International Journal of Pure and Applied Mathematics
Volume 106 No. 2 2016, 463-471
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i2.10

ijpam.eu

1-MOVABLE CLIQUE DOMINATING SETS OF A GRAPH

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Abstract: A clique (convex) dominating set S of G is a 1-movable clique dominating set (resp. 1-movable convex dominating set) of G if for every $v \in S$, either $S \setminus \{v\}$ is a clique (resp. convex) dominating set or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a clique (resp. convex) dominating set of G. The minimum cardinality of a 1-movable clique (resp. 1-movable convex) dominating set of G, denoted by $\gamma^1_{mcl}(G)$ (resp. $\gamma^1_{mcon}(G)$), is called the 1-movable clique domination number (resp. 1-movable convex domination number) of G. A 1-movable clique dominating set in G with cardinality $\gamma^1_{mcl}(G)$ is called a γ^1_{mcl}-set of G.

This paper aims to characterize the 1-movable clique dominating sets of some graphs including those resulting from the join and composition of two graphs. The corresponding 1-movable clique domination number of the resulting graph is then determined. Further, it is shown that the concepts of 1-movable clique domination and 1-movable convex domination are equivalent.

AMS Subject Classification: 05C69
Key Words: clique domination, convex domination, 1-movable clique domination, 1-movable convex domination

1. Introduction

Let $G = (V(G), E(G))$ be a graph with $n = |V(G)|$ and $m = |E(G)|$. For any vertex $v \in V(G)$, we define the *open neighborhood* of v as the set $N_G(v) =$

Received: September 2, 2015
Published: February 15, 2016

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$\{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v as the set $N_G[v] = N_G(v) \cup \{v\}$. If S is a nonempty subset of X, then $N_G(S) = \bigcup_{v \in S} N_G(v)$ and $N_G[S] = N_G(S) \cup S$. A nonempty subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G) \setminus S$, there exists $u \in S$ such that $uv \in E(G)$, that is $N_G[S] = V(G)$. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality among all dominating sets of G. A dominating set S of G with $|S| = \gamma(G)$ is called a γ-set of G. Domination in graph as well as its variants and the numerous applications of these related concepts in networks have been widely studied. The book by Haynes et al. (see [6]) contains a long list of some variations of the standard domination concept.

Now, given two vertices u and v of G, by $I_G[u, v]$ we mean the closed interval consisting of u, v and all vertices lying on some $u - v$ geodesic (shortest path connecting u and v) of G. A subset C of $V(G)$ is convex if $I_G(u, v) \subseteq C$ for every pair of vertices $u, v \in C$. A proper convex subset of $V(G)$ of largest cardinality is called a *maximum convex* set in G. The cardinality of a maximum convex set in G is called the *convexity number* of G and is denoted by $con(G)$. A dominating set S of $V(G)$ is a *convex dominating set* of G if S is convex.

A dominating set S of $V(G)$ is a *clique dominating set* of G if the induced subgraph $\langle S \rangle$ of S is complete. A clique (convex) dominating set S of G is a 1-*movable clique dominating set* (resp. 1-*movable convex dominating set*) of G if for every $v \in S$, either $S \setminus \{v\}$ is a clique (resp. convex) dominating set or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a clique (resp. convex) dominating set of G. The minimum cardinality of a 1-movable clique (resp. 1-movable convex) dominating set of G, denoted by $\gamma^1_{mcl}(G)$ (resp. $\gamma^1_{mcon}(G)$), is called the 1-*movable clique domination number* (resp. 1-*movable convex domination number*) of G. A 1-movable clique dominating set in G with cardinality $\gamma^1_{mcl}(G)$ is called a γ^1_{mcl}-set of G.

The concept of clique domination is studied by Cozzens and Kelleher in [4]. Convexity and some of its related concepts are studied and investigated in [2], [3], and [5] while convex domination in graphs are dealt with in [8] and [9]. Blair et al. introduced and studied movable domination in [1]. The concept of 1-movable domination is also studied in [7].

