

REMARKS ON GENERALIZED TOPOLOGIES INDUCED BY SUPRA-NEIGHBORHOOD SYSTEMS

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Abstract: We investigate some properties for the generalized topological space induced by a given supra-neighborhood system. And we introduce the notions of S_{I^*} -continuity and $(sn)^*$ -continuity, and study the relations among S_{I^*} -continuity, $(sn)^*$ -continuity and the other continui.

AMS Subject Classification: 54A05, 54C05

Key Words: supra-neighborhood system, supra-neighborhood space, S_{I^*} -continuity, $(sn)^*$ -continuity, s' -convergence of m -family

1. Introduction

The notion of generalized neighborhood system (briefly *GNS*) was introduced by Császár in [1]. He also introduced the notion of (ψ, ψ') -continuity on generalized neighborhood systems ψ, ψ' . In the same way, we introduced and studied the notion of the weak neighborhood systems in [7]. In [3], we investigated supra-neighborhood systems and supra-neighborhood spaces in order to generalize the notions of GNS, weak neighborhood system [7] and neighborhood structure [2]. In [3], we also introduced the notions of operators ι, γ, i and c in supra-neighborhood spaces, and studied basic properties for such operators.

Received: August 23, 2014

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Recently, we investigated two special operators I^* and CI^* induced by a given supra-neighborhood system in [4]. In this paper, we are going to investigate the collection Ψ_{I^*} induced by the operator I^* . In particular, we introduce the notions of S_{I^*} -continuity and $(sn)^*$ -continuity between the collections, and study the relations among S_{I^*} -continuity, $(sn)^*$ -continuity, S^*n -continuity and $s^*(\psi, \phi)$ -continuity.

2. Preliminaries

Let X be a nonempty set, $exp(X)$ the power set of X and $\psi : X \rightarrow exp(exp(X))$ satisfy $x \in V$ for $V \in \psi(x)$. Then $V \in \psi(x)$ is called a *generalized neighborhood* [1] of $x \in X$ and ψ is called a *generalized neighborhood system* (briefly *GNS*) on X . Let g be a collection of subsets of X . Then g is called a *generalized topology* [1] on X iff $\emptyset \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $G = \cup_{i \in I} G_i \in g$. The elements of g are called *g -open sets* and the complements are called *g -closed sets*. For $A \subseteq X$, we denote by $i_g A$ the union of all g -open sets contained in A , i.e. the largest g -open set contained in A . Any intersection of g -closed sets is g -closed, and for $A \subseteq X$, we denote by $c_g A$ the intersection of all g -closed sets containing A , i.e. the smallest g -closed set containing A . Then a function $f : X \rightarrow Y$ is said to be (g_X, g_Y) -continuous [5] if for each g_Y -open set V in Y , $f^{-1}(V)$ is g_X -open in X .

Let $\psi : X \rightarrow exp(exp(X))$. Then ψ is called a *supra-neighborhood system* [3] on X if it satisfies the following:

- (1) For $x \in X$, $\psi(x) \neq \emptyset$.
- (2) For $V \in \psi(x)$, $x \in V$.

Then the pair (X, ψ) is called a *supra-neighborhood space* (briefly *SNS*) on X . Then $V \in \psi(x)$ is called a *supra-neighborhood* of $x \in X$.

Let (X, ψ) be an SNS on X and $G \subseteq X$. Then G is called an *S^* -open set* [3] if for each $x \in G$, there is $V \in \psi(x)$ such that $V \subseteq G$. Let us denote $S^*(X)$ the collection of all *S^* -open sets* on an SNS (X, ψ) . The complements of *S^* -open sets* are called *S^* -closed sets*. Then we have shown that the collection $S^*(X)$ of all *S^* -open subsets* of X is a supratopology [5] on X , that is, $X \in S^*(X)$ and $S^*(X)$ is closed under arbitrary union.

