International Journal of Pure and Applied Mathematics
Volume 98 No. 3 2015, 389-398
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v98i3.11

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ON SOFT FUZZY G_δ PRE ALMOST COMPACTNESS IN SOFT FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper the concept of soft fuzzy G_δ pre almost compactness and soft fuzzy G_δ pre near compactness are introduced and studied. We give some characterizations of soft fuzzy G_δ pre almost compactness in terms of soft fuzzy regular G_δ pre open or soft fuzzy regular F_σ pre closed sets.

AMS Subject Classification: 54A40, 03E72
Key Words: soft topological spaces, soft fuzzy G_δ pre open set, soft fuzzy G_δ pre continuous, soft fuzzy G_δ pre compact space, soft fuzzy G_δ pre almost compactness, soft fuzzy G_δ pre nearly compact space

1. Introduction

Zadeh introduced the fundamental concept of a fuzzy set in [8]. Chang in [3] introduced and developed the concept of fuzzy topological spaces. A.Di Concilio and G.Gerla [4] and A.H.Eş [5] introduced and studied almost compactness for fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by I.U.Tiryaki [6]. Visalakshi, Uma and Roja [7] introduced the concept of soft fuzzy G_δ pre continuity, soft fuzzy G_δ pre connected space and soft fuzzy G_δ pre compact.

In this paper soft fuzzy G_δ pre almost compactness and G_δ pre near compactness are introduced and their properties are discussed.

Received: October 29, 2014

2. Preliminaries

Definition 1. ([1]). Let (X, τ) be a topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$, where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2. ([2]). Let (X, τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy pre open set if $\lambda \leq int(cl(\lambda))$. The complement of a pre open set is pre closed.

Definition 3. ([6]). Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then, the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 4. *([6]). If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcap_{j\in J}(\mu_j, M_j) \text{ such that } \sqcap_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where
$\mu(x) = \bigwedge_{j\in J} \mu_j(x), \forall x \in X.$
$M = \bigcap_{j\in J} M_j$

Proposition 5. *([6]). If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a join, that is least upper bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcup_{j\in J}(\mu_j, M_j) \text{ such that } \sqcup_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where
$\mu(x) = \bigvee_{j\in J} \mu_j(x), \forall x \in X.$
$M = \bigcup_{j\in J} M_j$

Definition 6. ([6]). Let X be a non-empty set and the soft fuzzy sets A and B be in the form,

$$A = \{(\mu, M)|\mu(x) \in I^X, \forall x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N)|\lambda(x) \in I^X, \forall x \in X, N \subseteq X\}$$

Then,
(1) $A \sqsubseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N.$
(2) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N.$
(3) $A' \Leftrightarrow 1 - \mu(x), \forall x \in X, X \setminus M.$
(4) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X, M \cap N.$
(4) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X, M \cup N.$

Definition 7. ([6]).

$$(0,\emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}$$
$$(1,X) = \{(\lambda, N) | \lambda = 1, N = X\}$$

Definition 8. ([6]). For $(\mu, M) \in SF(X)$ the soft fuzzy set $(\mu, M)' = (1-\mu, X \setminus M)$ is called the complement of (μ, M).

Definition 9. ([6]). A subset $\tau \subseteq SF(X)$ is called an SF-topology on X if
(1) $(0,\emptyset)$ and $(1,X) \in \tau$
(2) $(\mu_j, M_j) \in \tau$, $j = 1, 2, \dots, n \Rightarrow \sqcap_{j=1}^{n}(\mu_j, M_j) \in \tau$
(3) $(\mu_j, M_j), j \in J \Rightarrow \sqcup_{j\in J}(\mu_j, M_j) \in \tau$. The elements of τ are called soft fuzzy open, and those of $\tau' = \{(\mu, M) | (\mu, M)' \in \tau\}$ soft fuzzy closed.
If τ is SF-topology on X we call the pair (X, τ) SF-topological space (in short, SFTS).