This paper investigates, among others, the concept of 1-movable clique domination in the join and composition of graphs. Recall that the *join* of two graphs G and H is the graph $G + H$ with vertex set $V(G + H) = V(G) \cup V(H)$ and edge set $E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), u \in V(H)\}$. The *composition* (sometimes called *lexicographic product*) of G and H is the graph $G[H]$ with $V(G[H]) = V(G) \times V(H)$ and $(u, v)(u', v') \in E(G[H])$ if and only if either

$uu' \in E(G)$ or $u = u'$ and $vv' \in E(H)$.

2. Results

The first result shows that the concepts of 1-movable clique domination and 1-movable convex domination are equivalent.

Theorem 2.1. *Let G be a connected graph. Then G has a 1-movable convex dominating set if and only if it has a 1-movable clique dominating set. Moreover, S is a 1-movable convex dominating set of G if and only if it is a 1-movable clique dominating set.*

Proof. Suppose that S is a 1-movable convex dominating set of G. Suppose further that $\langle S \rangle$ is not complete. Then there exist vertices $x, y \in S$ such that $d(x, y) = 2$. Let $z \in S \cap N_G(x) \cap_G (y)$. Then $S \setminus \{z\}$ and $(S \setminus \{z\}) \cup \{v\}$ are not convex sets for all $v \in (V(G) \setminus S) \cap N_G(z)$. This implies that S is not a 1-movable convex dominating set of G, contrary to our assumption. Thus, S is a clique dominating set of G. Next, let $u \in S$. Since S is a 1-movable convex dominating set of G, either $S \smallsetminus \{v\}$ is a convex dominating set or there exists a vertex $w \in (V(G) \smallsetminus S) \cap N_G(u)$ such that $S_u = (S \smallsetminus \{u\}) \cup \{w\}$ is a convex dominating set of G. If $S \smallsetminus \{u\}$ is a convex dominating set, then it $S \smallsetminus \{v\}$ is a clique dominating set since $\langle S \setminus \{u\} \rangle$ is a clique (because $\langle S \rangle$ is a clique). Suppose $(S \smallsetminus \{v\}) \cup \{w\}$ is a convex dominating set of G for some $w \in (V(G) \smallsetminus S) \cap N_G(u)$. Suppose further that there exists $q \in (S \setminus \{u\}) \setminus N_G(w)$. Then $[q, u, w]$ is a q-w geodesic. Hence, $(S \smallsetminus \{v\}) \cup \{w\}$ is not a convex set, contrary to our assumption. Therefore $S \smallsetminus \{v\}) \cup \{w\}$ is a clique dominating set of G. Accordingly, S is a 1-movable clique dominating set of G.

The converse follows from the fact that every subset of $V(G)$ that induces a clique is a convex set. □

The next result is a direct consequence of Theorem 2.1.

Corollary 2.2. *Let G be a connected graph. If G has a 1-movable clique dominating set, then $\gamma^1_{mcl}(G) = \gamma^1_{mcon}(G)$.*

Remark 2.3. Let G be a connected graph. Then $\gamma_{cl}(G) = 1$ if and only if $\gamma(G) = 1$.

Theorem 2.4. *Let G be a connected nontrivial graph. Then the following are equivalent:*

(*i*) $\gamma^1_m(G) = 1$.

(*ii*) $\gamma^1_{mcl}(G) = 1$.

(*iii*) $G = K_2$ *or* $G = K_2 + H$ *for some graph* H.

Proof. $(i) \Longrightarrow (ii)$. Assume that $\gamma^1_m(G) = 1$ and S be a γ^1_m-set of G. Since $\langle S\rangle = K_1$, S is a 1-movable clique dominating set of G. Thus, $\gamma^1_{mcl}(G) = 1$.

$(ii) \Longrightarrow (iii)$. Suppose that $\gamma^1_{mcl}(G) = 1$, say $S = \{x\}$ is a γ^1_{mcl}-set of G. Since S is a 1-movable clique dominating set of G, there exists $y \in (V(G) \setminus \{x\}) \cap N_G(x)$ such that $(S \setminus \{x\}) \cup \{y\}$ is a dominating set of G. Let $H = \langle V(G) \setminus \{x, y\}\rangle$. Then $G = \langle\{x, y\}\rangle + H \cong K_2 + H$.

