A NOTE ON ORDERED QUASI- Γ -IDEALS IN INTRA-REGULAR PO- Γ -SEMIGROUPS

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Abstract: In this paper, we characterize intra-regular po- Γ -semigroups through ordered quasi- Γ -ideals, ordered right Γ -ideals and ordered left Γ -ideals. Also, we investigate the po- Γ -semigroup in which the regular and intra-regular sub- Γ -semigroups are idempotents, left regular idempotents, commutative and idempotents.

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Key Words: po- Γ -semigroup, intra-regular po- Γ -semigroup, ordered quasi- Γ -ideal

1. Introduction

The notion of Γ -semigroup was given by M. K. Sen [3] and that of the po- Γ -semigroup was introduced by Y. I. Kwon and S. K. Lee [16]. The notion of a quasi-ideal of semigroups was invented by O. Steinfield [13]. Thereafter, quasi-ideals have been studied in different algebraic structures [4, 14]. N. Kehayopulu et al [7] defined and studied an ordered quasi-ideal in ordered semigroups. For regular ordered semigroups, left regular ordered semigroups and intra-regular ordered semigroups, we refer [8, 9, 10, 11, 15].

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Recently, Abbasi and Basar in [4] gave some nice characterizations of regular ordered Γ -semigroups in terms of ordered quasi- Γ -ideals. In continuation of the study, we characterize regular and intra-regular po- Γ -semigroups using ordered quasi- Γ -ideals.

2. Preliminaries

Let S and Γ be two nonempty sets. Then a system of the form $(S, \Gamma, \cdot)$ is called a Γ -semigroup if $(a \cdot \gamma \cdot b) \cdot \mu \cdot c = a \cdot \gamma \cdot (b \cdot \mu \cdot c)$ for all $a, b, c \in S$ and $\gamma, \mu \in \Gamma$. Also, for the sake of smoothness, we denote $a \cdot \gamma \cdot b$ by $a\gamma b$. A nonempty subset N of a Γ -semigroup S is called a sub- Γ -semigroup of S if $a\alpha b \in N$ for all $a, b \in N$ and $\alpha \in \Gamma$. For any nonempty subsets A, B of S , the product set of A and B relative to S is defined by $A\Gamma B = \{a\alpha b : a \in A, b \in B \text{ and } \alpha \in \Gamma\}$. We denote $\{a\}\Gamma B$, $A\Gamma\{b\}$ and $\{a\}\Gamma\{b\}$ respectively by $a\Gamma B$, $A\Gamma b$ and $a\Gamma b$. By a po- Γ -semigroups S (also called an ordered Γ -semigroup), we mean an ordered set $(S, \leq)$, at the same time a Γ -semigroup satisfying the following conditions:

$$a \leq b \Rightarrow a\gamma c \leq b\gamma c$$

and

$$c\gamma a \leq c\gamma b$$

for all $a, b, c \in S$ and $\gamma \in \Gamma$.

For other basic concepts of Γ -semigroups and po- Γ -semigroups, we refer [1, 2, 5, 6, 12].

Throughout the paper, S will stand for a po- Γ -semigroup unless otherwise indicated. A po- Γ -semigroup S is called regular if for each $s \in S$ and for each $\alpha, \beta \in \Gamma$ there exists $a \in S$ such that $s \leq s\alpha a\beta s$. Equivalent definition of the regular po- Γ -semigroup is as follows: (i) $A \subseteq (A\Gamma S\Gamma A)$ for each $A \subseteq S$. (ii) $s \in (s\Gamma S\Gamma s)$ for each $s \in S$. A sub- Γ -semigroup N of S is called intra-regular if for each $a \in N$, there exist $x, y \in N$ such that $a \leq xa^2y$; in symbol, if $a \in (N\Gamma a^2\Gamma N)$ for every $a \in N$. Let $(S; \leq)$ be a po- Γ -semigroup, and N be a sub- Γ -semigroup of S , then $(N; \leq)$ is a po- Γ -semigroup. Let A be a nonempty subset of N , then we write $(A]_N = \{n \in N : n \leq a \text{ for some } a \in A\}$ and $A \cup a = A \cup \{a\}$. We also write $(A]_N$ by simply $(A]$ if $N = S$. A nonempty subset I of a po- Γ -semigroup S is called an ordered right Γ -ideal (resp. left Γ -ideal) of S if $I\Gamma S \subseteq I$ (resp. $S\Gamma I \subseteq I$), and for any $i \in I$, $(i] \subseteq I$. I is an ordered Γ -ideal of S if it is both an ordered left and an ordered right Γ -ideal of S . For any $s \in S$, we have that $(S\Gamma s]$ is an ordered left Γ -ideal of S , and $(s\Gamma S]$

