabla g + 2\varphi_g \otimes g = 0 \tag{2}$$

which is called *weak compatibility* of the affine connection with the metric. (2) can be expressed in local coordinates by

$$\nabla_k g_{ij} - 2 g_{ij} T_k = 0 \tag{3}$$

where T_k is a complementary covariant vector field (see [7]). Such a Weyl manifold will be denoted by $W_n(g_{ij}, T_k)$. If $T_k = 0$ or T_k is gradient, a Riemannian manifold is obtained.

One could also formulate above compatibility condition by

$$\Gamma -_g \Gamma = 1 \otimes \varphi_g + \varphi_g \otimes 1 - g \otimes \varphi_g^* \tag{4}$$

where 1 denotes identity in $Hom(V, V)$ for every $V = T_x M$, φ_g^* is the dual of φ_g with respect to g and ${}_g\Gamma$ is the Levi-Civita connection of g. In local coordinates, (4) is given by

$$\Gamma^i_{jk} = ijk - \delta^i_j T_k - \delta^i_k T_j + g_{jk} T^i \tag{5}$$

where ijk's are the coefficients of the Levi-Civita connection (see [7]).

The *curvature tensor* R^h_{ijk} of the symmetric connection Γ on the Weyl manifold is defined by

$$R^h_{ijk} = \partial_j \Gamma^h_{ik} - \partial_k \Gamma^h_{ij} + \Gamma^h_{rj}\Gamma^r_{ik} - \Gamma^h_{rk}\Gamma^r_{ij}. \tag{6}$$

In local coordinates, the *conformal curvature tensor* $C(X,Y,Z,U)$ and the *concircular curvature tensor* $\widetilde{C}(X,Y,Z,U)$ of the symmetric connection Γ on the Weyl manifold are expressed by

$$\begin{aligned} C^h_{ijk} = R^h_{ijk} - \frac{1}{n}\delta^h_i R^r_{rjk} + \frac{1}{n-2}\left(\delta^h_j R_{ik} - \delta^h_k R_{ij} + g_{ik}g^{mh}R_{mj} - g_{ij}g^{mh}R_{mk}\right) \\ - \frac{1}{n(n-2)}\left(\delta^h_j R^r_{rki} - \delta^h_k R^r_{rji} + g_{ik}g^{mh}R^r_{rjm} - g_{ij}g^{mh}R^r_{rkm}\right) \\ + \frac{r}{(n-1)(n-2)}\left(\delta^h_k g_{ij} - \delta^h_j g_{ik}\right), \end{aligned} \tag{7}$$

$$\widetilde{C}^h_{ijk} = R^h_{ijk} - \frac{r}{n(n-1)}(\delta^h_k g_{ij} - \delta^h_j g_{ik}) \tag{8}$$

with the help of (6), where R^h_{ijk}, R_{ij} and r denote the curvature tensor, the Ricci tensor and the scalar curvature tensor of Γ, respectively (see [5], [8]).

Let $\{e_i\}_{i=1}^n$ be an orthonormal basis of the tangent space at any point of the manifold. Then, from (7) and (8),

$$\sum_{i=1}^n C(e_i,Y,Z,e_i) = \sum_{i=1}^n C(Y,e_i,e_i,Z) = 0 \tag{9}$$

and

$$\sum_{i=1}^n \widetilde{C}(e_i,Y,Z,e_i) = \sum_{i=1}^n \widetilde{C}(Y,e_i,e_i,Z) = P(Y,Z) = S(Y,Z) - \frac{r}{n}g(Y,Z) \tag{10}$$

where $S(Y,Z)$ denotes the Ricci tensor of type $(0,2)$. From (10), it follows that

$$\sum_{i=1}^n P(e_i,e_i) = 0.$$

By using (7), and (8), the conformal curvature tensor C^h_{ijk} of the connection Γ is expressed in terms of the concircular curvature tensor $\widetilde{C}^h_{ijk}$ by the following equation:

$$C^h_{ijk} = \widetilde{C}^h_{ijk} - \frac{1}{n}\delta^h_i \widetilde{C}^r_{rjk} + \frac{1}{n-2}\{\delta^h_j \widetilde{C}_{ik} - \delta^h_k \widetilde{C}_{ij} + g_{ik} g^{mh} \widetilde{C}_{mj} - g_{ij} g^{mh} \widetilde{C}_{mk}\}$$
$$- \frac{1}{n(n-2)}\{\delta^h_j \widetilde{C}^r_{rki} - \delta^h_k \widetilde{C}^r_{rji} + g_{ik} g^{mh} \widetilde{C}^r_{rjm} - g_{ij} g^{mh} \widetilde{C}^r_{rkm}\}. \quad (11)$$

2. Quasi Conformally Symmetric Weyl Manifolds

In 1968, Yano and Sawaki defined and studied a new curvature tensor called *quasi conformal curvature tensor* on a Riemannian manifold of dimension n which includes both the conformal and concircular curvature tensor (see [14]).

The quasi conformal curvature tensor $W(X,Y,Z,U)$ of type $(0,4)$ on a Weyl manifold of dimension n $(n>3)$ is defined by

$$W(X,Y,Z,U) = -(n-2)\, bC(X,Y,Z,U) + [a+(n-2)\,b]\, \widetilde{C}(X,Y,Z,U) \quad (12)$$

where a, b are arbitrary constants not simultaneously zero, C and $\widetilde{C}$ are conformal curvature tensor and concircular curvature tensor of type $(0,4)$, respectively.

Substituting (9) and (10) in (12) yields

$$\sum_{i=1}^{n} W(e_i, Y, Z, e_i) = \sum_{i=1}^{n} W(Y, e_i, e_i, Z) = [a+(n-2)\,b]\, P(Y,Z). \quad (13)$$

In particular, if $a=1$ and $b=\frac{-1}{n-2}$, then the quasi conformal curvature tensor reduces to conformal curvature tensor. In a similar way, if $a=1$ and $b=0$, then the quasi conformal curvature reduces to concircular curvature tensor.

By substituting (7) and (8) in (12), the quasi conformal curvature tensor can be expressed by

$$W^h_{ijk} = aR^h_{ijk} + b\left\{\delta^h_k R_{ij} - \delta^h_j R_{ik} + g_{ij} g^{mh} R_{mk} - g_{ik} g^{mh} R_{mj}\right\} \quad (14)$$
$$+ \frac{b}{n}\left\{(n-2)\,\delta^h_i R^r_{rjk} + \delta^h_j R^r_{rki} - \delta^h_k R^r_{rji} + g_{ik} g^{mh} R^r_{rjm} - g_{ij} g^{mh} R^r_{rkm}\right\}$$
$$- \frac{r}{n}\left\{\frac{a}{n-1} + 2b\right\}\left(\delta^h_k g_{ij} - \delta^h_j g_{ik}\right)$$

in local coordinates.

An alternative definition of the quasi conformal curvature tensor is given in the form of

$$W^h_{ijk} = a\widetilde{C}^h_{ijk} + b\left\{\delta^h_k \widetilde{C}_{ij} - \delta^h_j \widetilde{C}_{ik} + g_{ij} g^{mh} \widetilde{C}_{mk} - g_{ik} g^{mh} \widetilde{C}_{mj}\right\} \tag{15}$$
$$+ \frac{b}{n}\left\{(n-2)\,\delta^h_i \widetilde{C}^r_{rjk} + \delta^h_j \widetilde{C}^r_{rki} - \delta^h_k \widetilde{C}^r_{rji} + g_{ik} g^{mh} \widetilde{C}^r_{rjm} - g_{ij} g^{mh} \widetilde{C}^r_{rkm}\right\}$$

by using (8) and (10) in (14).

The study of symmetric Riemannian manifolds began with the work of E.Cartan. A Riemannian manifold (M_n, g) is said to be *locally symmetric* due to Cartan (see [1]), if its curvature tensor $R\,(Y, Z, U, V)$ satisfies the condition $(\nabla_X R)\,(Y, Z, U, V) = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. This condition of local symmetry is equivalent to the fact that at every point $P \in M_n$, the local geodesic symmetry $F\,(P)$ is an isometry. The class of symmetric Riemannian manifolds is very natural generalization of the class of the manifolds of constant curvature.