$(iii) \Longrightarrow (i)$. Suppose first that $G = K_2$. Then $\gamma^1_{mcl}(G) = 1$. This implies that $\gamma^1_m(G) = 1$. Suppose that $G \cong K_2 + H$. Let $V(K_2) = \{a, b\}$ and let $S = \{a\}$. Then S is as 1-movable dominating set of G. Therefore, $\gamma^1_m(G) = 1$. □

Theorem 2.5. *Let G be a connected graph of order $n \geq 3$. Then $\gamma^1_{mcl}(G) = 2$ if and only if there exist adjacent vertices x and y of G satisfying the following properties:*

(*i*) $N_G(x) \cup N_G(y) = V(G)$

(*ii*) $N_G[x] \setminus N_G[y] \neq \varnothing$ or $N_G[y] \setminus N_G[x] \neq \varnothing$

(*iii*) $\{x\}$ *is a dominating set of* G *or there exists* $v \in (V(G) \setminus S) \cap N_G(y)$ *such that* $N_G[y] \setminus N_G(x) \subseteq N_G(v)$

(*iv*) $\{y\}$ *is a dominating set of* G *or there exists* $w \in (V(G) \setminus S) \cap N_G(x)$ *such that* $N_G[x] \setminus N_G(y) \subseteq N_G(w)$.

Proof. Let G be a connected graph of order $n \geq 3$ such that $\gamma^1_{mcl}(G) = 2$. Let $S = \{x, y\}$ be a γ^1_{mcl}-set of G. Then x and y are adjacent vertices. Clearly, $N_G(x) \cup N_G(y) \subseteq V(G)$. Let $v \in V(G)$. Since S is a 1-movable clique dominating set of G, either $vx \in E(G)$ or $vy \in E(G)$. This means that either $v \in N_G(x)$ or $v \in N_G(y)$, that is, $v \in N_G(x) \cup N_G(y)$. It follows that $V(G) \subseteq N_G(x) \cup N_G(y)$, and (i) is proved. Next, suppose that $N_G[x] \setminus N_G[y] = \varnothing$ and $N_G[y] \setminus N_G[x] = \varnothing$. This implies that $N_G[x] = N_G[y]$ and so there is some graph H such that $G \cong K_2 + H$. By Theorem 2.4, $\gamma^1_{mcl}(G) = 1$, a contradiction to the hypothesis. Therefore, either $N_G[x] \setminus N_G[y] \neq \varnothing$ or $N_G[y] \setminus N_G[x] \neq \varnothing$. This proves (ii). Since S is a 1-movable clique dominating set of G, either $S \setminus \{y\} = \{x\}$ is a dominating set of G or there exists $v \in (V(G) \setminus S) \cap N_G(y)$ such that $(S \setminus \{y\}) \cup \{v\} = \{x, v\}$ is a clique dominating set of G. Let $a \in N_G[y] \setminus N_G(x)$. Then $a \in N_G[y]$ but $a \notin N_G(x)$. Thus, a is a neighbor of v, that is, $a \in N_G(v)$. Hence, $N_G[y] \setminus N_G(x) \subseteq N_G(v)$. This proves (iii). Similarly, (iv) holds.

