1-MOVABLE CLIQUE DOMINATING SETS OF A GRAPH

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Abstract: A clique (convex) dominating set \( S \) of \( G \) is a 1-movable clique dominating set (resp. 1-movable convex dominating set) of \( G \) if for every \( v \in S \), either \( S \setminus \{v\} \) is a clique (resp. convex) dominating set or there exists a vertex \( u \in (V(G) \setminus S) \cap N_G(v) \) such that \( (S \setminus \{v\}) \cup \{u\} \) is a clique (resp. convex) dominating set of \( G \). The minimum cardinality of a 1-movable clique (resp. 1-movable convex) dominating set of \( G \), denoted by \( \gamma_{mcl}^1(G) \) (resp. \( \gamma_{mcon}^1(G) \)), is called the 1-movable clique domination number (resp. 1-movable convex domination number) of \( G \). A 1-movable clique dominating set in \( G \) with cardinality \( \gamma_{mcl}^1(G) \) is called a \( \gamma_{mcl}^1 \)-set of \( G \).

This paper aims to characterize the 1-movable clique dominating sets of some graphs including those resulting from the join and composition of two graphs. The corresponding 1-movable clique domination number of the resulting graph is then determined. Further, it is shown that the concepts of 1-movable clique domination and 1-movable convex domination are equivalent.

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1. Introduction

Let \( G = (V(G), E(G)) \) be a graph with \( n = |V(G)| \) and \( m = |E(G)| \). For any vertex \( v \in V(G) \), we define the open neighborhood of \( v \) as the set \( N_G(v) = \)
consisting of \( u \) and \( v \) list of some variations of the standard domination concept. Have been widely studied. The book by Haynes et al. (see [6]) contains a long list of some variations of the standard domination concept.

Now, given two vertices \( u \) and \( v \) of \( G \), by \( I_G[u, v] \) we mean the closed interval consisting of \( u \), \( v \) and all vertices lying on some \( u - v \) geodesic (shortest path connecting \( u \) and \( v \)) of \( G \). A subset \( C \) of \( V(G) \) is convex if \( I_G(u, v) \subseteq C \) for every pair of vertices \( u, v \in C \). A proper convex subset of \( V(G) \) of largest cardinality is called a maximum convex set in \( G \). The cardinality of a maximum convex set in \( G \) is called the convexity number of \( G \) and is denoted by \( \gamma(G) \). A dominating set \( S \) of \( V(G) \) is a convex dominating set of \( G \) if \( S \) is convex.

A dominating set \( S \) of \( V(G) \) is a clique dominating set of \( G \) if the induced subgraph \( \langle S \rangle \) of \( S \) is complete. A clique (convex) dominating set \( S \) of \( G \) is a 1-movable clique dominating set (resp. 1-movable convex dominating set) of \( G \) if for every \( v \in S \), either \( S \setminus \{v\} \) is a clique (resp. convex) dominating set or there exists a vertex \( u \in (V(G) \setminus S) \cap N_G(v) \) such that \( (S \setminus \{v\}) \cup \{u\} \) is a clique (resp. convex) dominating set of \( G \). The minimum cardinality of a 1-movable clique (resp. 1-movable convex) dominating set of \( G \), denoted by \( \gamma_{mcl}^1(G) \) (resp. \( \gamma_{mcom}^1(G) \)), is called the 1-movable clique domination number (resp. 1-movable convex domination number) of \( G \). A 1-movable clique dominating set in \( G \) with cardinality \( \gamma_{mcl}^1(G) \) is called a \( \gamma_{mcl}^1 \)-set of \( G \).

The concept of clique domination is studied by Cozzens and Kelleher in [4]. Convexity and some of its related concepts are studied and investigated in [2], [3], and [5] while convex domination in graphs are dealt with in [8] and [9]. Blair et al. introduced and studied movable domination in [1]. The concept of 1-movable domination is also studied in [7].

