ANALYSIS OF ECOLOGICAL MODEL WITH HOLLING TYPE IV FUNCTIONAL RESPONSE

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Abstract: In this paper a three-species of predator-prey food chain model with Holling type IV functional response is proposed. The boundedness of the solution of the model is discussed. Stability analysis of the system is carried out. The dynamics of the predator-prey food chain model with simplified Holling type IV functional response is investigated theoretically as well as numerically.

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1. Introduction

Ecological models of intracting population dynamics in nature, complex and non-linear. Many researchers have been reported the complex dynamics of a

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predator-prey food chains and food web, see [8],[9],[13] and [15]. May [17] proposed chaos and chaotic dynamics in a simple model which is well known by the logistic map and brought out unpredictable dynamics of chaotic. Rai [19] proposed a three species food chain model and explained the presence of a period-doubling scenario leading to chaos. Still the work of Hasting and Powell [13] among others [4-9],[12-16] and [18] is the most cited work. In their article, the model was first formulated, stability analysis and Hopf bifurcation were performed. An important factor discovered by Abrams [1], Abrams and Roth [2], and further discussed in De Feo and Rinaldi [7] is that the main value of density of the top population is maximum at the transition from cyclic to chaotic regimes. Haque [12] studied the impact of intraspecific competition of tri-trophic food chain model, theoretical and numerical results shows that intraspecific competition plays an important role in controlling the chaotic dynamics of the system. This will confirmed in this article.

A suitable mathematical model in ecology and biology cannot easily achieved because this model could never predict nature's behavior. To face this major problem we can search for analyzable model that can be close enough the reality. Various mathematical tools, like numerical simulation and bifurcation diagrams have been used to detect chaotic dynamics in predator-prey of multi species ecological models. The functional response defined by Lotka in 1925 and Volterra in 1926 for a predator-prey model is linear and unbounded, while studying the complexity in model ecosystems need reasonable functional responses that should be nonlinear and bounded, see [17] and [22]. In 1959 Holling [14] used Michaelis-Menten's formula $\frac{wx}{c+x}$ as one of predator functional response. It is know well-known as a Holling type II or Michaelis-Menten functional response. Andrews [3] proposed a response $\frac{wx}{c+dx+x^2}$ called the Monod-Haldane functional response. Collings [6] used this function and called it Holling type IV response. This response function describes a situation in which the predator's per capita rate of predation decreases at sufficiently high prey densities [20]. In their experiments about the kinetics of phenol oxidation Sokol and Howell [23] suggested a simplified Holling type IV function of the form $\frac{wx}{d+x^2}$ and found that it is simpler and better than the original function of Holling type IV. Many investigations with Holling type II functional response, while few investigations using Holling type IV functional response in population ecology [20-23].

This paper is organized as follows: In Section 2, the mathematical model is proposed and each parameter in the model is described. In Section 3, the boundedness of the solution of the model is established. Stability analysis of the equilibrium points of the model are derived in Section 4. In Section 5, numerical study are carried out to obtain the behavior of the model. Finally,

the paper is end with a discussion and results in Section 6.

2. The Mathematical Model

Consider the three species food chain model at time t consisting of the prey whose population density denoted by $x(t)$, the intermediate predator whose population density denoted by $y(t)$ and the top predator whose population density denoted by $z(t)$. The intermediate predator y preys on its sole food x at the lower level according to the simplified Holling type-IV functional response, while the top predator z preys on y at the second level according to Holling Type II functional response.

The dynamics of the above food chain model can be represented by the following set of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= x(a_1 - b_1x) - \frac{m_0xy}{h_1 + x^2} : x(0) \geq (0), \\ \frac{dy}{dt} &= \frac{m_1xy}{h_1 + x^2} - d_1y - \frac{m_2yz}{h_2 + y} : y(0) \geq (0), \\ \frac{dz}{dt} &= \frac{m_3yz}{h_2 + y} - d_2z : z(0) \geq 0.\end{aligned}\tag{1}$$