For the converse, suppose that there exist $x, y \in V(G)$ such that x and y are adjacent and satisfy conditions (i) to (iv). We claim that $S = \{x, y\}$ is a 1-movable clique dominating set of G. Clearly, $\langle S \rangle = K_2$, and so is complete. Let $a \in V(G) \setminus S$. Then by (i), $a \in (N_G(x) \cup N_G(y)) \setminus \{x, y\}$. This means that $a \neq x$, $a \neq y$ and $ax \in E(G)$ or $ay \in E(G)$. Since a is arbitrary, S is a dominating set of G. Consequently, S is a clique dominating set of G. Now consider $\{x\}$. If $\{x\}$ is a dominating set of G, then $\gamma(G) = 1$. By Theorem 2.4, $\gamma_{cl}(G) = 1$, that is, $\{x\}$ is a clique dominating set of G. Suppose not. Then there exists $v \in (V(G) \setminus \{x, y\}) \cap N_G(y)$ such that $N_G[y] \setminus N_G(x) \subseteq N_G(v)$. Note that $x \in N_G[y] \setminus N_G(x) \subseteq N_G(v)$. This shows that x is a neighbor of v. It follows that $\langle \{x, v\} \rangle = K_2$. Now, let $p \in V(G) \setminus \{x, y\}$. Then $p \in (N_G(x) \cup N_G(y)) \setminus \{x, y\}$ implying that $p \in N_G(x)$ or $p \in N_G(y)$. If $p \in N_G(x)$, then we are done. Suppose that $p \notin N_G(x)$. Then $p \in N_G(y)$. Thus, $p \in N_G(y) \setminus N_G(x) \subseteq N_G[y] \setminus N_G(x) \subseteq N_G(v)$. Hence, $pv \in E(G)$. Consequently, $\{x, v\}$ is a clique dominating set of G. Similarly, if we consider y, then either $\{y\}$ is a clique dominating set of G or there exists $w \in (V(G) \setminus S) \cap N_G(x)$ such that $\{y, w\}$ is a clique dominating set of G. This shows that S is a 1-movable clique dominating set of G. Suppose $\gamma^1_{mcl}(G) = 1$. Then either $G \cong K_2$ or $G \cong K_2 + H$ for some graph H. Since $n \geq 3$ and $\langle S \rangle = K_2$, $G = S + H$. Thus, $N_G[x] = N_G[y]$. Hence, $N_G[x] \setminus N_G[y] = \varnothing$ and $N_G[y] \setminus N_G[x] = \varnothing$, a contradiction. Since S is a 1-movable clique dominating set of G and $|S| = 2$, $\gamma^1_{mcl}(G) = 2$. □

Theorem 2.6. *Let G and H be any two nonempty graphs. A subset S of $V(G + H)$ is a 1-movable clique dominating set of $G + H$ if and only if one of the following statements holds:*

(i) *S is a clique dominating set of G such that if $|S| = 1$, then either S is a 1-movable dominating set of G or there exists $b \in V(H)$ such that $\{b\}$ is a dominating set of H.*

(ii) *S is a clique dominating set of H such that if $|S| = 1$, then either S is a 1-movable dominating set of H or there exists $a \in V(G)$ such that $\{a\}$ is a dominating set of G.*

(iii) *$S = \{a, b\}$, where one of the following is satisfied:*

(1) *a and b are not isolated vertices of G and H, respectively.*

(2) *$\{a\}$ is a dominating set of G.*

(3) *$\{a, v\}$ is a clique dominating set of G for some $v \in V(G)$.*

(4) *$\{b\}$ is a dominating set of H.*

(5) $\{b, w\}$ *is a clique dominating set of* H *for some* $w \in V(H)$.

(*iv*) $S = \{a\} \cup E$, *where* $|E| \geq 2$, a *is not an isolated vertex of* G *and* $\langle E \rangle$ *is a clique in* H, *or* E *is a clique dominating set of* H, *or* $E \cup \{v\}$ *is a clique dominating set of* H *for some* $v \in V(H) \setminus E$.

(*v*) $S = D \cup \{b\}$, *where* $|D| \geq 2$, b *is not an isolated vertex of* H *and* $\langle D \rangle$ *is a clique in* G, *or* D *is a clique dominating set of* G, *or* $D \cup \{w\}$ *is a clique dominating set of* G *for some* $w \in V(G) \setminus D$.

(*vi*) $|S_1| \geq 2$ *and* $|S_2| \geq 2$, *where* $\langle S_1 \rangle$ *and* $\langle S_2 \rangle$ *are cliques in* G *and* H, *respectively.*

Proof. Suppose S is a 1-movable clique dominating set of $G+H$. Consider the following cases:

Case 1. $S \cap V(H) = \varnothing$ or $S \cap V(G) = \varnothing$

Suppose that $S \cap V(H) = \varnothing$. Then $S \subseteq V(G)$ and is a clique dominating set of G. Suppose that $|S| = 1$, say $S = \{x\}$ for some $x \in V(G)$. Since S is a 1-movable clique dominating set of $G+H$, there exists $b \in V(G+H)$ such that $(S \setminus \{x\}) \cup \{b\} = \{b\}$ is a clique dominating set of $G+H$. If $b \in V(G)$, then $\{b\}$ is a dominating set of G. Hence, S is a 1-movable clique dominating set of G. If $b \in V(H)$, then $\{b\}$ is a dominating set of H. Thus, (*i*) holds. Similarly, (*ii*) holds if $S \cap V(G) = \varnothing$.