This paper investigates, among others, the concept of 1-movable clique domination in the join and composition of graphs. Recall that the join of two graphs \( G \) and \( H \) is the graph \( G + H \) with vertex set \( V(G + H) = V(G) \cup V(H) \) and edge set \( E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), u \in V(H)\} \). The composition (sometimes called lexicographic product) of \( G \) and \( H \) is the graph \( G[H] \) with \( V(G[H]) = V(G) \times V(H) \) and \( (u, v)(u', v') \in E(G[H]) \) if and only if either
$uu' \in E(G)$ or $u = u'$ and $vv' \in E(H)$.

2. Results

The first result shows that the concepts of 1-movable clique domination and 1-movable convex domination are equivalent.

**Theorem 2.1.** Let $G$ be a connected graph. Then $G$ has a 1-movable convex dominating set if and only if it has a 1-movable clique dominating set. Moreover, $S$ is a 1-movable convex dominating set of $G$ if and only if it is a 1-movable clique dominating set.

**Proof.** Suppose that $S$ is a 1-movable convex dominating set of $G$. Suppose further that $\langle S \rangle$ is not complete. Then there exist vertices $x, y \in S$ such that $d(x, y) = 2$. Let $z \in S \cap N_G(x) \cap G(y)$. Then $S \setminus \{z\}$ and $(S \setminus \{z\}) \cup \{v\}$ are not convex sets for all $v \in (V(G) \setminus S) \cap N_G(z)$. This implies that $S$ is not a 1-movable convex dominating set of $G$, contrary to our assumption. Thus, $S$ is a clique dominating set of $G$. Next, let $u \in S$. Since $S$ is a 1-movable convex dominating set of $G$, either $S \setminus \{v\}$ is a convex dominating set or there exists a vertex $w \in (V(G) \setminus S) \cap N_G(u)$ such that $S_u = (S \setminus \{u\}) \cup \{w\}$ is a convex dominating set of $G$. If $S \setminus \{u\}$ is a convex dominating set, then it $S \setminus \{v\}$ is a clique dominating set since $\langle S \setminus \{u\} \rangle$ is a clique (because $\langle S \rangle$ is a clique). Suppose $(S \setminus \{v\}) \cup \{w\}$ is a convex dominating set of $G$ for some $w \in (V(G) \setminus S) \cap N_G(u)$. Suppose further that there exists $q \in (S \setminus \{u\}) \setminus N_G(w)$. Then $[q, u, w]$ is a $q$-$w$ geodesic. Hence, $(S \setminus \{v\}) \cup \{w\}$ is not a convex set, contrary to our assumption. Therefore $S \setminus \{v\} \cup \{w\}$ is a clique dominating set of $G$. Accordingly, $S$ is a 1-movable clique dominating set of $G$.

The converse follows from the fact that every subset of $V(G)$ that induces a clique is a convex set.

The next result is a direct consequence of Theorem 2.1.

**Corollary 2.2.** Let $G$ be a connected graph. If $G$ has a 1-movable clique dominating set, then $\gamma_{1mcl}(G) = \gamma_{1mcon}(G)$.

**Remark 2.3.** Let $G$ be a connected graph. Then $\gamma_{cl}(G) = 1$ if and only if $\gamma(G) = 1$.

**Theorem 2.4.** Let $G$ be a connected nontrivial graph. Then the following are equivalent:

1. $\gamma_m^1(G) = 1$. 

(ii) $\gamma_{mcl}^1(G) = 1$.

(iii) $G = K_2$ or $G = K_2 + H$ for some graph $H$.

Proof. (i) $\Rightarrow$ (ii). Assume that $\gamma_{m}^1(G) = 1$ and $S$ be a $\gamma_{m}^1$-set of $G$. Since $\langle S \rangle = K_1$, $S$ is a 1-movable clique dominating set of $G$. Thus, $\gamma_{mcl}^1(G) = 1$.

(ii) $\Rightarrow$ (iii). Suppose that $\gamma_{mcl}^1(G) = 1$, say $S = \{x\}$ is a $\gamma_{mcl}^1$-set of $G$. Since $S$ is a 1-movable clique dominating set of $G$, there exists $y \in (V(G) \setminus \{x\}) \cap N_G(x)$ such that $(S \setminus \{x\}) \cup \{y\}$ is a dominating set of $G$. Let $H = \langle V(G) \setminus \{x, y\} \rangle$. Then $G = \langle \{x, y\} \rangle + H \cong K_2 + H$.