is an ordered right Γ -ideal of S . A nonempty subset Q of S is called an ordered quasi- Γ -ideal of S if (i) $(Q\Gamma S] \cap (S\Gamma Q] \subseteq Q$, and (ii) $(Q] \subseteq Q$.

Let X be a nonempty subset of S . Then the least right (resp. left) ordered Γ -ideal of S containing X is given by $R(X) = (X \cup X\Gamma S]$ (resp. $L(X) = (S\Gamma X \cup X]$). If $X = \{s\}$, $s \in S$, we write $R\{s\}$ and $L\{s\}$ respectively by $R(s)$ and $L(s)$, where $R(s) = (s \cup s\Gamma S]$ and $L(s) = (S\Gamma s \cup s]$ and the ideal generated by $s \in S$ is given by $I(s) = (s \cup S\Gamma s \cup s\Gamma S \cup S\Gamma s\Gamma S]$. Also, the least quasi- Γ -ideal of S containing X is denoted by $Q(X)$. Also the ordered quasi- Γ -ideal generated by $s \in S$ is given by $Q(s) = (s \cup ((s\Gamma S] \cap (S\Gamma s]))$. Now if $Q_1, Q_2 \in N_Q$, and $\star$ an operation in N_Q , then we write $Q_1 \star \Gamma \star Q_2 = (Q_1\Gamma Q_2]$. Also we use the following notations: (i) $N_Q = \{Q : Q \neq \emptyset \text{ where } Q \subseteq S \text{ and } (Q] \subseteq Q\}$, (ii) R_I is a set of ordered right Γ -ideals of S , (iii) L_I is a set of ordered left Γ -ideals of S , and (iv) Q_I is a set of ordered quasi- Γ -ideals of S .

3. Main Results

We start our study with some characterizations of intra-regular po- Γ -semigroups through ordered quasi- Γ -ideals, ordered right Γ -ideals and ordered left Γ -ideals of po- Γ -semigroups. We begin with proving the following Proposition.

Proposition 1. *Let S be a po- Γ -semigroup. Then the following assertions are equivalent:*

- (i) S is an intra-regular po- Γ -semigroup.
- (ii) $R \cap L \subseteq (L\Gamma R]$, where R and L are respectively an ordered right Γ -ideal and an ordered left Γ -ideal of S .
- (iii) $Q \subseteq (S\Gamma Q^2\Gamma S]$ for every ordered quasi- Γ -ideal Q of S .

Proof. (i) $\Rightarrow$ (ii). Suppose $s \in R \cap L$. As S is intra-regular, it follows that $s \in (S\Gamma s^2\Gamma S] = ((S\Gamma s)\Gamma(s\Gamma S)) \subseteq ((S\Gamma L)\Gamma(R\Gamma S)) \subseteq (L\Gamma R]$. Hence $R \cap L \subseteq (L\Gamma R]$.