Definition 10. ([6]). The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by

$$\overline{(\mu, M)} = \sqcap\{(\gamma, N) | (\mu, M) \sqsubseteq (\gamma, N), (\gamma, N) \in \tau'\}.$$

Likewise the interior is given by

$$(\mu, M)^\circ = \sqcup\{(\gamma, N) | (\gamma, N) \in \tau, (\gamma, N) \sqsubseteq (\mu, M)\}.$$

Note : $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^\circ = int(\mu, M)$.

Definition 11. ([6]). A soft fuzzy topological space (X, τ) is said to be a soft fuzzy compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X), (\lambda_i, M_i) \in \tau, i \in I$, there is a finite subset J of I with $\sqcup_{j\in J}(\lambda_j, \mu_j) = (1, X)$.

Definition 12. ([7]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ set if $(\lambda, N) = \sqcap_{i=1}^{\infty}(\mu_i, M_i)$, where each (μ_i, M_i) is soft fuzzy open set. The complement of a soft fuzzy G_δ set is soft fuzzy F_σ.

Definition 13. ([7]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy pre open set if $(\lambda, N) \sqsubseteq int(cl(\lambda, N))$. The complement of a soft fuzzy pre open set is soft fuzzy pre closed.

Definition 14. ([7]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ pre open set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$, where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy pre open set. The complement of a soft fuzzy G_δ pre open set is soft fuzzy F_σ pre closed.

Definition 15. ([7]). Let (X, τ) be a soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, τ). Then soft fuzzy G_δ pre interior of (λ, M) is defined as follows

$$\text{SF } G_\delta \text{ pre int}(\lambda, M) = \sqcup\{(\mu, N)|(\mu, N) \text{ is SF } G_\delta \text{ pre open and } (\mu, N) \sqsubseteq (\lambda, M)\}.$$

Proposition 16. *([7]). Let (X, τ) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then SF G_δ pre int(λ, N) is a soft fuzzy G_δ pre open set in (X, τ).*

Proposition 17. *([7]). Let (X, τ) be any soft fuzzy topological space and (λ, M), (μ, N) be soft fuzzy sets in (X, τ). Then the following properties hold:*

(i) SF G_δ pre int$(\lambda, M) \sqsubseteq (\lambda, M)$.
(ii) $(\lambda, M) \sqsubseteq (\mu, N) \Rightarrow$ SF G_δ pre int$(\lambda, M) \sqsubseteq$ SF G_δ pre int(μ, N).
(iii) SF G_δ pre int(SF G_δ pre int(λ, M)) =SF G_δ pre int(λ, M).
(iv) SF G_δ pre int$((\lambda, M) \sqcap (\mu, N)) \sqsubseteq$ SF G_δ pre int$(\lambda, M) \sqcap$ SF G_δ pre int(μ, N).
(v) SF G_δ pre int$(1, X) = (1, X)$.

Definition 18. ([7]). Let (X, τ) be any soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, τ). Then soft fuzzy F_σ pre closure of (λ, M) is defined as follows

$$\text{SF } F_\sigma \text{ pre cl}(\lambda, M) = \sqcap\{(\mu, N)|(\mu, N) \text{ is SF } F_\sigma \text{ pre closed and } (\lambda, M) \sqsubseteq (\mu, N)\}.$$

Proposition 19. *([7]). Let (X, τ) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then SF F_σ pre cl(λ, N) is a soft fuzzy F_σ pre closed set in (X, τ).*

Proposition 20. *([7]). Let (X, τ) be any soft fuzzy topological space and (λ, M), (μ, N) be soft fuzzy sets in (X, τ). Then the following properties hold:*

(i) $(\lambda, M) \sqsubseteq$SF F_σ pre cl(λ, M).
(ii) $(\lambda, M) \sqsubseteq (\mu, N) \Rightarrow$ SF F_σ pre cl$(\lambda, M) \sqsubseteq$ SF F_σ pre cl(μ, N).
(iii) SF F_σ pre cl(SF F_σ pre cl(λ, M)) =SF F_σ pre cl(λ, M).
(iv) SF F_σ pre cl$((\lambda, M) \sqcup (\mu, N)) =$ SF F_σ pre cl$(\lambda, M) \sqcup$ SF F_σ pre cl(μ, N).
(v) SF F_σ pre cl$(0, \emptyset) = (0, \emptyset)$.