During the last five decades, the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as *recurrent manifold* by A.G.Walker (see [13]), *semi-symmetric manifold* by Z.I.Szabo (see [10]), *pseudo symmetric manifold* by M.C.Chaki (see [2]), *generalized pseudo symmetric manifold* by M.C.Chaki (see [3]) and *weakly symmetric manifold* and *weakly projectively symmetric manifold* by L.Tamassy and T.Q.Binh (see [11]). On the other hand M.C.Chaki defined *conformally symmetric manifold* with B.Gupta in (see [4]).

The object of the present paper is to study a quasi conformally symmetric Weyl manifold which is defined by the following definition and is denoted by $(WSW)_n$:

Definition 2.1. A non-flat Weyl manifold is said to be conformally symmetric, concircularly symmetric and quasi conformally symmetric, if the conformal curvature tensor $C\,(Y, Z, U, V)$, the concircular curvature tensor $\widetilde{C}\,(Y, Z, U, V)$ and the quasi conformal curvature tensor $W\,(Y, Z, U, V)$ satisfy the conditions

$$(\nabla_X C)\,(Y, Z, U, V) = 0, \quad (\nabla_X \widetilde{C})\,(Y, Z, U, V) = 0, \quad (\nabla_X W)\,(Y, Z, U, V) = 0$$

respectively, for all vector fields $X, Y, Z, U, V \in \chi\,(M)$ which denotes the Lie algebra of all smooth vector fields on the manifold M and ∇ is the operator of covariant differentiation with respect to the Weylian metric g. Furthermore, in local coordinates, the same conditions are given by

$$\nabla_l C^h_{ijk} = 0, \quad \nabla_l \widetilde{C}^h_{ijk} = 0, \quad \nabla_l W^h_{ijk} = 0$$

respectively, where ∇ denotes the covariant derivative with respect to the symmetric connection Γ.

The paper is organized as follows:

Section 2 deals with some basic results of $(WSW)_n$. Section 3 gives some necessary and sufficient conditions on quasi conformally symmetric Weyl manifolds. Finally, in the last section a special condition is studied on a $(WSW)_n$ admitting a semi symmetric non-metric connection.

Theorem 2.1. *A concircularly symmetric Weyl manifold is quasi conformally symmetric.*

Proof. Assume that a Weyl manifold is concircularly symmetric. By differentiating (11) covariantly with respect to ∇ and using $\nabla_l \widetilde{C}^h_{ijk} = 0$ or equivalently $\nabla_l \widetilde{C}_{ij} = 0$ from Definition 2.1,

$$\begin{aligned}\nabla_l C^h_{ijk} = {} & \tfrac{1}{n-2}[\left\{ g^{mh}\nabla_l g_{ik} + g_{ik}\nabla_l g^{mh}\right\} \widetilde{C}_{mj} - \{g^{mh}\nabla_l g_{ij} \\ & + g_{ij}\nabla_l g^{mh}\}\widetilde{C}_{mk}] - \tfrac{1}{n(n-2)}[\left\{ g^{mh}\nabla_l g_{ik} + g_{ik}\nabla_l g^{mh}\right\} \widetilde{C}^r_{rjm} \\ & - \left\{ g^{mh}\nabla_l g_{ij} + g_{ij}\nabla_l g^{mh}\right\} \widetilde{C}^r_{rkm}]. \end{aligned} \tag{16}$$

By substituting the equations $\nabla_k g_{ij} = 2T_k g_{ij}$ and $\nabla_k g^{ij} = -2T_k g^{ij}$ by [7] into the formula (16), it is obtained as

$$\nabla_l C^h_{ijk} = 0$$

which states that a Weyl manifold is conformally symmetric. Then, by means of the covariant derivative of (12), it is seen that the manifold is quasi conformally symmetric. $\square$

