ON SOFT FUZZY $G_\delta$ PRE ALMOST COMPACTNESS IN SOFT FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper the concept of soft fuzzy $G_\delta$ pre almost compactness and soft fuzzy $G_\delta$ pre near compactness are introduced and studied. We give some characterizations of soft fuzzy $G_\delta$ pre almost compactness in terms of soft fuzzy regular $G_\delta$ pre open or soft fuzzy regular $F_\sigma$ pre closed sets.

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1. Introduction


In this paper soft fuzzy $G_\delta$ pre almost compactness and $G_\delta$ pre near compactness are introduced and their properties are discussed.
2. Preliminaries

Definition 1. ([1]). Let \((X, \tau)\) be a topological space. Let \(\lambda\) be any fuzzy set. Then \(\lambda\) is said to be fuzzy \(G_\delta\) set if \(\lambda = \bigwedge_{i=1}^{\infty} \mu_i\), where each \(\mu_i\) is fuzzy open set. The complement of a fuzzy \(G_\delta\) set is fuzzy \(F_{\sigma}\).

Definition 2. ([2]). Let \((X, \tau)\) be a fuzzy topological space. Let \(\lambda\) be any fuzzy set. Then \(\lambda\) is said to be fuzzy pre open set if \(\lambda \leq \text{int}(\text{cl}(\lambda))\). The complement of a pre open set is pre closed.

Definition 3. ([6]). Let \(X\) be a set, \(\mu\) be a fuzzy subset of \(X\) and \(M \subseteq X\). Then, the pair \((\mu, M)\) will be called a soft fuzzy subset of \(X\). The set of all soft fuzzy subsets of \(X\) will be denoted by \(SF(X)\).

Proposition 4. ([6]). If \((\mu_j, M_j)_{j \in J} \in SF(X)\), then the family \(\{ (\mu_j, M_j) | j \in J \}\) has a meet, that is greatest lower bound, in \((SF(X), \sqsubseteq)\), denoted by

\[ \sqcap_{j \in J}(\mu_j, M_j) \text{ such that } \sqcap_{j \in J}(\mu_j, M_j) = (\mu, M) \]

where
\[ \mu(x) = \bigwedge_{j \in J} \mu_j(x), \forall x \in X. \]
\[ M = \bigcap_{j \in J} M_j \]

Proposition 5. ([6]). If \((\mu_j, M_j)_{j \in J} \in SF(X)\), then the family \(\{ (\mu_j, M_j) | j \in J \}\) has a join, that is least upper bound, in \((SF(X), \sqsubseteq)\), denoted by

\[ \sqcup_{j \in J}(\mu_j, M_j) \text{ such that } \sqcup_{j \in J}(\mu_j, M_j) = (\mu, M) \]

where
\[ \mu(x) = \bigvee_{j \in J} \mu_j(x), \forall x \in X. \]
\[ M = \bigcup_{j \in J} M_j \]

Definition 6. ([6]). Let \(X\) be a non-empty set and the soft fuzzy sets \(A\) and \(B\) be in the form,

\[ A = \{ (\mu, M) | \mu(x) \in I_X, \forall x \in X, M \subseteq X \} \]
\[ B = \{ (\lambda, N) | \lambda(x) \in I_X, \forall x \in X, N \subseteq X \} \]

Then,
1. \(A \sqsubseteq B \iff \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N.\)
2. \(A = B \iff \mu(x) = \lambda(x), \forall x \in X, M = N.\)
3. \(A' \iff 1 - \mu(x), \forall x \in X, X \setminus M.\)
4. \(A \cap B \iff \mu(x) \land \lambda(x), \forall x \in X, M \cap N.\)
4. \(A \cup B \iff \mu(x) \lor \lambda(x), \forall x \in X, M \cup N.\)

Definition 7. ([6]).

\[(0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}\]
\[(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}\]

Definition 8. ([6]). For \((\mu, M) \in SF(X)\) the soft fuzzy set \((\mu, M)' = (1 - \mu, X \setminus M)\) is called the complement of \((\mu, M)\).