Proposition 21. *([7]). For any soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) the following hold:*

(i) SF F_σ pre cl$((1,X)-(\lambda,M)) = (1,X)-$*SF G_δ pre int*(λ,M).
(ii) SF G_δ pre int$((1,X)-(\lambda,M)) = (1,X)-$*SF F_σ pre cl*(λ,M).

Definition 22. ([7]). Let (X,τ) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,τ). Then (λ,N) is said to be soft fuzzy regular G_δ pre open if (λ,N) =SF G_δ pre int(SF F_σ pre cl(λ,N)).

Definition 23. ([7]). Let (X,τ) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,τ). Then (λ,N) is said to be soft fuzzy regular F_σ pre closed if (λ,N) =SF F_σ pre cl(SF G_δ pre int(λ,N)).

Proposition 24. *([7]). (i) The soft fuzzy F_σ pre closure of a soft fuzzy G_δ pre open set is soft fuzzy regular F_σ pre closed.*
(ii) The soft fuzzy G_δ pre interior of a soft fuzzy F_σ pre closed set is soft fuzzy regular G_δ pre open.

Definition 25. ([7]). Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\tau^*)$ is said to be soft fuzzy G_δ pre continuous, if the inverse image of every soft fuzzy open set in (Y,τ^*) is soft fuzzy G_δ pre open in (X,τ).

Definition 26. ([7]). Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\tau^*)$ is said to be soft fuzzy G_δ pre irresolute, if the inverse image of every soft fuzzy G_δ pre open set in (Y,τ^*) is soft fuzzy G_δ pre open in (X,τ).

Definition 27. ([7]). A soft fuzzy topological space (X,τ) is said to be a soft fuzzy G_δ pre compact if whenever $\sqcup_{i\in I}(\lambda_i,M_i)=(1,X)$, (λ_i,M_i) is soft fuzzy G_δ pre open, $i\in I$, there is a finite subset J of I with $\sqcup_{i\in J}(\lambda_i,M_i)=(1,X)$.

Proposition 28. *([7]). Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. If $f:(X,\tau)\to(Y,\tau^*)$ is soft fuzzy G_δ pre continuous bijection and (X,τ) is soft fuzzy G_δ pre compact, then (Y,τ^*) is soft fuzzy compact.*

Proposition 29. *([7]). Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. If $f:(X,\tau)\to(Y,\tau^*)$ is soft fuzzy G_δ pre irresolute bijection and (X,τ) is soft fuzzy G_δ pre compact, then (Y,τ^*) is soft fuzzy G_δ pre compact.*

3. Soft Fuzzy G_δ Pre Almost Compact Spaces

Definition 30. A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre almost compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft fuzzy G_δ pre open, $i \in I$, there is a finite subset J of I with $\sqcup_{i\in J}$SF F_σ pre cl$(\lambda_i, M_i) = (1, X)$.

Definition 31. Let (X, τ) be a soft fuzzy topological space. If (λ_i, M_i), $i \in I$, of soft fuzzy sets in (X, τ) satisfies the finite intersection property (FIP for short) iff every finite subfamily (λ_i, M_i), $i = 1, 2, \ldots, n$ of the family satisfies the condition $\sqcap_{i=1}^{n}(\lambda_i, M_i) \neq (0, \emptyset)$.