Now, suppose that a Weyl manifold be quasi conformally symmetric. Then, by taking derivative of (12) covariantly and using Definition 2.1 , it is obtained as

$$(n-2)\, b\, (\nabla_X C)\,(Y,Z,U,V) = [a + (n-2)\, b]\, (\nabla_X \widetilde{C})\,(Y,Z,U,V)\,. \tag{17}$$

From (17), we have:

Corollary 2.1. *A $(WSW)_n$ for which $a + (n-2)\, b = 0$ and $b \neq 0$ is conformally symmetric.*

Corollary 2.2. *The conformal curvature tensor $C(Y,Z,U,V)$ and the concircular curvature tensor $\widetilde{C}(Y,Z,U,V)$ of a $(WSW)_n$ for which $a+(n-2)b \neq 0$ and $a=0$ satisfy the following condition:*

$$(\nabla_X C)(Y,Z,U,V) = (\nabla_X \widetilde{C})(Y,Z,U,V). \tag{18}$$

Corollary 2.3. *A $(WSW)_n$ for which $a+(n-2)b \neq 0$ and $b=0$ is concircularly symmetric (and conformally symmetric).*

By (9) and (10), replacing Y and V by e_i in (17) gives

$$[a+(n-2)b](\nabla_X P)(Z,U) = 0$$

which leads us the following corollary:

Corollary 2.4. *A $(WSW)_n$ for which $a+(n-2)b \neq 0$ for any a, b is concircularly Ricci symmetric.*

3. Some Necessary and Sufficient Conditions On a Quasi Conformally Symmetric Weyl Manifold

The quasi conformal curvature tensor $W(X,Y,Z,U)$ contains both the concircular curvature tensor $\widetilde{C}(X,Y,Z,U)$ and the conformal curvature tensor $C(X,Y,Z,U)$. Therefore, in this section, we will improve some necessary and sufficient conditions for a $(WSW)_n$ to be concircularly symmetric and conformally symmetric, respectively.

Theorem 3.1. *A necessary and sufficient condition for a conformally symmetric Weyl manifold to be concircularly symmetric is that it is concircularly Ricci symmetric.*

Proof. Now, assume that a Weyl manifold is conformally symmetric. Then, by taking covariantly derivative of (7) with respect to ∇, using $\nabla_l C^h_{ijk} = 0$ from Definition 2.1 and remembering that $R_{ij} = R_{(ij)} + R_{[ij]}$, it is obtained that

$$\begin{aligned}\nabla_l R^h_{ijk} = {} & \frac{2}{n}\delta^h_i \nabla_l R_{[kj]} - \frac{1}{n-2}\Big\{\delta^h_j \nabla_l R_{(ik)} - \delta^h_k \nabla_l R_{(ij)} + g_{ik} g^{mh} \nabla_l R_{(mj)} \\ & - g_{ij} g^{mh} \nabla_l R_{(mk)}\Big\} - \frac{1}{n}\Big\{\delta^h_j \nabla_l R_{[ik]} - \delta^h_k \nabla_l R_{[ij]} + g_{ik} g^{mh} \nabla_l R_{[mj]} \\ & - g_{ij} g^{mh} \nabla_l R_{[mk]}\Big\} - \frac{(\nabla_l + 2T_l)\, r}{(n-1)(n-2)}\{\delta^h_k g_{ij} - \delta^h_j g_{ik}\}.\end{aligned} \tag{19}$$

Under consideration of a concircularly Ricci symmetric Weyl manifold, $\nabla_l \widetilde{C}_{ij} = 0$ implies

$$\nabla_l R_{ij} = \frac{(\nabla_l + 2T_l)\, r}{n} g_{ij}$$

by (10) and therefore $\nabla_l R_{[ij]} = 0$. So, (19) reduces to

$$\nabla_l R^h_{ijk} = \frac{(\nabla_l + 2T_l)\, r}{n\,(n-1)} \{\delta^h_k g_{ij} - \delta^h_j g_{ik}\}$$

which is equivalent to

$$\nabla_l \widetilde{C}^h_{ijk} = 0$$

is obtained by (8). This means that the Weyl manifold is concircularly symmetric.