Case 2. $S_1 = S \cap V(G) \neq \varnothing$ and $S_2 = S \cap V(H) \neq \varnothing$

Consider the following subcases.

Subcase 1. $|S_1| = |S_2| = 1$

Since S is a 1-movable clique dominating set of $G+H$, a and b cannot be both isolated vertices. If a and b are both non-isolated vertices, then (1) of (*iii*) holds. Suppose that a or b is an isolated vertex, say b is an isolated vertex of H. Since S is a 1-movable clique dominating set of $G+H$, $S \setminus \{b\} = \{a\}$ is a dominating set of $G+H$ or there exists $v \in (V(G+H) \setminus S) \cap N_{G+H}(b)$ such that $(S \setminus \{b\}) \cup \{v\} = \{a, v\}$ is a clique dominating set of $G+H$. Since b is an isolated vertex, $\{a, v\}$ is a clique dominating set of G. Hence, (2) or (3) of (*iii*) holds. Similarly, (4) or (5) of (*iii*) holds if a is an isolated vertex.

Subcase 2. $|S_1| = 1$ and $|S_2| \geq 2$ or $|S_1| \geq 2$ and $|S_2| = 1$

Let $S_1 = \{a\}$. Since $\langle S \rangle$ is a clique, $\langle S_2 \rangle$ is a clique in H. If a is not an isolated vertex of G, then we are done. Suppose a is an isolated vertex of G. Since S is a 1-movable clique dominating set $G+H$, either $S \setminus \{a\} = S_2$ is a clique dominating set of H or there exists $v \in V(H) \setminus S_2$ such that $(S \setminus \{a\}) \cup \{v\} = S_2 \cup \{v\}$ is a clique dominating set of H. Hence, (*iv*) holds. Similarly, (*v*) holds if $|S_1| \geq 2$ and $|S_2| = 1$.

Subcase 3. $|S_1| \geq 2$ and $|S_2| \geq 2$

Since $\langle S \rangle$ is a clique and $S = S_1 \cup S_2$, it follows that $\langle S_1 \rangle$ and $\langle S_2 \rangle$ are cliques in G and H, respectively.

The converse is easy. □

The next result is an immediate consequence of Theorem 2.6.

Corollary 2.7. *Let G and H be nonempty graphs. Then $1 \leq \gamma^1_{mcl}(G + H) \leq 2$. Moreover, $\gamma^1_{mcl}(G+H) = 1$ if and only if one of the following statements holds:*

(1) $\gamma^1_{mcl}(G) = 1$

(2) $\gamma(G) = 1$ *and* $\gamma(H) = 1$

(3) $\gamma^1_{mcl}(H) = 1$

We now characterize the 1-movable clique dominating sets in the composition of graphs.

Theorem 2.8. *Let G and H be connected nontrivial graphs such that G has a clique dominating set. A subset $C = \bigcup_{x \in S} [\{x\} \times T_x]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a 1-movable clique dominating set of $G[H]$ if and only if S is a clique dominating set of G such that*

(i) $\langle T_x \rangle$ *is a clique in* H *for each* $x \in S$

(ii) T_x *is a clique dominating set of* H *whenever* $S = \{x\}$ *and* $|T_x| \geq 2$.

(iii) T_x *is a 1-movable clique dominating set of* H *or* T_x *is a clique dominating set of* H *and* S *is a 1-movable clique dominating set of* G *whenever* $S = \{x\}$ *and* $|T_x| = 1$.