(iii) $\Rightarrow$ (i). Suppose first that $G = K_2$. Then $\gamma_{mcl}^1(G) = 1$. This implies that $\gamma_{m}^1(G) = 1$. Suppose that $G \cong K_2 + H$. Let $V(K_2) = \{a, b\}$ and let $S = \{a\}$. Then $S$ is as 1-movable dominating set of $G$. Therefore, $\gamma_{m}^1(G) = 1$. $\square$

Theorem 2.5. Let $G$ be a connected graph of order $n \geq 3$. Then $\gamma_{mcl}^1(G) = 2$ if and only if there exist adjacent vertices $x$ and $y$ of $G$ satisfying the following properties:

(i) $N_G(x) \cup N_G(y) = V(G)$

(ii) $N_G[x] \setminus N_G[y] \neq \emptyset$ or $N_G[y] \setminus N_G[x] \neq \emptyset$

(iii) $\{x\}$ is a dominating set of $G$ or there exists $v \in (V(G) \setminus S) \cap N_G(y)$ such that $N_G[y] \setminus N_G(x) \subseteq N_G(v)$

(iv) $\{y\}$ is a dominating set of $G$ or there exists $w \in (V(G) \setminus S) \cap N_G(x)$ such that $N_G[x] \setminus N_G(y) \subseteq N_G(w)$.

Proof. Let $G$ be a connected graph of order $n \geq 3$ such that $\gamma_{mcl}^1(G) = 2$. Let $S = \{x, y\}$ be a $\gamma_{mcl}^1$-set of $G$. Then $x$ and $y$ are adjacent vertices. Clearly, $N_G(x) \cup N_G(y) \subseteq V(G)$. Let $v \in V(G)$. Since $S$ is a 1-movable clique dominating set of $G$, either $vx \in E(G)$ or $vy \in E(G)$. This means that either $v \in N_G(x)$ or $v \in N_G(y)$, that is, $v \in N_G(x) \cup N_G(y)$. It follows that $V(G) \subseteq N_G(x) \cup N_G(y)$, and (i) is proved. Next, suppose that $N_G[x] \setminus N_G[y] = \emptyset$ and $N_G[y] \setminus N_G[x] = \emptyset$. This implies that $N_G[x] = N_G[y]$ and so there is some graph $H$ such that $G \cong K_2 + H$. By Theorem 2.4, $\gamma_{mcl}^1(G) = 1$, a contradiction to the hypothesis. Therefore, either $N_G[x] \setminus N_G[y] \neq \emptyset$ or $N_G[y] \setminus N_G[x] \neq \emptyset$. This proves (ii). Since $S$ is a 1-movable clique dominating set of $G$, either $S \setminus \{y\} = \{x\}$ is a dominating set of $G$ or there exists $v \in (V(G) \setminus S) \cap N_G(y)$ such that $(S \setminus \{y\}) \cup \{v\} = \{x, v\}$ is a clique dominating set of $G$. Let $a \in N_G[y] \setminus N_G(x)$. Then $a \in N_G[y]$ but $a \notin N_G(x)$. Thus, $a$ is a neighbor of $v$, that is, $a \in N_G(v)$. Hence, $N_G[y] \setminus N_G(x) \subseteq N_G(v)$. This proves (iii). Similarly, (iv) holds.
For the converse, suppose that there exist $x, y \in V(G)$ such that $x$ and $y$ are adjacent and satisfy conditions (i) to (iv). We claim that $S = \{x, y\}$ is a 1-movable clique dominating set of $G$. Clearly, $\langle S \rangle = K_2$, and so is complete. Let $a \in V(G) \setminus S$. Then by (i), $a \in (N_G(x) \cup N_G(y)) \setminus \{x, y\}$. This means that $a \neq x, a \neq y$ and $ax \in E(G)$ or $ay \in E(G)$. Since $a$ is arbitrary, $S$ is a dominating set of $G$. Consequently, $S$ is a clique dominating set of $G$. Now consider $\{x\}$. If $\{x\}$ is a dominating set of $G$, then $\gamma(G) = 1$. By Theorem 2.4, $\gamma_{cl}(G) = 1$, that is, $\{x\}$ is a clique dominating set of $G$. Suppose not. Then there exists $v \in (V(G) \setminus \{x, y\}) \cap N_G(y)$ such that $N_G[y] \setminus N_G(x) \subseteq N_G(v)$. Note that $x \in N_G[y] \setminus N_G(x) \subseteq N_G(v)$. This shows that $x$ is a neighbor of $v$. It follows that $\langle \{x, v\} \rangle = K_2$. Now, let $p \in V(G) \setminus \{x, y\}$. Then $p \in (N_G(x) \cup N_G(y)) \setminus \{x, y\}$ implying that $p \in N_G(x)$ or $p \in N_G(y)$. If $p \in N_G(x)$, then we are done. Suppose that $p \notin N_G(x)$. Then $p \in N_G(y)$. Thus, $p \in N_G(y) \setminus N_G(x) \subseteq N_G[y] \setminus N_G(x) \subseteq N_G(v)$. Hence, $pv \in E(G)$. Consequently, $\{x, v\}$ is a clique dominating set of $G$. Similarly, if we consider $y$, then either $\{y\}$ is a clique dominating set of $G$ or there exists $w \in (V(G) \setminus S) \cap N_G(x)$ such that $\{y, w\}$ is a clique dominating set of $G$. This shows that $S$ is a 1-movable clique dominating set of $G$. Suppose $\gamma_mcl(G) = 1$. Then either $G \cong K_2$ or $G \cong K_2 + H$ for some graph $H$. Since $n \geq 3$ and $\langle S \rangle = K_2$, $G = S + H$. Thus, $N_G[x] = N_G[y]$. Hence, $N_G[x] \setminus N_G[y] = \emptyset$ and $N_G[y] \setminus N_G[x] = \emptyset$, a contradiction. Since $S$ is a 1-movable clique dominating set of $G$ and $|S| = 2$, $\gamma_mcl(G) = 2$. 