Here the positive constants a_1 , b_1 , h_i , d_i ($i = 1, 2$) and m_j ($j = 0, 1, 2, 3$), denote to: a_1 is the growth rate of the prey x , b_1 represents the intraspecific competition among individuals of prey x , h_i 's are the half-saturation constant, d_i 's are the predator's death rate, m_0 is the maximum value which per capita reduction rate can x attain, m_1 has similar meaning to m_0 , m_2 is the maximum value which per capita reduction rate can y attain, m_3 has similar meaning to m_2 . Then the system of the three species food chain model given by (1) can be written as follows:

$$\begin{aligned}\frac{dx}{dt} &= x \left(a_1 - b_1x - \frac{m_0y}{h_1 + x^2} \right) = G_1(x, y, z), \\ \frac{dy}{dt} &= y \left(\frac{m_1x}{h_1 + x^2} - d_1 - \frac{m_2z}{h_2 + y} \right) = G_2(x, y, z), \\ \frac{dz}{dt} &= z \left(\frac{m_3y}{h_2 + y} - d_2 \right) = G_3(x, y, z),\end{aligned}\tag{2}$$

with $x(0) \geq 0$, $y(0) \geq 0$, $z(0) \geq 0$. Obviously the interaction functions G_i ($i = 1, 2, 3$) of system (2) are continuous and have continuous partial derivatives

on the positive octant $R_+^3 = \{(x, y, z) \in R_+^3 : x \geq 0, y \geq 0, z \geq 0\}$. Therefore these functions are Lipschitzian on R_+^3 , and hence the solution of the system (2) exists and is unique.

3. Boundedness of the Model

In this section, the boundedness of the solution of the system (2) in R_+^3 is established, as shown in the following theorem.

Theorem 1. *All the solutions of the three species food chain system (2) are uniformly bounded in R_+^3 , and ultimately entering the region*

$$\Omega = \left\{ (x, y, z) \in R_+^3 : 0 \leq x \leq \frac{a_1}{b_1}, 0 \leq M_1 \leq \frac{m_1 a_1}{m_0 b_1 d_1} (a_1 + d_1), \right. \\ \left. 0 \leq M_2 \leq \frac{m_1 a_1}{m_0 b_1 L} (a_1 + L) \right\}$$

where $M_1 = \frac{m_1}{m_0}x + y$, $M_2 = \frac{m_1}{m_0}x + y + \frac{m_2}{m_3}z$ and $L = \min\{d_1, d_2\}$.

Proof. Let $(x(t), y(t), z(t))$ be any solution of the system with non-negative initial condition.

From the first equation of system (2) we have

$$\frac{dx}{dt} \leq x(a_1 - b_1 x), \quad (3)$$

then according to comparison theorem [11], we obtain that

$$\lim_{t \rightarrow \infty} \text{Sup } x(t) \leq \frac{a_1}{b_1} \quad (4)$$

implies that $x(t) \leq \frac{a_1}{b_1}$, $\forall t \geq 0$. Now as $t \rightarrow \infty$.

$$\Rightarrow 0 \leq x(t) \leq \frac{a_1}{b_1}.$$

Consider now $M_2(t) = \frac{m_1}{m_0}x + y + \frac{m_2}{m_3}z$. Then

$$\begin{aligned} \frac{dM_2(t)}{dt} &= \frac{m_1}{m_0}a_1x - b_1x^2 - d_1y - \frac{m_2}{m_3}d_2z, \\ &\leq \frac{m_1 a_1}{m_0 b_1} (a_1 + L) - L \left(\frac{m_1}{m_0}x + y + \frac{m_2}{m_3}z \right), \end{aligned}$$

where $L = \min \{d_1, d_2\}$. Accordingly, we obtain the following differential inequality

$$\frac{dM_2(t)}{dt} + LM_2 \leq \frac{m_1 a_1}{m_0 b_1} (a_1 + L).$$

Again, by applying the comparison theorem [11] we obtain that for $t \rightarrow \infty$,

$$0 \leq M_2 \leq \frac{m_1 a_1}{m_0 b_1 L} (a_1 + L) \quad \forall t \geq 0. \quad (5)$$

Therefore, every solution initiating in non-negative octant are attracted in a bounded set Ω defined above (Note that M_1 is bounded similar to M_2).