Conversely, a conformally symmetric Weyl manifold which is concircularly symmetric is automatically concircularly Ricci symmetric. □

By using Theorem 2.1 and Theorem 3.1, we have:

Corollary 3.1. *A conformally symmetric Weyl manifold which is concircularly Ricci symmetric is quasi conformally symmetric .*

Theorem 3.2. *A necessary and sufficient condition for a $(WSW)_n$ for which $a + (n-2)\, b = 0$ and $b \neq 0$ to be concircularly symmetric is that it is concircularly Ricci symmetric.*

Proof. Suppose that a $(WSW)_n$ for which $a + (n-2)\, b = 0$ and $b \neq 0$ is concircularly symmetric. In this case, the manifold is automatically concircularly Ricci symmetric.

Conversely, if a $(WSW)_n$ for which $a + (n-2)\, b = 0$ and $b \neq 0$ which is conformally symmetric by Corollary 2.1 is concircularly Ricci symmetric, then the manifold is concircularly symmetric by Theorem 3.1. □

Theorem 3.3. *A necessary and sufficient condition for a $(WSW)_n$ for which $a + (n-2)\, b \neq 0$ for $a \in R$, $b \neq 0$ to be conformally symmetric (concircularly symmetric) is that it is concircularly symmetric (conformally symmetric).*

Proof. Firstly, we have to say that $a \in R$ is considered as the union of $a \neq 0$ and $a = 0$. Let's consider a $(WSW)_n$ for which $a + (n-2)\, b \neq 0$ *for* $a \in R$, $b \neq 0$. Under consideration of conformally symmetric Weyl manifold, $(WSW)_n$ is obtained as concircularly symmetric by Corollary 2.2, Corollary 2.4 and Theorem 3.1.

Conversely, under consideration of concircularly symmetric Weyl manifold, $(WSW)_n$ for which $a+(n-2)\,b \neq 0$ *for* $a \in R$, $b \neq 0$ is obtained as conformally symmetric by taking derivative of (11) covariantly with respect to ∇. □

4. A $(WSW)_n$ Admitting a Semi-Symmetric Non-Metric Connection With a Special Condition

A generalized connection $\overline{\nabla}$ on the Weyl manifold is defined by V.Murgescu [6] as follows:

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + a_{jkh}g^{hi} \tag{20}$$

where $$a_{jkh} = g_{jr}\Omega^r_{kh}+g_{rk}\Omega^r_{jh}+g_{rh}\Omega^r_{jk}$$

and Γ^i_{jk}'s are the coefficients of the symmetric connection ∇ defined in Section 1.

By choosing

$$\Omega^i_{jk} = \delta^i_j a_k - \delta^i_k a_j$$

in (20), the following equation denotes the coefficients $\overline{\Gamma}^i_{jk}$'s of a semi symmetric non-metric connection $\overline{\nabla}$ on the Weyl manifold:

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + \delta^i_k S_j - g_{jk}S^i \tag{21}$$

(see [12]) where $S_i = -2a_i$ and a_i is an arbitrary covariant vector field.

The curvature tensor $\overline{R}_{mijk}$ of the semi symmetric non-metric connection $\overline{\nabla}$ on the Weyl manifold is defined by

$$\overline{R}_{mijk} = R_{mijk} + g_{mk}S_{ij} - g_{mj}S_{ik} + g_{ij}S_{mk} - g_{ik}S_{mj}, \tag{22}$$

where $$S_{ij} = S_{i,j}-S_iS_j+\frac{1}{2}g_{ij}g^{kr}S_kS_r$$

and $S_{i,j}$ denotes the covariant derivative of S_i with respect to the symmetric connection ∇.