Theorem 32. *A soft fuzzy topological space (X, τ) is fuzzy G_δ pre almost compact iff for every collection of soft fuzzy G_δ pre open sets (λ_i, M_i) of $SF(X)$ having the finite intersection property we have $\sqcap_{i\in I}$SF F_σ pre cl$(\lambda_i, M_i) \neq (0, \emptyset)$.*

Proof. ($\Rightarrow$:) Let (λ_i, M_i), $i \in I$ be a collection of soft fuzzy G_δ pre open sets with finite intersection property. If $\sqcap_{i\in I}$ SF F_σ pre cl$(\lambda_i, M_i) = (0, \emptyset)$, then $(1, X) - \sqcap_{i\in I}$SF F_σ pre cl$(\lambda_i, M_i) = \sqcup_{i\in I}$SF G_δ pre int$((1, X) - (\lambda_i, M_i))$ From the fuzzy G_δ pre almost compactness it follows that there exists a finite subset F of I such that

$\sqcup_{i\in F}$SF F_σ pre cl(SF G_δ pre int $((1, X) - (\lambda_i, M_i))) = (1, X)$.

Hence

$\sqcap_{i\in F}$SF G_δ pre int$(\lambda_i, M_i) \sqsubseteq[(1, X)-$ SF F_σ pre cl(SF G_δ pre int $((1, X) - (\lambda_i, M_i))] = (0, \emptyset)$, which is a contradiction.

($\Leftarrow$:) Let (λ_i, M_i), $i \in I$ be a soft fuzzy G_δ pre open cover of $SF(X)$. If $\sqcup_{i\in F}$ SF F_σ pre cl(λ_i, M_i), for every finite F of I, does not cover $SF(X)$, then

$(1, X)-$SF F_σ pre cl$(\lambda_i, M_i) =$ SF G_δ pre int$((1, X) - (\lambda_i, M_i))$ and $\{(1, X)-$SF F_σ pre cl $(\lambda_i, M_i)\}$, $i \in I$, is a soft fuzzy G_δ pre open collection with the finite intersection property. Hence from the hypothesis it follows that

$\sqcap_{i\in I}$ SF F_σ pre cl$((1, X)-$ SF F_σ pre cl$(\lambda_i, M_i)) \neq (0, \emptyset)$

and

$\sqcup_{i\in I}[(1, X)-$ SF F_σ pre cl$((1, X)-$ SF F_σ pre cl$(\lambda_i, M_i))] \neq (1, X)$,

and hence contradiction

$\sqcup_{i\in I}$ SF G_δ pre int$(\lambda_i, M_i) \neq (1, X)$. □

Definition 33. Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre almost continuous, if the inverse image of every soft fuzzy G_δ pre regular open set in (Y, τ^*) is soft fuzzy G_δ pre open in (X, τ).

Theorem 34. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre almost continuous surjection and (X, τ) is soft fuzzy G_δ pre almost compact, then (Y, τ^*) is soft fuzzy G_δ pre almost compact.*

Proof. Let (λ_i, M_i), $i \in I$ is a soft fuzzy G_δ pre open cover of (Y, τ^*), then SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i))$, $i \in I$ is a fuzzy G_δ pre regular open set in (Y, τ^*). From the soft fuzzy G_δ pre almost continuity of f it follows that

f^{-1}(SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i)))$, $i \in I$ is a soft fuzzy G_δ pre open in (X, τ). Hence there exists a finite F of I such that

$\sqcup_{i \in F}$SF F_σ pre cl(f^{-1}(SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i))) = (1, X)$.

From the surjectivity of f we have

$f(\sqcup_{i \in F}$SF F_σ pre cl(f^{-1}(SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i)))) =$
$\sqcup_{i \in F} f$(SF F_σ pre cl(SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i)))) = (1, Y)$.
But, from SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i)) \sqsubseteq$ SF F_σ pre cl(λ_i, M_i) and soft fuzzy G_δ pre almost continuity,

f(SF F_σ pre cl(f^{-1}(SF G_δ pre int(SF F_σ pre cl$(\lambda_i, M_i)))) \sqsubseteq$
SF F_σ pre cl(λ_i, M_i), $i \in I$.
Hence, $\sqcup_{i \in F}$SF F_σ pre cl$(\lambda_i, M_i) = (1, Y)$ □

Theorem 35. *In a soft fuzzy topological space (X, τ) the following conditions are equivalent:*