(ii) $\Rightarrow$ (iii) Suppose that Q is an ordered quasi- Γ -ideal of S . Then using [4, Lemma 4], we obtain

$$\begin{aligned}
 Q = L(Q) \cap R(Q) &\subseteq (L(Q)\Gamma R(Q)) = ((S^1\Gamma Q]\Gamma(Q\Gamma S^1]) \\
 &= (S^1\Gamma Q\Gamma Q\Gamma S^1] \\
 &\subseteq (S^1\Gamma Q\Gamma (S^1\Gamma Q\Gamma Q\Gamma S^1]\Gamma S^1] \\
 &= (S^1\Gamma Q\Gamma S^1\Gamma Q\Gamma Q\Gamma S^1\Gamma S^1]
 \end{aligned}$$

$$\begin{aligned}
&\subseteq (S^1\Gamma Q\Gamma S^1\Gamma(S^1\Gamma Q\Gamma Q\Gamma S^1)\Gamma Q\Gamma S^1\Gamma S^1) \\
&= ((S^1\Gamma Q\Gamma S^1\Gamma S^1)\Gamma Q^2\Gamma(S^1\Gamma Q\Gamma S^1\Gamma S^1)) \\
&\subseteq (S\Gamma Q^2\Gamma S].
\end{aligned}$$

(iii) $\Rightarrow$ (i). Suppose $s \in S$. Using [4, Theorem 5 (i)], it follows that $Q(s) = L(s) \cap R(s)$ and therefore

$$\begin{aligned}
s \in Q(s) \subseteq (S\Gamma Q(s)\Gamma Q(s)\Gamma S] &\subseteq (S\Gamma L(s)\Gamma R(s)\Gamma S] \\
&= (S\Gamma(S^1\Gamma s]\Gamma(s\Gamma S^1)\Gamma S] \\
&= ((S\Gamma S^1)\Gamma s^2(S^1\Gamma S)) \\
&\subseteq (S\Gamma s^2\Gamma S].
\end{aligned}$$

Hence S is an intra-regular po- Γ -semigroup.

Theorem 2. Let S be a po- Γ -semigroup. Then the following assertions are equivalent:

- (i) S is a regular and an intra-regular po- Γ -semigroup.
- (ii) $(R\Gamma L] = R \cap L \subseteq (L\Gamma R]$, (a) where R and L are respectively an ordered right Γ -ideal and an ordered left Γ -ideal of S .
- (iii) $(Q_I, *)$ is an idempotent po- Γ -semigroup.
- (iv) $(Q^2] = Q$ for every ordered quasi- Γ -ideal Q of S .

Proof. (i) $\Leftrightarrow$ (ii). It is a consequence of [4, Theorem 8] and Proposition 1.

(ii) $\Rightarrow$ (iii) Applying [4, Theorem 8] and the first part of the assertion (a), it follows that $(Q_I, *)$ is a regular po- Γ -semigroup and therefore we need only verify that every ordered quasi- Γ -ideal Q of S is idempotent in $(Q_I, *)$. By [4, Lemma 2], it is clear that $(Q^2] \subseteq (Q] \subseteq Q$.

Again using [4, Theorem 8], Proposition 1 and assertion (a), we obtain $Q = (Q\Gamma S\Gamma Q]$ and $Q \subseteq (S\Gamma Q^2\Gamma S]$ respectively.

Consequently,

$$\begin{aligned}
Q = (Q\Gamma S\Gamma Q] &= (Q\Gamma S\Gamma(Q\Gamma S\Gamma Q]) \\
&\subseteq (Q\Gamma S\Gamma Q\Gamma S\Gamma Q] \\
&\subseteq (Q\Gamma S\Gamma(S\Gamma Q^2\Gamma S)\Gamma S\Gamma Q] \\
&\subseteq ((Q\Gamma S^2\Gamma Q)\Gamma(Q\Gamma S^2\Gamma Q))
\end{aligned}$$

$$\subseteq ((Q\Gamma STQ)\Gamma(Q\Gamma STQ)) \subseteq (Q^2].$$

Therefore $Q = (Q^2] = Q*\Gamma*Q$. Hence $(Q_I, *)$ is an idempotent po- Γ -semigroup.