4. Stability of the Model

In this section, the local stability analysis of the non-negative equilibrium points of system (2) are discussed. There are at most four non-negative equilibrium points of system (2), existence and stability conditions of them are given below:

- The trivial equilibrium point $E_0 = (0, 0, 0)$ always exists.
- The equilibrium point $E_1 = (\frac{a_1}{b_1}, 0, 0)$ always exists on the boundary of the first octant.
- The intermediate predator can exists and survive depending on its prey. Therefore, the equilibrium point $E_2 = (\bar{x}, \bar{y}, 0)$ exists uniqlly in the positive quadrant of $x - y$ plane, where $\bar{x}$ and $\bar{y}$ are given by:

$$\bar{x} = \frac{m_1}{2d_1}, \quad \bar{y} = \frac{1}{m_0} (a_1 - b_1 \frac{m_1}{2d_1}) (h_1 + \bar{x}^2), \quad (6)$$

provided that the following conditions hold

$$\frac{m_1}{2d_1} < \frac{a_1}{b_1}, \quad m_1^2 - 4d_1^2 h_1 = 0. \quad (7)$$

- In the absence of prey x , then both y and z can not survive, so there is no equilibrium point in the $y - z$ plane. In addition to that, if the intermediate predator y is absence, then there is no equilibrium point in $x - z$ plane.

- The positive equilibrium point $E_3 = (x^*, y^*, z^*)$ exists in the interior of the first octant if and only if there is a positive solution to the following equations:

$$\begin{aligned} f_1 &= a_1 - b_1x - \frac{m_0y}{h_1 + x^2} = 0, \\ f_2 &= \frac{m_1x}{h_1 + x^2} - d_1 - \frac{m_2z}{h_2 + y} = 0, \\ f_3 &= \frac{m_3y}{h_2 + y} - d_2 = 0. \end{aligned} \quad (8)$$

Straightforward computation shows that

$$y^* = \frac{d_2h_2}{m_3 - d_2}, \quad (9)$$

while x^* is the positive root of the following equation

$$x^3 - \frac{a_1}{b_1}x^2 + h_1x + \left[\frac{1}{b_1} (m_0y^* - a_1h_1) \right] = 0, \quad (10)$$

Now since $0 \leq x^* \leq \frac{a_1}{b_1}$, then $f(0) = \left[\frac{1}{b_1} (m_0y^* - a_1h_1) \right] < 0$, if

$$y^* < \frac{a_1h_1}{m_0}, \quad (11)$$

Also, $f(\frac{a_1}{b_1}) = \frac{m_0}{b_1}y^* > 0$. Thus, $f(0)f(\frac{a_1}{b_1}) < 0$, and then there is a positive root of eq.(10) lies in $(0, \frac{a_1}{b_1})$ provided that condition (11) satisfied.

Further, the second equation of (8) gives

$$z^* = \frac{(h_2 + y^*)}{m_2} \left(\frac{m_1x^*}{h_1 + x^{*2}} - d_1 \right) \quad (12)$$

Therefore, the positive equilibrium point $E_3 = (x^*, y^*, z^*)$ exists if in addition to condition (11) the following conditions hold:

$$m_3 > d_2, \quad (13)$$

$$\frac{m_1x^*}{(h_1 + x^{*2})} > d_1. \quad (14)$$

Now in order to investigate the dynamical behavior of the three species food chain system (2) near the above equilibrium points. The variational matrix V of the system (2) at (x, y, z) is computed as:

$$V(x, y, z) = \begin{bmatrix} x \frac{\partial f_1}{\partial x} + f_1 & x \frac{\partial f_1}{\partial y} & x \frac{\partial f_1}{\partial z} \\ y \frac{\partial f_2}{\partial x} & y \frac{\partial f_2}{\partial y} + f_2 & y \frac{\partial f_2}{\partial z} \\ z \frac{\partial f_3}{\partial x} & z \frac{\partial f_3}{\partial y} & z \frac{\partial f_3}{\partial z} + f_3 \end{bmatrix},$$

where

$$\begin{aligned} \frac{\partial f_1}{\partial x} &= -b_1 + \frac{2m_0xy}{(h_1 + x^2)}, & \frac{\partial f_1}{\partial y} &= -\frac{m_0}{(h_1 + x^2)}, & \frac{\partial f_1}{\partial z} &= 0, \\ \frac{\partial f_2}{\partial x} &= \frac{m_1(h_1 - x^2)}{(h_1 + x^2)^2}, & \frac{\partial f_2}{\partial y} &= \frac{m_2z}{(h_2 + y)^2}, & \frac{\partial f_2}{\partial z} &= -\frac{m_2}{(h_2 + y)}, \\ \frac{\partial f_3}{\partial x} &= 0, & \frac{\partial f_3}{\partial y} &= \frac{h_2m_3}{(h_2+y)^2}, & \frac{\partial f_3}{\partial z} &= 0. \end{aligned}$$