Definition 9. ([6]). A subset \(\tau \subseteq SF(X)\) is called an SF-topology on \(X\) if
\[(1) (0, \emptyset) \text{ and } (1, X) \in \tau\]
\[(2) (\mu_j, M_j), j = 1, 2, \ldots, n \Rightarrow \bigcap_{j=1}^{n}(\mu_j, M_j) \in \tau\]
\[(3) (\mu_j, M_j), j \in J \Rightarrow \bigcup_{j \in J}(\mu_j, M_j) \in \tau.\]
The elements of \(\tau\) are called soft fuzzy open, and those of \(\tau' = \{((\mu, M)|(\mu, M)' \in \tau\}\) soft fuzzy closed.

If \(\tau\) is SF-topology on \(X\) we call the pair \((X, \tau)\) SF-topological space (in short, SFTS).

Definition 10. ([6]). The closure of a soft fuzzy set \((\mu, M)\) will be denoted by \((\mu, M)\). It is given by
\[\overline{(\mu, M)} = \bigcap\{(\gamma, N)|(\mu, M) \subseteq (\gamma, N), (\gamma, N) \in \tau'\}\]
Likewise the interior is given by
\[(\mu, M) = \bigcup\{(\gamma, N)|(\gamma, N) \in \tau, (\gamma, N) \subseteq (\mu, M)\}\]
Note : \((\mu, M) = cl(\mu, M)\) and \((\mu, M) = int(\mu, M)\).

Definition 11. ([6]). A soft fuzzy topological space \((X, \tau)\) is said to be a soft fuzzy compact if whenever \(\bigcup_{i \in I}(\lambda_i, M_i) = (1, X)\), \((\lambda_i, M_i) \in \tau, i \in I\), there is a finite subset \(J\) of \(I\) with \(\bigcup_{j \in J}(\lambda_j, \mu_j) = (1, X)\).

Definition 12. ([7]). Let \((X, \tau)\) be a soft fuzzy topological space. Let \((\lambda, N)\) be any soft fuzzy set. Then \((\lambda, N)\) is said to be soft fuzzy \(G_\delta\) set if \((\lambda, N) = \bigcap_{i=1}^{\infty}(\mu_i, M_i)\), where each \((\mu_i, M_i)\) is soft fuzzy open set. The complement of a soft fuzzy \(G_\delta\) set is soft fuzzy \(F_\sigma\).

Definition 13. ([7]). Let \((X, \tau)\) be a soft fuzzy topological space. Let \((\lambda, N)\) be any soft fuzzy set. Then \((\lambda, N)\) is said to be soft fuzzy pre open set if \((\lambda, N) \subseteq int(cl(\lambda, N))\). The complement of a soft fuzzy pre open set is soft fuzzy pre closed.

Definition 14. ([7]). Let \((X, \tau)\) be a soft fuzzy topological space. Let \((\lambda, N)\) be any soft fuzzy set. Then \((\lambda, N)\) is said to be soft fuzzy \(G_\delta\) pre open set if \((\lambda, N) = (\mu, M) \cap (\gamma, L)\), where \((\mu, M)\) is soft fuzzy \(G_\delta\) set and \((\gamma, L)\) is soft fuzzy pre open set. The complement of a soft fuzzy \(G_\delta\) pre open set is soft fuzzy \(F_\sigma\) pre closed.
**Definition 15.** ([7]). Let \((X, \tau)\) be a soft fuzzy topological space and let \((\lambda, M)\) be any soft fuzzy set in \((X, \tau)\). Then soft fuzzy \(G_\delta\) pre interior of \((\lambda, M)\) is defined as follows

\[
\text{SF } G_\delta \text{ pre int}(\lambda, M) = \bigsqcup \{ (\mu, N) | (\mu, N) \text{ is SF } G_\delta \text{ pre open and } (\mu, N) \sqsubseteq (\lambda, M) \}.
\]

**Proposition 16.** ([7]). Let \((X, \tau)\) be any soft fuzzy topological space. Let \((\lambda, N)\) be any soft fuzzy set in \((X, \tau)\). Then \(\text{SF } G_\delta \text{ pre int}(\lambda, N)\) is a soft fuzzy \(G_\delta\) pre open set in \((X, \tau)\).