**Theorem 2.6.** Let $G$ and $H$ be any two nonempty graphs. A subset $S$ of $V(G + H)$ is a 1-movable clique dominating set of $G + H$ if and only if one of the following statements holds:

(i) $S$ is a clique dominating set of $G$ such that if $|S| = 1$, then either $S$ is a 1-movable dominating set of $G$ or there exists $b \in V(H)$ such that $\{b\}$ is a dominating set of $H$.

(ii) $S$ is a clique dominating set of $H$ such that if $|S| = 1$, then either $S$ is a 1-movable dominating set of $H$ or there exists $a \in V(G)$ such that $\{a\}$ is a dominating set of $G$.

(iii) $S = \{a, b\}$, where one of the following is satisfied:

1. $a$ and $b$ are not isolated vertices of $G$ and $H$, respectively.
2. $\{a\}$ is a dominating set of $G$.
3. $\{a, v\}$ is a clique dominating set of $G$ for some $v \in V(G)$.
4. $\{b\}$ is a dominating set of $H$. 


Similarly, (iii) holds. Similarly, (iv) or (v) of (G) holds if S is an isolated vertex of S. Hence, S is a clique dominating set of G for some w ∈ V(H) \ E.

(v) S = D ∪ {b}, where |D| ≥ 2, b is not an isolated vertex of H and ⟨D⟩ is a clique in G, or D is a clique dominating set of G, or D ∪ {w} is a clique dominating set of G for some w ∈ V(G) \ D.

(vi) |S_1| ≥ 2 and |S_2| ≥ 2, where ⟨S_1⟩ and ⟨S_2⟩ are cliques in G and H, respectively.

Proof. Suppose S is a 1-movable clique dominating set of G + H. Consider the following cases:

Case 1. S ∩ V(H) = ∅ or S ∩ V(G) = ∅
Suppose that S ∩ V(H) = ∅. Then S ⊆ V(G) and is a clique dominating set of G. Suppose that |S| = 1, say S = {x} for some x ∈ V(G). Since S is a 1-movable clique dominating set of G + H, there exists b ∈ V(G + H) such that (S \ {x}) ∪ {b} = {b} is a clique dominating set of G + H. If b ∈ V(G), then {b} is a dominating set of G. Hence, S is a 1-movable clique dominating set of G. If b ∈ V(H), then {b} is a dominating set of H. Thus, (i) holds. Similarly, (ii) holds if S ∩ V(G) = ∅.