(iii) $\Rightarrow$ (iv). It is straightforward.

(iv) $\Rightarrow$ (ii). Suppose R and L are respectively an ordered right and an ordered left Γ -ideals of S . Now $R \cap L$ is an ordered quasi- Γ -ideal of S [4, Lemma 3]. Therefore by hypothesis (iv), we have

$$R \cap L = ((R \cap L)^2] = ((R \cap L)\Gamma(R \cap L)) \subseteq (R\Gamma L],$$

and as $(R\Gamma L) \subseteq R \cap L$ is always true, we obtain $(R\Gamma L) = R \cap L$. Also $R \cap L = ((R \cap L)\Gamma(R \cap L)) \subseteq (L\Gamma R]$.

Hence the assertion (ii) is proved. This completes the proof.

Proposition 3. A po- Γ -semigroup S is left duo if and only if every ordered quasi- Γ -ideal of S is an ordered right Γ -ideal of S .

Proof. Let S be left duo and Q be an ordered quasi- Γ -ideal of S . Then there exist an ordered right Γ -ideal R and an ordered left Γ -ideal L of S such that $Q = R \cap L$ by [4, Lemma 4]. As S is left duo, L is an ordered right Γ -ideal of S and so Q is an ordered right Γ -ideal of S .

Conversely, since every ordered left Γ -ideal of S is an ordered quasi- Γ -ideal of S and hence the converse follows. This completes the proof.

Proposition 4. A po- Γ -semigroup S is duo if and only if every ordered quasi- Γ -ideal of S is an ordered two-sided Γ -ideal of S .

Theorem 5. The following assertions are equivalent for a po- Γ -semigroup S :

- (i) S is a regular left duo po- Γ -semigroup.
- (ii) $(Q_I, *)$ is a left regular idempotent po- Γ -semigroup.
- (iii) Let R be an ordered right Γ -ideal and L_1, L_2, L_3 be ordered left Γ -ideals of S . Then

$$(R\Gamma L_1] = (R \cap L_1] \text{ and } (L_2\Gamma L_3] = L_2 \cap L_3.$$

- (iv) $(L_I, *)$ is a commutative idempotent po- Γ -semigroup, $(R_I, *)$ is an idempotent po- Γ -semigroup and $(Q_I, *)$ is the sub- Γ -semigroup of the po- Γ -semigroup $(N_Q, *)$ generated by $(L_I, *)$ and $(R_I, *)$.

Proof. (i) $\Rightarrow$ (ii) Suppose Q_1 and Q_2 are ordered quasi- Γ -ideals of S . As S is left duo, by Proposition 3, Q_1 and Q_2 are ordered right Γ -ideals of S . As S is regular, using [4, Theorem 8 and Lemma 2], we obtain

$$Q_1 = (Q_1 \Gamma S \Gamma Q_1] = ((Q_1 \Gamma S) \Gamma Q_1] \subseteq (Q_1 \Gamma Q_1] \subseteq (Q_1] \subseteq Q_1.$$

This implies $Q_1 * \Gamma * Q_1 = (Q_1^2] = Q_1$.

So $(Q_I, *)$ is an idempotent po- Γ -semigroup. As Q_2 is also an ordered right Γ -ideal of S , we obtain

$$(Q_1 \Gamma Q_2 \Gamma Q_1] = (Q_1 \Gamma (Q_2 \Gamma Q_1)] \subseteq (Q_1 \Gamma (Q_2 \Gamma S)] \subseteq (Q_1 \Gamma Q_2].$$

Applying [4, Theorem 8], $(Q_1 \Gamma Q_2]$ is an ordered quasi- Γ -ideal of S .