**Definition 18.** ([7]). Let \((X, \tau)\) be any soft fuzzy topological space and let \((\lambda, M)\) be any soft fuzzy set in \((X, \tau)\). Then soft fuzzy \(F_\sigma\) pre closure of \((\lambda, M)\) is defined as follows

\[
\text{SF } F_\sigma \text{ pre cl}(\lambda, M) = \bigsqcap \{ (\mu, N) | (\mu, N) \text{ is SF } F_\sigma \text{ pre closed and } (\lambda, M) \sqsubseteq (\mu, N) \}.
\]

**Proposition 19.** ([7]). Let \((X, \tau)\) be any soft fuzzy topological space. Let \((\lambda, N)\) be any soft fuzzy set in \((X, \tau)\). Then \(\text{SF } F_\sigma \text{ pre cl}(\lambda, N)\) is a soft fuzzy \(F_\sigma\) pre closed set in \((X, \tau)\).
(i) $SF F_{\sigma} \text{ pre } cl((1, X) - (\lambda, M)) = (1, X) - SF G_{\delta} \text{ pre } int(\lambda, M)$.
(ii) $SF G_{\delta} \text{ pre } int((1, X) - (\lambda, M)) = (1, X) - SF F_{\sigma} \text{ pre } cl(\lambda, M)$.

Definition 22. ([7]). Let $(X, \tau)$ be a soft fuzzy topological space. Let $(\lambda, N)$ be a soft fuzzy set in $(X, \tau)$. Then $(\lambda, N)$ is said to be soft fuzzy regular $G_{\delta}$ pre open if $(\lambda, N) = SF G_{\delta} \text{ pre } int(SF F_{\sigma} \text{ pre cl}(\lambda, N))$.

Definition 23. ([7]). Let $(X, \tau)$ be a soft fuzzy topological space. Let $(\lambda, N)$ be a soft fuzzy set in $(X, \tau)$. Then $(\lambda, N)$ is said to be soft fuzzy regular $F_{\sigma}$ pre closed if $(\lambda, N) = SF F_{\sigma} \text{ pre cl}(SF G_{\delta} \text{ pre } int(\lambda, N))$.

Proposition 24. ([7]). (i) The soft fuzzy $F_{\sigma}$ pre closure of a soft fuzzy $G_{\delta}$ pre open set is soft fuzzy regular $F_{\sigma}$ pre closed.
(ii) The soft fuzzy $G_{\delta}$ pre interior of a soft fuzzy $F_{\sigma}$ pre closed set is soft fuzzy regular $G_{\delta}$ pre open.

Definition 25. ([7]). Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. A function $f : (X, \tau) \rightarrow (Y, \tau^*)$ is said to be soft fuzzy $G_{\delta}$ pre continuous, if the inverse image of every soft fuzzy open set in $(Y, \tau^*)$ is soft fuzzy $G_{\delta}$ pre open in $(X, \tau)$.

Definition 26. ([7]). Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. A function $f : (X, \tau) \rightarrow (Y, \tau^*)$ is said to be soft fuzzy $G_{\delta}$ pre irresolute, if the inverse image of every soft fuzzy $G_{\delta}$ pre open set in $(Y, \tau^*)$ is soft fuzzy $G_{\delta}$ pre open in $(X, \tau)$.

Definition 27. ([7]). A soft fuzzy topological space $(X, \tau)$ is said to be a soft fuzzy $G_{\delta}$ pre compact if whenever $\sqcup_{i \in I}(\lambda_i, M_i) = (1, X)$, $(\lambda_i, M_i)$ is soft fuzzy $G_{\delta}$ pre open, $i \in I$, there is a finite subset $J$ of $I$ with $\sqcup_{i \in J}(\lambda_i, M_i) = (1, X)$.