Case 2. S_1 = S ∩ V(G) ≠ ∅ and S_2 = S ∩ V(H) ≠ ∅
Consider the following subcases.
Subcase 1. |S_1| = |S_2| = 1
Since S is a 1-movable clique dominating set of G + H, a and b cannot both be isolated vertices. If a and b are both non-isolated vertices, then (1) of (iii) holds. Suppose that a or b is an isolated vertex, say b is an isolated vertex of H. Since S is a 1-movable clique dominating set of G + H, S \ {b} = {a} is a dominating set of G + H or there exists v ∈ (V(G + H) \ S) ∩ N_{G+H}(b) such that (S \ {b}) ∪ {v} = {a, v} is a clique dominating set of G + H. Since b is an isolated vertex, {a, v} is a clique dominating set of G. Hence, (2) or (3) of (iii) holds. Similarly, (4) or (5) of (iii) holds if a is an isolated vertex.

Subcase 2. |S_1| = 1 and |S_2| ≥ 2 or |S_1| ≥ 2 and |S_2| = 1
Let S_1 = {a}. Since ⟨S⟩ is a clique, ⟨S_2⟩ is a clique in H. If a is not an isolated vertex of G, then we are done. Suppose a is an isolated vertex of G. Since S is a 1-movable clique dominating set of G + H, either S \ {a} = S_2 is a clique dominating set of H or there exists v ∈ V(H) \ S_2 such that (S \ {a}) ∪ {v} = S_2 ∪ {v} is a clique dominating set of H. Hence, (iv) holds. Similarly, (v) holds if |S_1| ≥ 2 and |S_2| = 1.
Subcase 3. $|S_1| \geq 2$ and $|S_2| \geq 2$

Since $\langle S \rangle$ is a clique and $S = S_1 \cup S_2$, it follows that $\langle S_1 \rangle$ and $\langle S_2 \rangle$ are cliques in $G$ and $H$, respectively.

The converse is easy. \hfill \Box

The next result is an immediate consequence of Theorem 2.6.

**Corollary 2.7.** Let $G$ and $H$ be nonempty graphs. Then $1 \leq \gamma_{mcl}^1(G + H) \leq 2$. Moreover, $\gamma_{mcl}^1(G + H) = 1$ if and only if one of the following statements holds:

1. $\gamma_{mcl}^1(G) = 1$
2. $\gamma(G) = 1$ and $\gamma(H) = 1$
3. $\gamma_{mcl}^1(H) = 1$

We now characterize the 1-movable clique dominating sets in the composition of graphs.

**Theorem 2.8.** Let $G$ and $H$ be connected nontrivial graphs such that $G$ has a clique dominating set. A subset $C = \bigcup_{x \in S} \{x\} \times T_x$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a 1-movable clique dominating set of $G[H]$ if and only if $S$ is a clique dominating set of $G$ such that

(i) $\langle T_x \rangle$ is a clique in $H$ for each $x \in S$

(ii) $T_x$ is a clique dominating set of $H$ whenever $S = \{x\}$ and $|T_x| \geq 2$.

(iii) $T_x$ is a 1-movable clique dominating set of $H$ or $T_x$ is a clique dominating set of $H$ and $S$ is a 1-movable clique dominating set of $G$ whenever $S = \{x\}$ and $|T_x| = 1$.