Therefore

$$(Q_1 \Gamma Q_2] = ((Q_1 \Gamma Q_2) \Gamma S \Gamma (Q_1 \Gamma Q_2)] = (Q_1 \Gamma (Q_2 \Gamma S) \Gamma (Q_1 \Gamma Q_2)] \subseteq (Q_1 \Gamma Q_2 \Gamma Q_1].$$

So $(Q_1 \Gamma Q_2] = (Q_1 \Gamma Q_2 \Gamma Q_1]$.

This means $Q_1 * \Gamma * Q_2 = Q_1 * \Gamma * Q_2 * \Gamma * Q_1$, where $Q_1, Q_2 \in (Q_I, *)$.

Hence $(Q_I, *)$ is a left regular idempotent po- Γ -semigroup.

(ii) $\Rightarrow$ (iii) As a left regular idempotent po- Γ -semigroup is a regular po- Γ -semigroup, by [4, Theorem 8], the first assertion in (iii) is proved.

Now suppose L_2, L_3 are ordered left Γ -ideals of S . So $(L_2 \Gamma L_3] \subseteq (S \Gamma L_3] \subseteq L_3$. Now by condition (ii), it follows that

$$(L_2 \Gamma L_3] = (L_2 \Gamma L_3 \Gamma L_2] = ((L_2 \Gamma L_3) \Gamma L_2] \subseteq (S \Gamma L_2] \subseteq (L_2] \subseteq L_2,$$

therefore $(L_2 \Gamma L_3] \subseteq L_2 \cap L_3$. This together with [4, Theorem 10(v)], we obtain $(L_2 \Gamma L_3] = L_2 \cap L_3$.

(iii) $\Rightarrow$ (iv) This is a consequence of [4, Theorem 8] and the hypothesis (iii).

(iv) $\Rightarrow$ (i) Applying [4, Theorem 8], we have S is a regular po- Γ -semigroup. Let L be an ordered left Γ -ideal of S . As $(L_I, *)$ is a commutative idempotent po- Γ -semigroup and $S \in L_I$, it follows that

$$L \Gamma S \subseteq (L \Gamma S] = L * \Gamma * S = S * \Gamma * L = (S \Gamma L] \subseteq L.$$

This implies that L is an ordered right Γ -ideal of S . Hence S is a left duo po- Γ -semigroup. This completes the proof.

Theorem 6. The following assertions are equivalent for a po- Γ -semigroup S :

- (i) S is a regular duo po- Γ -semigroup.
- (ii) $(Q_I, *)$ is a commutative idempotent po- Γ -semigroup.
- (iii) $(L_1 \Gamma L_2] = L_1 \cap L_2$ and $(R_1 \Gamma R_2] = R_1 \cap R_2$, where L_1, L_2 are ordered left Γ -ideals and R_1, R_2 are ordered right Γ -ideals of S .
- (iv) $(L_I, *)$ and $(R_I, *)$ are commutative idempotent po- Γ -semigroups and $(Q_I, *)$ is the sub- Γ -semigroup of $(N_Q, *)$ generated by $(L_I, *)$ and $(R_I, *)$.
- (v) $(Q_1 \Gamma Q_2] = Q_1 \cap Q_2$ for any arbitrary ordered quasi- Γ -ideals Q_1, Q_2 of S .
- (vi) $((R(Q))^2] = L(Q)$ and $((L(Q))^2] = R(Q)$ for every ordered quasi- Γ -ideal Q of S .
- (vii) $R \cap L = (L \Gamma R]$ for every ordered right Γ -ideal R and every ordered left Γ -ideal L of S .

Proof. (i) $\Rightarrow$ (ii) Using Theorem 5 and its dual part, the assertion (i) shows that $(Q_I, *)$ is both a left and right idempotent po- Γ -semigroup. Hence $(Q_I, *)$ is a commutative idempotent po- Γ -semigroup.

(ii) $\Rightarrow$ (iii) In the light of the assumption, $(Q_I, *)$ is an idempotent po- Γ -semigroup. This along with Theorem 2 leads to S being regular. As a commutative idempotent po- Γ -semigroup is both a left and a right regular idempotent po- Γ -semigroup, the condition (iii) holds by Theorem 5 and its dual part.