By means of (22), the Ricci tensor $\overline{R}_{ij}$ and the scalar curvature $\overline{r}$ of the connection $\overline{\nabla}$ are obtained as

$$\overline{R}_{ij} = R_{ij} + (n-2)\,S_{ij} + g_{ij}S, \tag{23}$$

$$\overline{r} = r + 2\,(n-1)\,S. \tag{24}$$

Theorem 4.1. *The quasi conformal curvature tensor of a Weyl manifold with a vanishing curvature tensor with respect to a semi symmetric non-metric connection* $\overline{\nabla}$ *is of the form*

$$W(X,Y,Z,U) = [a + (n-2)\, b]\, \widetilde{C}(X,Y,Z,U). \tag{25}$$

Proof. Suppose that the curvature tensor $\overline{R}_{mijk}$ with respect to $\overline{\nabla}$ vanishes on the manifold, i.e., $\overline{R}_{mijk} = 0$. Then the curvature tensor R_{mijk} of the connection ∇ is in the form of

$$R_{mijk} = g_{mj} S_{ik} - g_{mk} S_{ij} + g_{ik} S_{mj} - g_{ij} S_{mk}. \tag{26}$$

Furthermore, from (26), we have $R^{h}_{hjk} = 0$ which means that R_{kj} is symmetric. By using this fact and (26) in (7), the conformal curvature tensor C_{mijk} of the connection ∇ is obtained as

$$C_{mijk} = 0.$$

So, by using this result in (12), we obtain (25). □

By taking covariant differentiation of (25), under consideration of a quasi conformally symmetric Weyl manifold, it is obtained that

$$[a + (n-2)\, b]\, (\nabla_X \widetilde{C})(Y,Z,U,V) = 0$$

which leads us the following:

Corollary 4.1. *A* $(WSW)_n$ *for which* $a + (n-2)\, b \neq 0$ *for any* a, b *with a vanishing curvature tensor with respect to a semi symmetric non-metric connection* $\overline{\nabla}$ *is concircularly symmetric.*

References

[1] E. Cartan, Sur une classe remarquable d'espaces de Riemann, *Bull. Soc. Math. France*, **54** (1926), 214-264.

[2] M. C. Chaki, On pseudo-symmetric manifolds, *An. Stiint. Ale Univ., "AL I CUZA" Din Iasi Romania*, **33** (1987), 53-58.

[3] M. C. Chaki, On generalized pseudo-symmetric manifolds, *Publ. Math. Debrecen*, **45** (1994), 305-312.

[4] M. C. Chaki and B. Gupta, On conformally symmetric spaces, *Indian J. Math.*, **5** (1963), 113-122.

[5] R. Miron, Mouvements conformes dans les espaces W_n and N_n, *Tensor N.S.*, **19** (1968), 33-41.

[6] V. Murgescu, Espaces de Weyl a torsion et leurs representations conformes, *Ann. Sci. Univ. Timisoara*, (1968), 221-228.

[7] A. Norden, *Affinely connected spaces*, GRMFL, Moscow, (1976).

[8] A. Ozdeger and Z. Senturk, Generalized Circles in Weyl Spaces and their conformal mapping, *Publ. Math. Debrecen*, **60** (2002), 75-87.

[9] E. Scholz, Weyl Geometry in Late 20th Century Physics, *arXiv.org/math arXiv:1111.3220*, (2008).

[10] Z. I. Szabo, Structure theorems on Riemannian spaces satisfying $R(X,Y).R = 0$, *J.Diff.Geom.*, **17** (1982), 531-582.

[11] L. Tamassy and T. Q. Binh, On weakly symmetric and weakly projective symmetric Riemannian manifolds, *Coll. Math. Soc. J. Bolyai*, **56** (1989), 663-670.

[12] F. Unal (Nurcan) and S. A. Uysal, Weyl Manifolds with semi-symmetric connections, *Mathematical and Computational Applications*, **10(3)** (2005), 351-358.

[13] A. G. Walker, On Ruses spaces of recurrent curvature, *Proc. London Math. Soc.*, **52** (1950), 36-64.

[14] K. Yano and S. Sawaki, Riemannian Manifolds admitting a conformal transformation group, *J. Diff. Geom.*, **2** (1968), 161-184.