**Proof.** Suppose that $C$ is a 1-movable clique dominating set of $G[H]$. Then $C$ is a clique dominating set of $G[H]$. Hence, $S$ is a clique dominating set of $G$. Let $x \in S$ and let $a, b \in T_x$, where $a \neq b$. Since $\langle C \rangle$ is a clique in $G[H]$, $(x, a)(x, b) \in E(G[H])$. It follows that $ab \in E(H)$, showing that $\langle T_x \rangle$ is a clique in $H$. Next, suppose that $S = \{x\}$. Let $c \in V(H) \setminus T_x$. Since $C$ is a dominating set and $(x, c) \notin C$, it follows that there exists $(x, d) \in C \cap N_{G[H]}((x, c))$. This implies that $d \in T_x \cap N_H(c)$. Therefore, $T_x$ is a (clique) dominating set of $H$. Hence, (ii) holds if $|T_x| \geq 2$. Suppose $|T_x| = 1$. Let $a \in T_x$. Then $\{(x, a)\} = C$. Since $C$ is a 1-movable clique dominating set of $G[H]$, there exists $(y, b) \in (V(G[H]) \setminus C) \cap N_{G[H]}((x, a))$
such that \((C \setminus \{(x, a)\}) \cup \{(y, b)\}\) is a clique dominating set of \(G[H]\). It follows that \(T_x\) is a clique dominating set of \(H\). If \(x = y\), then \(b \in (V(H) \setminus T_x) \cap N_H(a)\). This means that \((T_x \setminus \{a\}) \cup \{b\}\) is a clique dominating set of \(H\). Thus, \(T_x\) is a 1-movable clique dominating set of \(H\). If \(x \neq y\), then \(y \in (V(G) \setminus S) \cap N_G(x)\). Hence, \((S \setminus \{x\}) \cup \{y\}\) is a clique dominating set of \(G\). Thus, \(S\) is a 1-movable clique dominating set of \(G\) showing that \((iii)\) holds.

For the converse, suppose that \(S\) is a clique dominating set of \(G\) and \((i)\), \((ii)\) and \((iii)\) hold. Then, clearly, \(C = \bigcup_{x \in S} \{(x) \times T_x\}\) induces a clique in \(G[H]\). Let \((z, d) \notin C\). If \(z \notin S\), then there exists \(w \in S\) such that \(wz \in E(G)\). Choose any \(q \in T_w\). Then \((w, q) \in C \cap N_{G[H]}((z, d))\). Suppose \(z \in S\). If \(|S| \geq 2\), then there exists \(y \in S \cap N_G(z)\). Pick any \(p \in T_y\). Then \((y, p) \in C \cap N_{G[H]}((z, d))\). Suppose \(S = \{z\}\). Then, by assumption, \(T_z\) is a dominating set in \(H\). Hence, there exists \(t \in T_z \cap N_H(d)\). This implies that \((z, t) \in C \cap N_{G[H]}((z, d))\). Therefore, \(C\) is a (clique) dominating set of \(G[H]\).

Let \((x, a) \in C\). Consider the following cases:

Case 1. \(S = \{x\}\)

If \(|T_x| = 1\), that is, \(T_x = \{a\}\), then \(T_x\) is a 1-movable clique dominating set of \(H\) or \(T_x\) is a clique dominating set of \(H\) and \(S\) is a 1-movable clique dominating set of \(G\). Hence, \(C\) is a 1-movable clique dominating set of \(G[H]\). Suppose \(|T_x| \geq 2\). Then there exists \(y \in (V(G) \setminus \{x\}) \cap N_G(x)\). Thus, \((C \setminus \{(x, a)\}) \cup \{(y, b)\}\) is a clique dominating set of \(G[H]\). Hence, \(C\) is a 1-movable clique dominating set of \(G[H]\).

Case 2. \(|S| \geq 2\)

If \(T_x = \{a\}\), then there exists \(b \in (V(H) \setminus T_x) \cap N_H(a)\). This means that \((C \setminus \{(x, a)\}) \cup \{(x, b)\}\) is a clique dominating set of \(G[H]\). If \(|T_x| \geq 2\), then \(C \setminus \{(x, a)\}\) is a clique dominating set of \(G[H]\). Therefore, \(C\) is a 1-movable clique dominating set of \(G[H]\). \(\square\)

**Corollary 2.9.** Let \(G\) and \(H\) be connected nontrivial graphs such that \(G\) has a clique dominating set. Then

\[
\gamma^1_{mcl}(G[H]) = \begin{cases} 
1, & \text{if } \gamma(G) = 1 \text{ and } \gamma^1_{mcl}(H) = 1 \\
 & \text{or } \gamma_{mcl}(G) = 1 \text{ and } \gamma(H) = 1 \\
2, & \text{if } \gamma(G) = 1 \text{ and } \gamma(H) \neq 1 \\
\gamma_{cl}(G), & \text{if } \gamma(G) \neq 1.
\end{cases}
\]
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