Further, the stability analysis of the system (2) is carried out and according to the variational matrix $V_i; i = 0, 1, 2, 3$ of $E_i; i = 0, 1, 2, 3$ respectively the following results are obtained:

$$\begin{aligned} V_0 &= \begin{bmatrix} a_1 & 0 & 0 \\ 0 & -d_1 & 0 \\ 0 & 0 & -d_2 \end{bmatrix}, \\ V_1 &= \begin{bmatrix} -a_1 & -\frac{m_0x}{h_1+x^2} & 0 \\ 0 & \frac{m_1x}{h_1+x^2} - d_1 & 0 \\ 0 & 0 & -d_2 \end{bmatrix}, \\ V_2 &= \begin{bmatrix} \bar{x} \left(-b_1 + \frac{2\bar{x}(a_1 - b_1\bar{x})}{h_1 + \bar{x}^2} \right) & -\frac{m_0\bar{x}}{(h_1 + \bar{x}^2)} & 0 \\ \frac{m_1\bar{y}(h_1 - \bar{x}^2)}{(h_1 + \bar{x}^2)^2} & \frac{m_1\bar{x}}{h_1 + \bar{x}^2} - d_1 & \frac{-m_2\bar{y}}{h_2 + \bar{y}} \\ 0 & 0 & \frac{m_3\bar{y}}{h_2 + \bar{y}} - d_2 \end{bmatrix}. \end{aligned}$$

- Here the trivial equilibrium point E_0 is saddle-node, having an unstable manifold along x -direction and stable manifold along $x - y$ plane.
- From variational matrix V_1 , it is observed equilibrium point E_1 is a saddle point having stable manifold along x and z -direction if the following condition holds

$$\frac{m_1x}{h_1 + x^2} > d_1, \quad (15)$$

while the equilibrium point E_1 having stable manifold along x , y , and z -direction if the following condition holds

$$\frac{m_1x}{h_1 + x^2} < d_1. \quad (16)$$

- The equilibrium point $E_2 = (\bar{x}, \bar{y}, 0)$ according to the characteristic equation of $V(E_2)$ having stable manifold along x and y -direction if the following conditions hold respectively

$$\frac{2\bar{x}(a_1 - b_1\bar{x})}{h_1 + \bar{x}^2} < b_1, \quad (17)$$

$$\frac{m_1\bar{x}}{h_1 + \bar{x}^2} < d_1. \quad (18)$$

It is easy to verify that the root of this equation satisfy the following relations:

$$\begin{aligned} \lambda_1 + \lambda_2 &= \frac{\bar{x}(m_1 + 2(a_1 - b_1\bar{x})) - (b_1\bar{x} + d_1)(h_1 + \bar{x}^2)}{h_1 + \bar{x}^2}, \\ \lambda_1\lambda_2 &= \frac{2\bar{x}^2(a_1 - b_1\bar{x})(m_1\bar{x} - d_1) + b_1\bar{x}(d_1(h_1 + \bar{x}^2) - m_1\bar{x})}{h_1 + \bar{x}^2}, \\ \lambda_3 &= \frac{m_3\bar{y}}{h_2 + \bar{y}} - d_2. \end{aligned}$$

Notice that $\lambda_1 + \lambda_2 < 0$ and $\lambda_1\lambda_2 > 0$ (*i.e* condition (17) and (18) hold), hence E_2 is locally asymptotically stable in the $IntR_+^2$ of the $x - y$ plane. In addition, since the eigenvalue λ_3 describes the dynamics in the z -direction orthogonal on the $x - y$ plane. Therefore, if $\frac{m_3\bar{y}}{h_2 + \bar{y}} < d_2$ holds, then E_2 is locally asymptotically stable in the $IntR_+^3$. Moreover, if $\frac{m_3\bar{y}}{h_2 + \bar{y}} > d_2$ holds, then E_2 is a saddle point in the $IntR_+^3$ with locally stable manifold in $x - y$ plane and with locally unstable manifold in z -direction.