Proposition 28. ([7]). Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. If $f : (X, \tau) \rightarrow (Y, \tau^*)$ is soft fuzzy $G_{\delta}$ pre continuous bijection and $(X, \tau)$ is soft fuzzy $G_{\delta}$ pre compact, then $(Y, \tau^*)$ is soft fuzzy compact.

Proposition 29. ([7]). Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. If $f : (X, \tau) \rightarrow (Y, \tau^*)$ is soft fuzzy $G_{\delta}$ pre irresolute bijection and $(X, \tau)$ is soft fuzzy $G_{\delta}$ pre compact, then $(Y, \tau^*)$ is soft fuzzy $G_{\delta}$ pre compact.
3. Soft Fuzzy $G_\delta$ Pre Almost Compact Spaces

**Definition 30.** A soft fuzzy topological space $(X, \tau)$ is said to be a soft fuzzy $G_\delta$ pre almost compact if whenever $\bigcup_{i \in I} (\lambda_i, M_i) = (1, X)$, $(\lambda_i, M_i)$ is soft fuzzy $G_\delta$ pre open, $i \in I$, there is a finite subset $J$ of $I$ with $\bigcup_{i \in J} SF F_\sigma$ pre cl$(\lambda_i, M_i) = (1, X)$.

**Definition 31.** Let $(X, \tau)$ be a soft fuzzy topological space. If $(\lambda_i, M_i)$, $i \in I$, of soft fuzzy sets in $(X, \tau)$ satisfies the finite intersection property (FIP for short) iff every finite subfamily $(\lambda_i, M_i)$, $i = 1, 2, \ldots, n$ of the family satisfies the condition $\bigcap_{i=1}^n (\lambda_i, M_i) \neq (0, \emptyset)$.

**Theorem 32.** A soft fuzzy topological space $(X, \tau)$ is fuzzy $G_\delta$ pre almost compact iff for every collection of soft fuzzy $G_\delta$ pre open sets $(\lambda_i, M_i)$ of SF$(X)$ having the finite intersection property we have $\bigcap_{i \in I} SF F_\sigma$ pre cl$(\lambda_i, M_i) \neq (0, \emptyset)$.

**Proof.** ($\Rightarrow$) Let $(\lambda_i, M_i)$, $i \in I$ be a collection of soft fuzzy $G_\delta$ pre open sets with finite intersection property. If $\bigcap_{i \in I} SF F_\sigma$ pre cl$(\lambda_i, M_i) = (0, \emptyset)$, then $(1, X) - \bigcap_{i \in I} SF F_\sigma$ pre cl$(\lambda_i, M_i) = \bigcup_{i \in I} SF G_\delta$ pre int$((1, X) - (\lambda_i, M_i))$

From the fuzzy $G_\delta$ pre almost compactness it follows that there exists a finite subset $F$ of $I$ such that

$\bigcup_{i \in F} SF F_\sigma$ pre cl$(SF G_\delta$ pre int$((1, X) - (\lambda_i, M_i))) = (1, X)$.

Hence

$\bigcap_{i \in F} SF G_\delta$ pre int$(\lambda_i, M_i) \subseteq [(1, X) - SF F_\sigma$ pre cl$(SF G_\delta$ pre int$((1, X) - (\lambda_i, M_i)))] = (0, \emptyset)$, which is a contradiction.