We recall the notions of interior operators i, ι and closure operators c, γ introduced in [3], respectively: Let (X, ψ) be an SNS on X and $A \subseteq X$. Then

- (1) $i(A) = \cup \{G \subseteq A \text{ and } G \in S^*(X)\}$;
- (2) $c(A) = \cap \{F \subseteq X \text{ and } X - F \in S^*(X)\}$;
- (3) $\iota(A) = \{x \in A : \text{there exists } V \in \psi(x) \text{ such that } V \subseteq A\}$;

$$(4) \gamma(A) = \{x \in X : V \cap A \neq \emptyset \text{ for all } V \in \psi(x)\}.$$

Theorem 2.1 ([3]). *Let (X, ψ) be an SNS on X and $A \subseteq X$. Then*

- (1) $i(A) \subseteq \iota(A) \subseteq A$;
- (2) $A \subseteq \gamma(A) \subseteq c(A)$;
- (3) A is S^* -open iff $i(A) = A$;
- (4) A is S^* -closed iff $c(A) = A$.

Remark 2.2. Let (X, ψ) and (Y, ϕ) be two SNS's. Then $f : X \rightarrow Y$ is said to be

- (1) $s^*(\psi, \phi)$ -continuous [3] if for each $x \in X$ and $V \in \phi(f(x))$, there is $U \in \psi(x)$ such that $f(U) \subseteq V$;
- (2) sn -continuous [3] if for every $A \in S^*(Y)$, $f^{-1}(A)$ is in $S^*(X)$;
- (3) S^*n -continuous [4] if for every $A \in \phi(f(x))$, $f^{-1}(A)$ is in $\psi(x)$.

$$S^*n\text{-continuous} \Rightarrow s^*(\psi, \phi)\text{-continuous} \Rightarrow sn\text{-continuous}$$

3. Main Results

We recall two special operators I^* and Cl^* introduced in [4]: Let (X, ψ) be an SNS and $A \subseteq X$.

$$I^*(A) = \{x \in A : A \in \psi(x)\}; \quad Cl^*(A) = \{x \in X : X - A \notin \psi(x)\}.$$

In [4], we showed that the operators satisfy $I^*(A) \subseteq A \subseteq Cl^*(A)$ but the following statements are not always true:

- (1) If $A \subseteq B$, then $I^*(A) \subseteq I^*(B)$; $Cl^*(A) \subseteq Cl^*(B)$.
- (2) $I^*(A) \cap I^*(B) = I^*(A \cap B)$; $Cl^*(A) \cup Cl^*(B) = Cl^*(A \cup B)$.
- (3) $I^*(I^*(A)) = I^*(A)$; $Cl^*(Cl^*(A)) = Cl^*(A)$.

From now on, we study the collection induced by the operator I^* :

Definition 3.1. *Let (X, ψ) be an SNS on X and $\Psi_{I^*}(X) = \{\cup \sigma : \sigma \subseteq \mathbf{B}\}$ where $\mathbf{B} = \{A \subseteq X : I^*(A) = A\}$. For $G \subseteq X$, G is called an S_{I^*} -open set if $G \in \Psi_{I^*}(X)$.*

The complements of S_{I^} -open sets are called S_{I^*} -closed sets. Set $\Psi_{I^*}(x) = \{G \in \Psi_{I^*}(X) : x \in G\}$.*

Remark 3.2. Let (X, ψ) be an SNS on X . Then clearly $\Psi_{I^*}(X)$ is a generalized topology on X but it need not be a supra-topology (See Example 3.16 in [4]).

Definition 3.3. Let (X, ψ) be an SNS on X and $A \subseteq X$. The S_{I^*} -interior of A , denoted by $i^*(A)$, is the union of all $G \subseteq A$, $G \in \Psi_{I^*}(X)$, and the S_{I^*} -closure of A , denoted by $c^*(A)$, is the intersection of all S_{I^*} -closed sets containing A .

Theorem 3.4. Let (X, ψ) be an SNS on X and $A \subseteq X$. Then the following hold.

- (1) $i^*(A) \subseteq i(A) \subseteq A$;
- (2) $A \subseteq c^*(A) \subseteq c(A)$;
- (3) A is S_{I^*} -open if and only if $i^*(A) = A$;
- (4) A is S_{I^*} -closed if and only if $c^*(A) = A$.