Proof. Suppose that C is a 1-movable clique dominating set of $G[H]$. Then C is a clique dominating set of $G[H]$. Hence, S is a clique dominating set of G. Let $x \in S$ and let $a, b \in T_x$, where $a \neq b$. Since $\langle C \rangle$ is a clique in $G[H]$, $(x, a)(x, b) \in E(G[H])$. It follows that $ab \in E(H)$, showing that $\langle T_x \rangle$ is a clique in H. Next, suppose that $S = \{x\}$. Let $c \in V(H) \setminus T_x$. Since C is a dominating set and $(x, c) \notin C$, it follows that there exists $(x, d) \in C \cap N_{G[H]}((x, c))$. This implies that $d \in T_x \cap N_H(c)$. Therefore, T_x is a (clique) dominating set of H. Hence, (ii) holds if $|T_x| \geq 2$. Suppose $|T_x| = 1$. Let $a \in T_x$. Then $\{(x, a)\} = C$. Since C is a 1-movable clique dominating set of $G[H]$, there exists

$$(y, b) \in (V(G[H]) \setminus C) \cap N_{G[H]}((x, a))$$

such that $(C \setminus \{(x,a)\}) \cup \{(y,b)\}$ is a clique dominating set of $G[H]$. It follows that T_x is a clique dominating set of H. If $x = y$, then $b \in (V(H) \setminus T_x) \cap N_H(a)$. This means that $(T_x \setminus \{a\}) \cup \{b\} = \{b\}$ is a clique dominating set of H. Thus, T_x is a 1-movable clique dominating set of H. If $x \neq y$, then $y \in (V(G) \setminus S) \cap N_G(x)$. Hence, $(S \setminus \{x\}) \cup \{y\} = \{y\}$ is a clique dominating set of G. Thus, S is a 1-movable clique dominating set of G showing that (iii) holds.

For the converse, suppose that S is a clique dominating set of G and (i), (ii) and (iii) hold. Then, clearly, $C = \bigcup_{x \in S} [\{x\} \times T_x]$ induces a clique in $G[H]$. Let $(z,d) \notin C$. If $z \notin S$, then there exists $w \in S$ such that $wz \in E(G)$. Choose any $q \in T_w$. Then $(w,q) \in C \cap N_{G[H]}((z,d))$. Suppose $z \in S$. If $|S| \geq 2$, then there exists $y \in S \cap N_G(z)$. Pick any $p \in T_y$. Then $(y,p) \in C \cap N_{G[H]}((z,d))$. Suppose $S = \{z\}$. Then, by assumption, T_z is a dominating set in H. Hence, there exists $t \in T_z \cap N_H(d)$. This implies that $(z,t) \in C \cap N_{G[H]}((z,d))$. Therefore, C is a (clique) dominating set of $G[H]$.

Let $(x,a) \in C$. Consider the following cases:

Case 1. $S = \{x\}$

If $|T_x| = 1$, that is, $T_x = \{a\}$, then T_x is a 1-movable clique dominating set of H or T_x is a clique dominating set of H and S is a 1-movable clique dominating set of G. Hence, C is a 1-movable clique dominating set of $G[H]$. Suppose $|T_x| \geq 2$. Then there exists $y \in (V(G) \setminus \{x\}) \cap N_G(x)$. Thus, $(C \setminus \{(x,a)\}) \cup \{(y,b)\}$ is a clique dominating set of $G[H]$. Hence, C is a 1-movable clique dominating set of $G[H]$.

Case 2. $|S| \geq 2$

If $T_x = \{a\}$, then there exists $b \in (V(H) \setminus T_x) \cap N_H(a)$. This means that $(C \setminus \{(x,a)\}) \cup \{(x,b)\}$ is a clique dominating set of $G[H]$. If $|T_x| \geq 2$, then $C \setminus \{(x,a)\}$ is a clique dominating set of $G[H]$. Therefore, C is a 1-movable clique dominating set of $G[H]$. □

Corollary 2.9. *Let G and H be connected nontrivial graphs such that G has a clique dominating set. Then*

$$\gamma^1_{mcl}(G[H]) = \begin{cases} 1, & \text{if } \gamma(G) = 1 \text{ and } \gamma^1_{mcl}(H) = 1 \\ & \quad \text{or } \gamma^1_{mcl}(G) = 1 \text{ and } \gamma(H) = 1 \\ 2, & \text{if } \gamma(G) = 1 \text{ and } \gamma(H) \neq 1 \\ \gamma_{cl}(G), & \text{if } \gamma(G) \neq 1. \end{cases}$$

Acknowledgments

This research is funded by the DOST-ASTHRDP-NSC-SRSF, Philippines. The authors also acknowledge the reviewers for the comments and suggestions they made which contributed much for the improvement of the paper.

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