($\Leftarrow$) Let $(\lambda_i, M_i)$, $i \in I$ be a soft fuzzy $G_\delta$ pre open cover of SF$(X)$. If $\bigcup_{i \in F} SF F_\sigma$ pre cl$(\lambda_i, M_i)$, for every finite $F$ of $I$, does not cover SF$(X)$, then $(1, X) - SF F_\sigma$ pre cl$(\lambda_i, M_i) = SF G_\delta$ pre int$((1, X) - (\lambda_i, M_i))$ and

$\{(1, X) - SF F_\sigma$ pre cl$(\lambda_i, M_i)\}$, $i \in I$, is a soft fuzzy $G_\delta$ pre open collection with the finite intersection property. Hence from the hypothesis it follows that

$\bigcap_{i \in I} SF F_\sigma$ pre cl$((1, X) - SF F_\sigma$ pre cl$(\lambda_i, M_i)) \neq (0, \emptyset)$

and

$\bigcup_{i \in I} [(1, X) - SF F_\sigma$ pre cl$((1, X) - SF F_\sigma$ pre cl$(\lambda_i, M_i))] \neq (1, X)$, and hence contradiction

$\bigcup_{i \in I} SF G_\delta$ pre int$(\lambda_i, M_i) \neq (1, X)$. \qed

**Definition 33.** Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy $G_\delta$ pre almost continuous, if the inverse image of every soft fuzzy $G_\delta$ pre regular open set in $(Y, \tau^*)$ is soft fuzzy $G_\delta$ pre open in $(X, \tau)$.
Theorem 34. Let \((X, \tau)\) and \((Y, \tau^*)\) be any two soft fuzzy topological spaces. If \(f : (X, \tau) \to (Y, \tau^*)\) is soft fuzzy \(G_\delta\) pre almost continuous surjection and \((X, \tau)\) is soft fuzzy \(G_\delta\) pre almost compact, then \((Y, \tau^*)\) is soft fuzzy \(G_\delta\) pre almost compact.

Proof. Let \((\lambda_i, M_i), i \in I\) is a soft fuzzy \(G_\delta\) pre open cover of \((Y, \tau^*)\), then SF \(G_\delta\) pre int(SF \(F_\sigma\) pre cl(\(\lambda_i, M_i\))), \(i \in I\) is a fuzzy \(G_\delta\) pre regular open set in \((Y, \tau^*)\). From the soft fuzzy \(G_\delta\) pre almost continuity of \(f\) it follows that
\[
f^{-1}(\text{SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}(\lambda_i, M_i))) = \text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i), \quad i \in I,
\]
and soft fuzzy \(G_\delta\) pre almost continuity,
\[
f(\text{SF } F_\sigma \text{ pre cl}(f^{-1}(\text{SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}(\lambda_i, M_i)))) \subseteq \text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i), \quad i \in I.
\]
Hence, \(\sqcup_{i \in F} \text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i) = (1, Y)\)
\[
\square
\]

Theorem 35. In a soft fuzzy topological space \((X, \tau)\) the following conditions are equivalent:

(i) \((X, \tau)\) is soft fuzzy \(G_\delta\) pre almost compact.

(ii) For every collection \((\lambda_i, M_i)\) of soft fuzzy regular \(F_\sigma\) pre closed sets such that \(\cap_{i \in I} (\lambda_i, M_i) = (0, \emptyset)\), there exists a finite subset \(F\) of \(I\) such that
\[
\cap_{i \in F} \text{SF } G_\delta \text{ pre int}(\lambda_i, M_i) = (0, \emptyset).
\]

(iii) \(\cap_{i \in F} \text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i) \neq (0, \emptyset)\) holds for every collection of soft regular \(G_\delta\) pre open sets \((\lambda_i, M_i), i \in I\) with the finite intersection property.

(iv) If whenever \(\sqcup_{i \in I} (\lambda_i, M_i) = (1, X)\), \((\lambda_i, M_i)\) is soft fuzzy regular \(G_\delta\) pre open, \(i \in I\), there is a finite subset \(F\) of \(I\) with \(\sqcup_{i \in F} \text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i) = (1, X)\)