Proof. (1) We show that $i^*(A) \subseteq i(A)$. Let $x \in i^*(A)$. Then there exists an S_{I^*} -open set G such that $x \in G \subseteq A$. From the definition of S_{I^*} -openness, there is a family $\sigma = \{U \subseteq X : I^*(U) = U\}$ such that $G = \cup \sigma$. Let $U \in \sigma$. For each $z \in U$, from $z \in U = I^*(U)$, it follows $U \in \psi(z)$ and $z \in i(U)$, and so U is S^* -open. This implies G is S^* -open and $x \in i(A)$. So from Theorem 2.1, the inclusion relations are completed.

(2) It is similar to (1).

(3) and (4) are obvious. □

Example 3.5. Let $X = \{a, b, c, d\}$ and $\psi : X \rightarrow \exp(\exp(X))$ a supra-neighborhood system defined as the following: $\psi(a) = \{\{a, c\}\}$, $\psi(b) = \{\{b\}\}$, $\psi(c) = \psi(d) = \{X\}$. Then for $A = \{a, b, c\} \subseteq X$, $i(A) = \{a, b\}$ and $i^*(A) = \emptyset$, and so $i^*(A) \neq i(A)$.

Theorem 3.6. Let (X, ψ) be an SNS on X . Then every S_{I^*} -open set is S^* -open.

Proof. Let $A \subseteq X$ be an S_{I^*} -open set. From (1) and (3) of Theorem 3.4, $i(A) = A$ is obtained. It implies A is S^* -open. □

We recall the notion of m -family [6]: A collection \mathbf{H} of subsets of X is called an m -family on X if $\cap \mathbf{H} = \cap \{G : G \in \mathbf{H}\} \neq \emptyset$.

Definition 3.7. Let (X, ψ) be an SNS and let \mathbf{H} be an m -family on X . Then we say that an m -family \mathbf{H} s^* -converges to $x \in X$ if $\Psi_{I^*}(x) \subseteq \mathbf{H}$.

Theorem 3.8. Let (X, ψ) be an SNS and $A \subseteq X$. Then the following hold.

- (1) $i^*(A) = \{x \in A : A \in \mathbf{H}, \text{ for every } m\text{-family } \mathbf{H} \text{ } s^*\text{-converging to } x\}$.
- (2) $c^*(A) = \{x \in X : \text{there exists an } m\text{-family } \mathbf{H} \text{ such that } \mathbf{H} \text{ } s^*\text{-converges to } x \text{ and } A \in \mathbf{H}\}$.

Proof. (1) Let $x \in i^*(A)$ and an m -family \mathbf{H} s^* -converge to x . Then it is obvious that $A \in \Psi_{I^*}(x) \subseteq \mathbf{H}$.

Suppose that for every m -family \mathbf{H} s^* -converging to x , $A \in \mathbf{H}$. Since $\Psi_{I^*}(x)$ s^* -converges to x , by hypothesis, $A \in \Psi_{I^*}(x)$, so that $x \in i^*(A)$.

(2) Let $x \in Cl^*(A)$; then for all $V \in \Psi_{I^*}(x)$, $V \cap A \neq \emptyset$. Set $\mathbf{H} = \Psi_{I^*}(x) \cup \{A\}$; then \mathbf{H} is an m -family s^* -converging to x such that $A \in \mathbf{H}$.

For the converse, let \mathbf{H} be an m -family s^* -converging to x and $A \in \mathbf{H}$; then since $\Psi_{I^*}(x)$ is contained in \mathbf{H} , by definition of m -family, $V \cap A \neq \emptyset$ for all $V \in \Psi_{I^*}(x)$. Hence $x \in c^*(A)$. \square

Theorem 3.9. *Let (X, ψ) be a generalized topological space and $A \subseteq X$. Then the following hold.*

(1) $i_g(A) = \{x \in A : A \in \mathbf{H}, \text{ for every } m\text{-family } \mathbf{H} \text{ } s^*\text{-converging to } x\}$.