(i) (X, τ) is soft fuzzy G_δ pre almost compact.
(ii) For every collection (λ_i, M_i) of soft fuzzy regular F_σ pre closed sets such that $\sqcap_{i \in I}(\lambda_i, M_i) = (0, \emptyset)$, there exists a finite subset F of I such that $\sqcap_{i \in F}$SF G_δ pre int$(\lambda_i, M_i) = (0, \emptyset)$.
(iii) $\sqcap_{i \in F}$SF F_σ pre cl$(\lambda_i, M_i) \neq (0, \emptyset)$ holds for every collection of soft regular G_δ pre open sets (λ_i, M_i), $i \in I$ with the finite intersection property.
(iv) If whenever $\sqcup_{i \in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft fuzzy regular G_δ pre open, $i \in I$, there is a finite subset F of I with $\sqcup_{i \in F}$SF F_σ pre cl$(\lambda_i, M_i) = (1, X)$

Proof. $(i \Rightarrow ii)$: Let (X, τ) be soft fuzzy G_δ pre almost compact and (λ_i, M_i), $i \in I$ a collection of soft fuzzy regular F_σ pre closed sets with $\sqcap_{i \in I}(\lambda_i, M_i) = (0, \emptyset)$.
Then $\sqcup_{i \in I}((1, X) - (\lambda_i, M_i)) = (1, X)$. Since

$(1, X) - (\lambda_i, M_i) =$ SF G_δ pre int(SF F_σ pre cl$((1, X) - (\lambda_i, M_i)))$,
we have $\sqcup_{i \in I}$SF G_δ pre int(SF F_σ pre cl$((1, X) - (\lambda_i, M_i))) = (1, X)$.
From the soft fuzzy G_δ pre almost compactness, it follows that there exists a finite subset F of I such that

$\sqcup_{i\in F}$SF F_σ pre cl(SF G_δ pre int(SF F_σ pre cl($(1,X)-(\lambda_i,M_i)$))) = $(1,X)$, and then

$(1,X)-\sqcup_{i\in F}$SF F_σ pre cl(SF G_δ pre int(SF F_σ pre cl($(1,X)-(\lambda_i,M_i)$))) = $(0,\emptyset)$.

Now we have

$(1,X)-$(SF F_σ pre cl(SF G_δ pre int(SF F_σ pre cl($(1,X)-(\lambda_i,M_i)$)))) = SF G_δ pre int(λ_i,M_i) and thus $\sqcap_{i\in F}$SF G_δ pre int$(\lambda_i,M_i)=(0,\emptyset)$.

($ii\Rightarrow iii$): It follows from Theorem 32.

($iii\Rightarrow iv$): Proof is similar to the Theorem 32.

($iv\Rightarrow i$): Let (λ_i,M_i) be a soft fuzzy G_δ pre open cover of $SF(X)$. Then {SF G_δ pre int(SF F_σ pre cl(λ_i,M_i))}, $i\in I$, is a soft regular G_δ pre open cover of $SF(X)$. From the hypothesis it follows that there exists a finite subset F of I with

$\sqcup_{i\in F}$SF F_σ pre cl(SF G_δ pre int(SF F_σ pre cl(λ_i,M_i))) = $(1,X)$.

Since $(\lambda_i,M_i)\sqsubseteq$ SF G_δ pre int(SF F_σ pre cl(λ_i,M_i)) $\sqsubseteq$ SF F_σ pre cl(λ_i,M_i), then $\sqcup_{i\in F}$SF F_σ pre cl$(\lambda_i,M_i)=(1,X)$. □

Definition 36. Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\tau^*)$ is said to be soft fuzzy G_δ pre weakly continuous, if we have

$f^{-1}(\mu,N)\sqsubseteq$ SF G_δ pre int(f^{-1}(SF F_σ pre cl(μ,N)))

for each G_δ pre open $(\mu,N)\in SF(Y)$.