Proof. \((i \Rightarrow ii):\) Let \((X, \tau)\) be soft fuzzy \(G_\delta\) pre almost compact and \((\lambda_i, M_i), i \in I\) a collection of soft fuzzy regular \(F_\sigma\) pre closed sets with \(\cap_{i \in I} (\lambda_i, M_i) = (0, \emptyset)\).
Then \(\sqcup_{i \in I} ((1, X) - (\lambda_i, M_i)) = (1, X)\). Since
\[
(1, X) - (\lambda_i, M_i) = \text{SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}((1, X) - (\lambda_i, M_i))),
\]
we have \(\sqcup_{i \in I} \text{SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}((1, X) - (\lambda_i, M_i))) = (1, X)\).
From the soft fuzzy \(G_\delta\) pre almost compactness, it follows that there exists a finite subset \(F\) of \(I\) such that
\[ \bigcup_{i \in F} SF F_{\sigma} \text{ pre cl}(SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}((1, X) - (\lambda_i, M_i)))) = (1, X), \]
and then
\[ (1, X) - \bigcup_{i \in F} SF F_{\sigma} \text{ pre cl}(SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}((1, X) - (\lambda_i, M_i)))) = (0, \emptyset). \]

Now we have
\[ (1, X) - (SF F_{\sigma} \text{ pre cl}(SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}((1, X) - (\lambda_i, M_i)))) = SF G_{\delta} \text{ pre int}(\lambda_i, M_i) \] and thus \( \cap_{i \in F} SF G_{\delta} \text{ pre int}(\lambda_i, M_i) = (0, \emptyset). \)

\((ii \Rightarrow iii):\) It follows from Theorem 32.

\((iv \Rightarrow i):\) Let \((\lambda_i, M_i)\) be a soft fuzzy \(G_{\delta}\) pre open cover of \(SF(X)\). Then \(\{SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))\}, \), \(i \in I\), is a soft regular \(G_{\delta}\) pre open cover of \(SF(X)\). From the hypothesis it follows that there exists a finite subset \(F\) of \(I\) with
\[ \bigcup_{i \in F} SF F_{\sigma} \text{ pre cl}(SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))) = (1, X). \]
Since \((\lambda_i, M_i) \subseteq SF G_{\delta} \text{ pre int}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i)) \subseteq SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i),\) then \(\bigcup_{i \in F} SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i) = (1, X). \)

**Definition 36.** Let \((X, \tau)\) and \((Y, \tau^*)\) be any two soft fuzzy topological spaces. A function \(f : (X, \tau) \rightarrow (Y, \tau^*)\) is said to be soft fuzzy \(G_{\delta}\) pre weakly continuous, if we have
\[ f^{-1}(\mu, N) \subseteq SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\mu, N))) \]
for each \(G_{\delta}\) pre open \((\mu, N) \in SF(Y)\).

**Theorem 37.** A soft fuzzy \(G_{\delta}\) pre weakly continuous image of a soft fuzzy \(G_{\delta}\) pre compact space is soft fuzzy \(G_{\delta}\) pre almost compact.

**Proof.** Let \((X, \tau)\) be a soft fuzzy \(G_{\delta}\) pre compact space, \(f : (X, \tau) \rightarrow (Y, \tau^*)\) a soft fuzzy \(G_{\delta}\) pre weakly continuous and bijective function. If \((\lambda_i, M_i), i \in I,\) is a soft fuzzy \(G_{\delta}\) pre open cover of \(SF(Y)\), then \(\bigcup_{i \in I} f^{-1}(\lambda_i, M_i) = (1, X)\). Since \(f\) is a soft fuzzy \(G_{\delta}\) pre weakly continuous, then for each \(i \in I\) we have
\[ f^{-1}(\lambda_i, M_i) \subseteq SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))). \]
Hence \(\bigcup_{i \in I} SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))) = (1, X).\)
Since \((X, \tau)\) is soft fuzzy \(G_{\delta}\) pre compact, there exists a finite subset \(F\) of \(I\) with
\[ \bigcup_{i \in F} SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))) = (1, X). \]
From the hypothesis it follows that
\[ f(\bigcup_{i \in F} SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i)))) = \bigcup_{i \in F} f(SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i)))) = f(1, X) = (1, Y). \]
Since \(f(SF G_{\delta} \text{ pre int}(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i)))) \subseteq f(f^{-1}(SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i))) \) \(= SF F_{\sigma} \text{ pre cl}(\lambda_i, M_i),\)
we deduce $\sqcup_{i \in F} SF F_\sigma^{pre} \text{cl}(\lambda_i, M_i) = (1, Y)$, i.e. $(Y, \tau^*)$ is soft fuzzy $G_\delta$ pre almost compact.