(2) $c_g(A) = \{x \in X : \text{there exists an } m\text{-family } \mathbf{H} \text{ such that } \mathbf{H} \text{ } s^*\text{-converges to } x \text{ and } A \in \mathbf{H}\}$.

Proof. First, for $A \subseteq X$, we know that $i^*(A)$ ($c^*(A)$) is generalized open (generalized close), since the family $\Psi_{I^*} = \{\cup \sigma : \sigma \subseteq \mathbf{B}\}$ is a generalized topology. So from Theorem 3.8, the above statements (1) and (2) are directly obtained. \square

Definition 3.10. *Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then f is said to be S_{I^*} -continuous if for every $A \in \Phi_{I^*}(Y)$, $f^{-1}(A)$ is in $\Psi_{I^*}(X)$.*

Theorem 3.11. *Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then the following things are equivalent:*

(1) f is S_{I^*} -continuous.

(2) For each S_{I^*} -closed set F in Y , $f^{-1}(F)$ is S_{I^*} -closed in X .

(3) $f(c^*(A)) \subseteq c^*(f(A))$ for all $A \subseteq X$.

(4) $c^*(f^{-1}(B)) \subseteq f^{-1}(c^*(B))$ for all $B \subseteq Y$.

(5) $f^{-1}(i^*(B)) \subseteq i^*(f^{-1}(B))$ for all $B \subseteq Y$.

(6) $f : (X, \Psi_{I^*}(X)) \rightarrow (Y, \Phi_{I^*}(Y))$ is $(\Psi_{I^*}(X), \Phi_{I^*}(Y))$ -continuous.

Proof. Obvious. \square

Corollary 3.12. *Let $f : X \rightarrow Y$ be a function on generalized topological spaces (X, g_X) and (Y, g_Y) . Then the following things are equivalent:*

(1) f is (g_X, g_Y) -continuous.

(2) For each g -closed set F in Y , $f^{-1}(F)$ is g -closed in X .

- (3) $f(c_g(A)) \subseteq c_g(f(A))$ for all $A \subseteq X$.
- (4) $c_g(f^{-1}(B)) \subseteq f^{-1}(c_g(B))$ for all $B \subseteq Y$.
- (5) $f^{-1}(i_g(B)) \subseteq i_g(f^{-1}(B))$ for all $B \subseteq Y$.

Proof. Since $\Psi_{I^*}(X)$ and $\Phi_{I^*}(Y)$ are generalized topologies on X and Y , respectively, the corollary is obtained from Theorem 3.11. □

Theorem 3.13. *Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . If f is S^*n -continuous, then it is also S_{I^*} -continuous.*

Proof. Let $A \in \Phi_{I^*}(Y)$. Then $A = \cup A_\alpha$ where $I^*(A_\alpha) = A_\alpha$. First, we show that for each α , $I^*(f^{-1}(A_\alpha)) = f^{-1}(A_\alpha)$. For the proof, let $x \in f^{-1}(A_\alpha)$; then $f(x) \in A_\alpha = I^*(A_\alpha)$, and $A_\alpha \in \phi(f(x))$. From S^*n -continuity of f , it is obtained $f^{-1}(A_\alpha) \in \psi(x)$. It implies $x \in I^*(f^{-1}(A_\alpha))$ and $I^*(f^{-1}(A_\alpha)) = f^{-1}(A_\alpha)$. From this fact, we have $f^{-1}(A) = \cup f^{-1}(A_\alpha)$ such that $I^*(f^{-1}(A)) = f^{-1}(A)$. Consequently, $f^{-1}(A)$ is S_{I^*} -open, that is, $f^{-1}(A) \in \Psi_{I^*}(X)$. □

In the next example, we can show that the converse is not always true:

Example 3.14. Let $X = \{a, b, c, d\}$ and let $\psi : X \rightarrow \exp(\exp(X))$ be a supra-neighborhood system defined as Example 3.5. Consider a function $f : (X, \psi) \rightarrow (X, \psi)$ as follows $f(a) = a, f(b) = b, f(c) = d, f(d) = c$.

Then f is S_{I^*} -continuous but not S^*n -continuous.

