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BONDAGE AND STRONG-WEAK BONDAGE NUMBERS OF TRANSFORMATION GRAPHS G^{xyz}

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Abstract: Let $G(V(G), E(G))$ be a simple undirected graph. A dominating set of G is a subset $D \subseteq V(G)$ such that every vertex in $V(G) - D$ is adjacent to at least one vertex in D. The minimum cardinality taken over all dominating sets of G is called the domination number of G and also is denoted by $\gamma(G)$. There are a lot of vulnerability parameters depending upon dominating set. These parameters are *strong and weak domination numbers*, *reinforcement number*, *bondage number*, *strong and weak bondage numbers*, etc. The bondage parameters are important in these parameters. The bondage number $b(G)$ of a nonempty graph G is the cardinality of a smallest set of edges whose removal from G results in a graph with domination number greater than $\gamma(G)$. In this paper, the bondage parameters have been examined of transformation graphs, then exact values and upper bounds have been obtained.

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1. Introduction

In a communication network, the vulnerability measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. Networks are modeled with graphs. Robustness of the network topology is a key aspect in the design of computer networks (see [15]-[16]). If we think of a graph as modeling a network, then there are many graph theoretical parameters such as domination number, strong and weak domination numbers, bondage number, strong and weak bondage numbers (see [1]-[2]-[3]-[4]).

We begin by recalling some standard definitions that we need throughout this paper. For any vertex $v \in V(G)$, the *open neighborhood* of v is $N_G(v) = \{u \in V(G) | uv \in E(G)\}$ and *closed neighborhood* of v is $N_G[v] = N_G(v) \cup \{v\}$. The *degree of a vertex* v in G, denoted by $d_G(v)$, is the size of its open neighborhood. The *maximum degree* of the graph G is $max\{d_G(v)|v \in V(G)\}$, also is denoted by $\Delta(G)$. The *minimum degree* of the graph G is $min\{d_G(v)|v \in V(G)\}$, also is denoted by $\delta(G)$. A vertex v is said to be *end vertex* if $d_G(v) = 1$. The graph G is called *r-regular graph* if $d_G(v) = r$ for every vertex $v \in V(G)$. The *complement* $\overline{G}$ of a graph G has $V(G)$ as its vertex sets, but two vertices are adjacent in $\overline{G}$ if only if they are not adjacent in G (see [5]-[13]).

A set D is *dominating set* of G if $N_G[D] = V(G)$, or equivalently, every vertex in $V(G)-D$ is adjacent to at least one vertex of D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G (see [17]). The concept of strong and weak dominating sets were introduced by Sampathkumar and Latha (see [9]). If $e_{uv} \in E(G)$, then u and v dominate each other. Furthermore, u *strongly dominates* v and v *weakly dominates* u if $d_G(u) \geq d_G(v)$. A set $S \subseteq V(G)$ is *strong dominating set*, if every vertex $v \in V(G) - S$ is strongly dominated by some u in S. The *strong domination number* $\gamma_s(G)$ of G is the minimum cardinality of a strong dominating set. Similarly, a set $W \subseteq V(G)$ is *weak dominating set*, if every vertex $v \in V(G) - W$ is weakly dominated by some u in W. The *weak domination number* $\gamma_w(G)$ of G is the minimum cardinality of a weak dominating set. A subset X of $E(G)$ is called an *edge dominating set* of G if every edge not in X is adjacent to some edge in X. The *edge domination number* $\gamma'(G)$ of G is the minimum cardinality taken over all edge dominating sets of G (see [17]).

The bondage number was introduced by Fink et al. (see [10]) and has been further studied by Bauer et al. (see [8]) and Hartnell et al. (see [6]). The *bondage number* $b(G)$ of a nonempty graph G is the cardinality of a smallest set of edges whose removal from G results in a graph with domination number greater than $\gamma(G)$. Then, the strong and weak bondage numbers were introduced, respectively (see [11]-[12]). The *strong bondage number* of G, as the minimum cardinality among all sets of edges $E' \subseteq E(G)$ such that $\gamma_s(G - E') > \gamma_s(G)$ and it is denoted by $b_s(G)$. Similarly, the *weak bondage number* of G as the minimum cardinality of among all sets of edges $E' \subseteq E(G)$ such that $\gamma_w(G - E') > \gamma_w(G)$ and it is denoted by $b_w(G)$.

The *line graph* $L(G)$ of G is the graph whose vertex set is $E(G)$ and in which two vertices are adjacent if and only if they are adjacent in G. The *total graph* $T(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$, and in which two vertices are adjacent if and only if they are adjacent or incident in G (see [5]). Wu and Meng generalized the concept of total graph and introduced some new transformations graphs G^{xyz} (see [7]).

Let x, y, z be three variables taking value $+$ or $-$. The transformation graph of G, G^{xyz} is a simple graph having as the vertex set $V(G) \cup E(G)$, and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent or incident in G^{xyz} if and only if one of the following holds (see [1]-[2]-[14]-[18]-[19]):

(i) Let $\alpha, \beta \in V(G)$. α and β are adjacent in G if $x = +$; α and β are not adjacent in G if $x = -$.

(ii) Let $\alpha, \beta \in E(G)$. α and β are adjacent in G if $y = +$; α and β are not adjacent in G if $y = -$.

(iii) Let $\alpha \in V(G)$, $\beta \in E(G)$. α and β are incident in G if $z = +$; α and β are not incident in G if $z = -$.