**Definition 38.** Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy $G_\delta$ pre strongly continuous iff $f(SF F_\sigma^{pre} \text{cl}(\lambda, M)) \subseteq f(\lambda, M)$ for every soft fuzzy set $(\lambda, M)$ in $SF(X)$.

**Theorem 39.** A soft fuzzy $G_\delta$ pre strongly continuous image of a soft fuzzy $G_\delta$ pre almost compact space is soft fuzzy $G_\delta$ pre compact.

**Proof.** This follows from Definition 38.

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4. Soft Fuzzy $G_\delta$ Pre Nearly Compact Spaces

**Definition 40.** A soft fuzzy topological space $(X, \tau)$ is said to be a soft fuzzy $G_\delta$ pre nearly compact if whenever $\sqcup_{i \in I}(\lambda_i, M_i) = (1, X)$, $(\lambda_i, M_i)$ is soft regular fuzzy $G_\delta$ pre open, $i \in I$, there is a finite subset $F$ of $I$ with $\sqcup_{i \in F}(\lambda_i, M_i) = (1, X)$.

It is clear that in soft fuzzy topological spaces, we have the following implications:

- soft fuzzy $G_\delta$ pre compactness
  - $\downarrow$
  - soft fuzzy $G_\delta$ pre near compactness
  - $\downarrow$
  - soft fuzzy $G_\delta$ pre almost compactness

**Theorem 41.** A soft fuzzy topological space $(X, \tau)$ is $G_\delta$ pre nearly compact iff for every collection of soft fuzzy regular $F_\sigma^{pre}$ closed sets $(\lambda_i, M_i)$, $i \in I$ of $SF(X)$ having the finite intersection property we have $\cap_{i \in I}(\lambda_i, M_i) \neq (0, \emptyset)$.

**Proof.** Proof is similar to the Theorem 32.

**Theorem 42.** Let $(X, \tau)$ and $(Y, \tau^*)$ be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy $G_\delta$ pre almost continuous bijection and $(X, \tau)$ is soft fuzzy $G_\delta$ pre compact, then $(Y, \tau^*)$ is soft fuzzy $G_\delta$ pre nearly compact.
Proof. Proof is similar to the Theorem 34.

Definition 43. A soft fuzzy topological space \((X, \tau)\) is said to be a soft fuzzy \(G_\delta\) pre lightly compact if whenever \(\bigsqcup_{n \in \mathbb{N}} (\lambda_n, M_n) = (1, X)\), \((\lambda_n, M_n)\) is soft \(G_\delta\) pre open, \(n \in \mathbb{N}\), there is a finite \(F\) of \(\mathbb{N}\) with \(\bigsqcup_{n \in F} SF F_{\sigma}^{\precl} (\lambda_n, M_n) = (1, X)\).

Theorem 44. A soft fuzzy topological space \((X, \tau)\) is fuzzy \(G_\delta\) pre lightly compact iff for every countable collection of soft fuzzy \(G_\delta\) pre open sets \((\lambda_n, M_n)\) of \(SF(X)\) having the finite intersection property we have \(\bigcap_{n \in \mathbb{N}} SF F_{\sigma}^{\precl} (\lambda_n, M_n) \neq (0, \emptyset)\).

Proof. It follows from Theorem 32.

It is clear that in soft fuzzy topological spaces, we have the following implications: soft fuzzy \(G_\delta\) pre almost compactness \(\Rightarrow\) soft fuzzy \(G_\delta\) pre lightly compactness.

References