Moreover, we can show that there is no any relation between S_{I^*} -continuity and $s^*(\psi, \phi)$ -continuity as in the next example:

Example 3.15. Let $X = \{a, b, c, d\}$ and let $\psi : X \rightarrow \exp(\exp(X))$ be a supra-neighborhood system defined as Example 3.5.

- (1) Let us define the function $f : (X, \psi) \rightarrow (X, \psi)$ as follows

$$f(a) = a, f(b) = b, f(c) = d, f(d) = c.$$

Then f is S_{I^*} -continuous. For $a \in X$ and $V = \{a, c\} \in \psi(f(a))$, since $\{a, c\} \in \psi(a)$ and $f(\{a, c\}) = \{a, d\}$, there is no $U \in \psi(a)$ such that $f(U) \subseteq V$, so f is not $s^*(\psi, \psi)$ -continuous.

- (2) Let us define the function $f : (X, \psi) \rightarrow (X, \psi)$ as follows

$$f(a) = f(c) = b, f(b) = c, f(d) = d.$$

Then f is $s^*(\psi, \psi)$ -continuous. For the S_{I^*} -open set $G = \{b\}$, $f^{-1}(G) = \{a, c\}$ and it is not S_{I^*} -open. So f is not S_{I^*} -continuous.

In the next examples, we show that there is no any relationship between S_{I^*} -continuity and sn -continuity:

Example 3.16. Let $X = \{a, b, c, d\}$. Consider a supra-neighborhood system $\psi : X \rightarrow \exp(\exp(X))$ defined as the following: $\psi(a) = \{\{a, b\}\}$, $\psi(b) = \psi(c) = \{\{b, c\}\}$, and $\psi(d) = \emptyset$. Note that $\Psi_{I^*}(X) = \{\emptyset, \{b, c\}\}$ and $S^*(X) = \{\emptyset, \{b, c\}, \{a, b, c\}\}$.

(1) Let us define the function $f : (X, \psi) \rightarrow (X, \psi)$ as the following:

$$f(a) = a, f(b) = c, f(c) = b, f(d) = a.$$

Then f is S_{I^*} -continuous, but it is not sn -continuous.

(2) Let us define the function $g : (X, \psi) \rightarrow (X, \psi)$ as the following:

$$g(a) = c, g(b) = c, g(c) = b, g(d) = d.$$

Then g is sn -continuous, but it is not S_{I^*} -continuous.

Finally, from the above examples and Remark 2.3, the following diagram is obtained:

$$\begin{array}{c} S^*n\text{-continuous} \Rightarrow s^*(\psi, \phi)\text{-continuous} \Rightarrow sn\text{-continuous} \\ \Downarrow \\ S_{I^*}\text{-continuous} \end{array}$$

Theorem 3.17. Let $f : X \rightarrow Y$ be a bijective function between SNS's (X, ψ) and (Y, ϕ) . Then f is S_{I^*} -continuous iff for an m -family \mathbf{H} s^* -converging to $x \in X$, $f(\mathbf{H})$ s^* -converges to $f(x)$.

Proof. Suppose f is S_{I^*} -continuous and \mathbf{H} is an m -family s^* -converging to $x \in X$. It is obvious $f(\mathbf{H}) = \{f(F) : F \in \mathbf{H}\}$ is an m -family on Y . By hypothesis and surjectivity, we get $\Phi_{I^*}(f(x)) \subseteq f(\Psi_{I^*}(x)) \subseteq f(\mathbf{H})$, so that $f(\mathbf{H})$ s^* -converges to $f(x)$.

For the converse, let $G \in \Phi_{I^*}(f(x))$ for $G \subseteq Y$. Since $\Psi_{I^*}(x)$ s^* -converges to x , by hypothesis, we get $\Phi_{I^*}(f(x)) \subseteq f(\Psi_{I^*}(x))$ for $x \in X$. Since f is injective, $f^{-1}(G) \in \Psi_{I^*}(x)$. Hence f is S_{I^*} -continuous. \square

From Theorem 3.17, we have the following:

Corollary 3.18. Let $f : X \rightarrow Y$ be a bijective function between generalized topological spaces (X, g_X) , (Y, g_Y) . Then f is (g_X, g_Y) -continuous iff for an m -family \mathbf{H} s^* -converging to $x \in X$, $f(\mathbf{H})$ s^* -converges to $f(x)$.