Theorem 37. *A soft fuzzy G_δ pre weakly continuous image of a soft fuzzy G_δ pre compact space is soft fuzzy G_δ pre almost compact.*

Proof. Let (X,τ) be a soft fuzzy G_δ pre compact space, $f:(X,\tau)\to(Y,\tau^*)$ a soft fuzzy G_δ pre weakly continuous and bijective function. If (λ_i,M_i), $i\in I$, is a soft fuzzy G_δ pre open cover of $SF(Y)$, then $\sqcup_{i\in I}f^{-1}(\lambda_i,M_i)=(1,X)$. Since f is a soft fuzzy G_δ pre weakly continuous, then for each $i\in I$ we have

$f^{-1}(\lambda_i,M_i)\sqsubseteq$ SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i))).

Hence $\sqcup_{i\in I}$SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i))) = $(1,X)$.

Since (X,τ) is soft fuzzy G_δ pre compact, there exists a finite subset F of I with

$\sqcup_{i\in F}$SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i))) = $(1,X)$.

From the hypothesis it follows that

$f(\sqcup_{i\in F}$SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i)))) =
$\sqcup_{i\in F}\,f$(SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i)))) = $f(1,X)=(1,Y)$.

Since f(SF G_δ pre int(f^{-1}(SF F_σ pre cl(λ_i,M_i)))) $\sqsubseteq f(f^{-1}$(SF F_σ pre cl(λ_i,M_i))) = SF F_σ pre cl(λ_i,M_i), we deduce $\sqcup_{i\in F}$SF F_σ pre cl$(\lambda_i, M_i) = (1, Y)$, i.e. (Y, τ^*) is soft fuzzy G_δ pre almost compact. □

Definition 38. Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre strongly continuous iff f(SF F_σ pre cl$(\lambda, M)) \sqsubseteq f(\lambda, M)$ for every soft fuzzy set (λ, M) in $SF(X)$.

Theorem 39. *A soft fuzzy G_δ pre strongly continuous image of a soft fuzzy G_δ pre almost compact space is soft fuzzy G_δ pre compact.*

Proof. This follows from Definition 38. □

4. Soft Fuzzy G_δ Pre Nearly Compact Spaces

Definition 40. A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre nearly compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft regular fuzzy G_δ pre open, $i \in I$, there is a finite subset F of I with $\sqcup_{i\in F}(\lambda_i, M_i) = (1, X)$.

It is clear that in soft fuzzy topological spaces, we have the following implications:

soft fuzzy G_δ pre compactness
$\Downarrow$
soft fuzzy G_δ pre near compactness
$\Downarrow$
soft fuzzy G_δ pre almost compactness

Theorem 41. *A soft fuzzy topological space (X, τ) is G_δ pre nearly compact iff for every collection of soft fuzzy regular F_σ pre closed sets (λ_i, M_i), $i \in I$ of $SF(X)$ having the finite intersection property we have $\sqcap_{i\in I}(\lambda_i, M_i) \neq (0, \emptyset)$.*

Proof. Proof is similar to the Theorem 32. □

Theorem 42. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre almost continuous bijection and (X, τ) is soft fuzzy G_δ pre compact, then (Y, τ^*) is soft fuzzy G_δ pre nearly compact.*

Proof. Proof is similar to the Theorem 34. □

Definition 43. A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre lightly compact if whenever $\sqcup_{n \in \mathbb{N}}(\lambda_n, M_n) = (1, X)$, (λ_n, M_n) is soft G_δ pre open, $n \in \mathbb{N}$, there is a finite F of $\mathbb{N}$ with $\sqcup_{n \in F}$SF F_σ pre cl$(\lambda_n, M_n) = (1, X)$.

Theorem 44. *A soft fuzzy topological space (X, τ) is fuzzy G_δ pre lightly compact iff for every countable collection of soft fuzzy G_δ pre open sets (λ_n, M_n) of $SF(X)$ having the finite intersection property we have $\sqcap_{n \in \mathbb{N}}SF$ F_σ pre cl$(\lambda_n, M_n) \neq (0, \emptyset)$.*

Proof. It follows from Theorem 32. □

It is clear that in soft fuzzy topological spaces, we have the following implications: soft fuzzy G_δ pre almost compactness $\Rightarrow$ soft fuzzy G_δ pre lightly compactness.

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