However, for the positive equilibrium point $E_3 = (x^*, y^*, z^*)$ the variational matrix is:

$$\begin{aligned} V_3 &= \begin{bmatrix} a_1 - 2b_1x^* - \frac{m_0y^*(h_1 - x^{*2})}{R^2} & -\frac{m_0x^*}{R} & 0 \\ \frac{m_1y^*(h_1 - x^{*2})}{R^2} & \frac{m_1x^*}{R} - d_1 - \frac{h_2m_2z^*}{Q^2} & -\frac{m_2y^*}{Q} \\ 0 & \frac{h_2m_3z^*}{Q^2} & 0 \end{bmatrix} \\ &= [a_{ij}]. \end{aligned} \quad (19)$$

Here $R = (h_1 + x^{*2})$ and $Q = (h_2 + y^*)$. The characteristic equation of the variational matrix (19) can written as:

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$$

where

$$A_1 = -(a_{11} + a_{22}), A_2 = a_{11}a_{22} - a_{12}a_{21} - a_{23}a_{32}, A_3 = a_{11}a_{23}a_{32}.$$

According to Routh-Hurwitz criterion, $E_3 = (x^*, y^*, z^*)$ is locally asymptotically stable provided $A_1 > 0$, $A_3 > 0$ and $\Delta = A_1 A_2 - A_3 > 0$.

Now straightforward computations show that, $A_1 > 0$ and $A_3 > 0$ if and only if the following conditions are satisfied:

$$a_1 < 2b_1x + \frac{m_0y^*(h_1 - x^{*2}) - m_1x^*R}{R^2} + \left(d_1 + \frac{h_2m_2z^*}{Q^2}\right), \quad (20)$$

with

$$x^{*2} < h_1, \quad m_1 < \frac{m_0y^*(h_1 - x^{*2})}{x^*R}. \quad (21)$$

In addition to that, since

$$A_1 A_2 - A_3 = (a_{11} + a_{22})(a_{12}a_{21} - a_{11}a_{22}) + a_{22}a_{23}a_{32}.$$

Hence, the necessary condition for $A_1 A_2 - A_3 > 0$ is $(a_{12}a_{21} - a_{11}a_{22}) < 0$ satisfied, provided that the following condition holds

$$\begin{aligned} & (a_1 - 2b_1x^*) \left[\frac{m_1x^*}{R} - \left(a_1 + \frac{h_2m_2z^*}{Q^2} \right) \right] \\ & + \frac{m_0y^*(h_1 - x^{*2})}{R^2} \left(d_1 + \frac{h_2m_2z^*}{Q^2} \right) > 0. \end{aligned} \quad (22)$$

Therefore, depending on the above analysis, the locally asymptotically stable in $IntR_+^3$ of the positive equilibrium point $E_3 = (x^*, y^*, z^*)$ is discussed in the following theorem.

Theorem 2. Suppose that positive equilibrium point $E_3 = (x^*, y^*, z^*)$ exists in $IntR_+^3$, then E_3 is locally asymptotically stable provided conditions (20-22) hold.

Proof. Follows directly from Routh-Hurwitz criterion [17].

5. Numerical Simulation

In this section the global dynamics of system (2) is studied numerically. The food chain system is solved numerically using predictor-corrector method with six order Runge-Kutta method [10]. System (2) run for 40000 time steps and

the first 25000 time steps are deleted to eliminate the transient effect. For the following set of fixed parameters values

$$\begin{aligned} b_1 = 0.3, m_0 = 0.1, m_1 = 0.27, m_2 = 0.29, m_3 = 0.095, \\ d_1 = 0.017, d_2 = 0.025, h_1 = 1.0, h_2 = 2.0 \end{aligned} \quad (23)$$

The attractor of system (2) in 3D and 2D are drawn for the initial condition (1.2, 1.2, 1.2). The main objective is to explore the possibility of chaotic behavior of system (2) by varying the control parameter and keeping other parameters of (23) fixed, and there are three cases to explain the behavior of the model in addition to bifurcation diagrams.

The bifurcation diagram with respect to sensitive parameter a_1 is drawn in Fig.1. The cascades of period doubling is observed in Fig.1 for $a_1 \in (0.25, 0.5)$ and with data in (23) which is the route of chaos in system (2). The complex dynamics of system (2) is easily observed from bifurcation diagrams.