Definition 3.19. Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then f is said to be $(sn)^*$ -continuous if for every S_{I^*} -open set A in Y , $f^{-1}(A)$ is S^* -open in X .

Theorem 3.20 ([3]). Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then the following things are equivalent:

- (1) f is sn -continuous.
- (2) For each S^* -closed set F in Y , $f^{-1}(F)$ is S^* -closed in X .
- (3) $f(c(A)) \subseteq c(f(A))$ for all $A \subseteq X$.
- (4) $c(f^{-1}(B)) \subseteq f^{-1}(c(B))$ for all $B \subseteq Y$.
- (5) $f^{-1}(i(B)) \subseteq i(f^{-1}(B))$ for all $B \subseteq Y$.

Theorem 3.21. Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then the following things are equivalent:

- (1) f is $(sn)^*$ -continuous.
- (2) For each S_{I^*} -closed set F in Y , $f^{-1}(F)$ is S^* -closed in X .
- (3) $f(c(A)) \subseteq c^*(f(A))$ for all $A \subseteq X$.
- (4) $c(f^{-1}(B)) \subseteq f^{-1}(c^*(B))$ for all $B \subseteq Y$.
- (5) $f^{-1}(i^*(B)) \subseteq i(f^{-1}(B))$ for all $B \subseteq Y$.

Proof. It follows from Theorem 3.11 and Theorem 3.20. □

Theorem 3.22. Let $f : X \rightarrow Y$ be a function on two SNS's (X, ψ) and (Y, ϕ) . Then:

- (1) If f is S_{I^*} -continuous, then it is also $(sn)^*$ -continuous.
- (2) If f is sn -continuous, then it is also $(sn)^*$ -continuous.

Proof. It is obvious from Theorem 3.6 and definitions of such continuous functions. □

In Theorem 3.22, the converses may not be true as the next example:

Example 3.23. Let $X = \{a, b, c, d\}$. Consider a supra-neighborhood system $\psi : X \rightarrow \exp(\exp(X))$ defined as the following: $\psi(a) = \{\{a, b\}\}$, $\psi(b) = \psi(c) = \{\{b, c\}\}$, and $\psi(d) = \emptyset$. Note that $\Psi_{I^*}(X) = \{\emptyset, \{b, c\}\}$ and $S^*(X) = \{\emptyset, \{b, c\}, \{a, b, c\}\}$.

- (1) Let us define the function $f : (X, \psi) \rightarrow (X, \psi)$ as follows

$$f(a) = b, f(b) = b, f(c) = b, f(d) = a.$$

Then f is $(sn)^*$ -continuous, but it is not sn -continuous.

- (2) Let us define the function $g : (X, \psi) \rightarrow (X, \psi)$ as follows

$$g(a) = c, g(b) = c, g(c) = b, g(d) = d.$$

Then g is $(sn)^*$ -continuous, but it is not S_{I^*} -continuous.

In summary, we have the following implications:

$$\begin{array}{ccc}
 S^*n\text{-continuous} & \Rightarrow & s^*(\psi, \phi)\text{-continuous} & \Rightarrow & sn\text{-continuous} \\
 \downarrow & & & & \downarrow \\
 S_{I^*}\text{-continuous} & & \Rightarrow & & (sn)^*\text{-continuous}
 \end{array}$$

4. Conclusion

We investigated some properties for the generalized topology induced by a given supra-neighborhood system on X . And we introduced the notions of S_{I^*} -continuity and $(sn)^*$ -continuity, and studied the relations among S_{I^*} -continuity, $(sn)^*$ -continuity, S^*n -continuity and $s^*(\psi, \phi)$ -continuity. In the next research, we are going to study a soft generalized topological universe induced by soft supra-neighborhood systems on a universe set, and investigate soft S_{I^*} -continuity and soft $(sn)^*$ -continuity.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(Grant No. NRF-2012R1A1A4A01004765)

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