(iii) $\Rightarrow$ (i) Suppose L and R are ordered left and right Γ -ideals of S . Using the condition (iii) and the fact that $S \in L_I$, we obtain

$$L \Gamma S \subseteq (L \Gamma S] = L \cap S = L.$$

This proves that L is an ordered right Γ -ideal of S . The hypothesis symmetrically shows that R is an ordered left Γ -ideal of S . Therefore L and R are ordered two-sided Γ -ideals of S . So S is duo, and

$$S \Gamma (R \Gamma L] \subseteq (S \Gamma (R \Gamma L]) \subseteq ((S \Gamma R) \Gamma L] \subseteq (R \Gamma L],$$

$$(R \Gamma L] \Gamma S \subseteq (R \Gamma (L \Gamma S]) \subseteq (R \Gamma L].$$

This proves that $(R \Gamma L]$ is an ordered two-sided Γ -ideal of S . Using the assertion (iii) again, it follows that $(L^2] = L$ and $(R^2] = R$. Therefore S satisfies [4, Theorem 8(iii)]. So S is regular by [4, Theorem 8].

(i) $\Leftrightarrow$ (iv) It is a consequence of Theorem 5 and its dual part.

(i) $\Rightarrow$ (v). Suppose Q_1, Q_2 are ordered quasi- Γ -ideals of S . As S is duo, using Proposition 4, we observe that Q_1, Q_2 are ordered Γ -ideals of S and so $(Q_1 \Gamma Q_2] \subseteq Q_1 \cap Q_2$.

As $Q_1 \cap Q_2$ is an ordered two-sided Γ -ideal of S and S is regular, using [4, Theorem 8], we have

$$Q_1 \cap Q_2 = ((Q_1 \Gamma Q_2) \Gamma S \Gamma (Q_1 \Gamma Q_2)] \subseteq (Q_1 \Gamma S \Gamma Q_2] \subseteq (Q_1 \Gamma Q_2].$$

Hence $(Q_1 \Gamma Q_2] = Q_1 \cap Q_2$.

(v) $\Rightarrow$ (vi). Suppose Q is any ordered quasi- Γ -ideal of S . So by assertion (v), we have

$$(Q^2] = Q, Q \Gamma S \subseteq (Q \Gamma S] = Q \cap S = Q \text{ and } S \Gamma Q \subseteq (S \Gamma Q] = S \cap Q = Q.$$

Therefore Q is an ordered two-sided Γ -ideal of S .

Consequently $L(Q) = R(Q) = Q$, and hence

$$((R(Q))^2] = (Q^2] = Q = (L(Q)], ((L(Q))^2] = (Q^2] = Q = R(Q).$$

(vi) $\Rightarrow$ (i) Suppose L is an ordered left Γ -ideal of S . So the assertion (vi) shows that

$$L \Gamma S \subseteq (L \cup L \Gamma S] = ((L \cup S \Gamma L]^2] \subseteq (L^2] \subseteq (L] \subseteq L.$$

Thus L is also an ordered right Γ -ideal of S . Dually we can show that every ordered right Γ -ideal of S is also an ordered left Γ -ideal of S . Therefore S is a duo po- Γ -semigroup.

For arbitrary ordered right Γ -ideal R and ordered left Γ -ideal L , as S is duo, it follows that R and L are ordered two-sided Γ -ideals of S . Therefore $(R \Gamma L]$ is an ordered two-sided Γ -ideal of S and the assertion (vi) shows that $(L^2] = L$ and $(R^2] = R$. Applying [4, Theorem 8], we infer that S is regular.

(v) $\Rightarrow$ (vii) Straightforward.

(vii) $\Rightarrow$ (i) Similar to that of (iii) $\Rightarrow$ (i). This completes the proof.

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