Figure 1: Bifurcation diagram of system (2) for data in (23) the maxima of y as function of a_1 and $a_1 \in (0.25, 0.5)$.

Now the first case by varying the value of a_1 in the range 0.35-0.20, it is observed that the system approaches to chaotic dynamics of system (2) as shown in Fig.2. Decreasing the value of $a_1 = 0.27$ change the behavior of system (2) from chaotic to period doubling as shown in Fig.3(A), while decreasing $a_1 = 0.25$ change the behavior of the system from chaotic to limit cycle as shown in Fig.3(B). Further decreasing the value of $a_1 = 0.20$ change the dynamics of system (2) from chaotic to stable point (0.472, 0.714, 0.816).

The second case about fixing the controlling parameter $a_1 = 0.35$ with same data in (23), and by increasing the value of b_1 about 2-times to $b_1 = 0.6$ change

Figure 2: (A) 3D of system (2) chaotic attractor with data in (23) and $a_1 = 0.35$. (B) 2D $x - y$ plane of Fig.2(A).

Figure 3: (A) 3D of system (2) period doubling attractor with data in (23) and $a_1 = 0.27$. (B) limit cycle attractor with data in (23) and $a_1 = 0.25$.

the behavior of system (2) dynamics from chaotic to limit cycle as shown in fig.4.(A), while increasing the value of b_1 about 4-times for the typical value of $b_1 = 1.2$ change the behavior of system (2) from chaotic to stable point $(0.235, 0.710, 0.402)$ as show in fig.4(B).

The third case about fixing $a_1 = 0.35$ with same data in (23), it is observed by increasing the death rate of the middle predator $d_1 = 0.034$ about 2-times then system (2) approaches to periodic attractor as shown in Fig.5(A) and bifurcation diagram in Fig.6, while increasing the death rate of the top predator about 2-times $d_2 = 0.053$ change the behavior of system (2) from chaotic to

Figure 4: (A) 3D of system (2) periodic with data set (23) and $b_1 = 0.60$. (B) 3D of system (2) stable point with data set (23) and $b_1 = 1.20$.

Figure 5: (A) 2D $y - z$ plane of system (2) period doubling with data in (23) and $d_1 = 0.034$. (B) 2D $y - z$ plane of system (2) stable point with data in (23) and $d_2 = 0.053$.

stable at the point $(0.482, 2.524, 1.392)$ as shown in Fig.5(B).

The bifurcation diagram with respect to a_1 with data in (23) is plotted in Fig.6 for $a_1 \in (0.25, 0.5)$ and $d_1 = 0.034$, the change in the dynamics of system (2) comparing with Fig.1 when $d_1 = 0.017$ is observed.

Figure 6: (A) Bifurcation diagram of system (2) for data in (23) the max. of (y) as function of a_1 and $d_1 = 0.034$ with $a_1 \in (0.25, 0.5)$.

6. Discussion and Results

In order to explain the dynamical behavior of the proposed food chain systems (2), boundedness of the system is discussed. stability analysis of the model are carried out.

In addition to that, to confirm the analytical results the system is solved numerically for different sets of biologically feasible parameter values, and then the attracting sets are drawn in order to explain the dynamical behavior of the model as in Figs.(1-6) with data the same as in (23). According to our study the following results are obtained:

- 1) The bifurcation diagrams indicate the existence of rich dynamics (chaotic, periodic, period doubling, asymptotically stable) for parameter range in a three species food chain model system (2).
- 2) System (2) has a chaotic dynamics as in Fig.2, but if we decreasing the values of intrinsic growth of the prey species a_1 , then system (2) approaches to periodic attractor as in Fig.3, and then to stable point. So decreasing a_1 has a stabilizing effect on the dynamics of system (2).
- 3) Increasing the intraspecific competition change system (2) from chaotic to asymptotically stable, so increasing b_1 has a stabilizing effect on the dynamics of food chain system (2), see Fig.4.
- 4) Increasing the death rate d_1 in the middle predator y about 2-times with

data (23) change the dynamics of system (2) from chaotic to periodic as it was plotted in Fig.5(A). Moreover increasing the death rate d_2 in the top predator z about 2-times change system (2) behavior from chaotic to stable as shown in fig.5(B), so d_2 is more effected than d_1 in the behavior of system (2).

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