Since there are eight distinct 3-permutations of $\{+, -\}$, we may obtain eight kinds of transformation graphs, in which G^{+++} is the total graph of G, and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--}, respectively.

In this paper, we study about bondage numbers, strong bondage numbers and weak bondage numbers of the transformation graph G^{xyz}. Then exact values and upper bounds are obtained. Some notations are used in order to make the proof of the given theorems understandable. Let u and v be any two vertices of the graph G. If these two vertices are adjacent in the graph G, then the edge between these two vertices is denoted by e_{uv} in graph the G. Furthermore, this edge is represented as the vertex e_{uv} in the graph G^{xyz}.

2. Basic Results

Theorem 1. [1] *Let G be a connected graph that has only one end vertex of order n and size m. If $d_G(u) = 1$, $v \in N_G(u)$ and $\gamma^{\square}(G) > 3$, then*

$$\gamma(G^{-+-}) = \begin{cases} 3 & , d_G(v) = n - 1; \\ 2 & , d_G(v) < n - 1. \end{cases}$$

Theorem 2. [1] *Let G be a connected graph of order n and size m. If the minimum vertex degree $\delta(\overline{L(G)}) > m - n + 2$, then $\gamma_w(G^{+--}) = \gamma(G)$.*

Theorem 3. [1] *Let G be a connected graph of order n and r-regular. If $n > 2r + 1$, then $\gamma_w(G^{+--}) = \gamma(G)$.*

Theorem 4. [1] *Let G be a connected graph of order n and size m. If G includes only one star subgraph and $d_G(u) + d_G(v) < m - n + 4$ for every $e_{uv} \in L(G)$, then $\gamma(G^{++-}) = \gamma_s(G^{++-}) = 2$.*

Theorem 5. [1] *Let G be a connected graph of order n and r-regular. If $r > 2$, then $\gamma_w(G^{++-}) = \gamma^{\square}(G)$.*

Theorem 6. [1] *Let G be a connected graph of order n and size m. If G includes more than one end vertices, then $\gamma(G^{---}) = 2$.*

Theorem 7. [2] *Let G be a connected graph of order n and r-regular. If $2\gamma^{\square}(G) < n < 2r + 1$, then $\gamma_s(G^{-++}) = \gamma^{\square}(G) + 1$.*

Theorem 8. [2] *Let G be a connected graph of order n and r-regular. If $n > 2\gamma^{\square}(G)$ and $n > 2r + 1$, then $\gamma_w(G^{-++}) = \gamma^{\square}(G) + 1$.*

Theorem 9. [2] *Let G be a connected graph of order n and size m. If G includes only one pendant vertex, the maximum vertex degree $\Delta(G) = n - 1$ and $m > n + 1$, then $\gamma(G^{+-+}) = 2$.*

Theorem 10. [2] *Let $G = K_{1,n}$ be a star graph of order n+1. Then, $\gamma(G^{+++}) = \gamma_s(G^{+++}) = 1$ and $\gamma_w(G^{+++}) = n$.*

Theorem 11. [2] *Let $G = C_n$ be a cycle graph of order n. If $n \geq 6$, then $\gamma(G^{+++}) = \gamma_s(G^{+++}) = \gamma_w(G^{+++}) = \lceil \frac{2n}{5} \rceil$.*

3. Some Exact Values and Bounds for the Graph G^{xyz}

In this section, we have obtained some exact values and bounds for the bondage number, strong bondage number and weak bondage number of the transformation graph G^{xyz}.

Theorem 12. *Let G be a connected graph that has only one end vertex of order n and size m. If $d_G(u) = 1$, $v \in N_G(u)$ and $\gamma^{\square}(G) > 3$, then*

(a) *If $d_G(v) = n - 1$, then $b(G^{-+-}) \le \delta(G^{-+-} - \{u, v, e_{uv}\})$;*

(b) *If $d_G(v) < n - 1$, then $b(G^{-+-}) = n - 1 - d_G(v)$.*

Proof. **(a)** If $d_G(v) = n - 1$, then $N_{G^{-+-}}[u, v, e_{uv}] = V(G^{-+-})$ by the Theorem 1. In worst case, any vertex with minimum degree of $V(G^{-+-}) - \{u, v, e_{uv}\}$ is isolated for increasing the domination number. So, the domination number increases by 1. As a result, we have $b(G^{-+-}) \le \delta(G^{-+-} - \{u, v, e_{uv}\})$.

(b) If $d_G(v) < n - 1$, then $\gamma(G^{-+-}) = 2$ by the Theorem 1 and $\gamma(G^{-+-})$-dominating set includes the vertex u and any vertex of $V(\overline{G}) - \{u\}$. The vertices v and e_{uv} are not dominated by the vertex u. Due to $d_{G^{-+-}}(v) = d_{G^{-+-}}(e_{uv})$, if the edges which incident to the vertices v or e_{uv} is deleted from the graph G^{-+-}, we obtain $\gamma(G^{-+-} - S(v)) > \gamma(G^{-+-})$ or $\gamma(G^{-+-} - S(e_{uv})) > \gamma(G^{-+-})$, where $S(v)$ and $S(e_{uv})$ are the set which include the edges that incident to vertices v and e_{uv}, respectively. Hence, $b(G^{-+-}) = n - 1 - d_G(v)$.

The proof is completed. □

Theorem 13. [1] *Let G be a connected graph of order n and size m. If the minimum vertex degree $\delta(\overline{L(G)}) > m - n + 2$, then $1 \le b_w(G^{+--}) \le \delta(G)$.*

Proof. By the Theorem 2, we have $\gamma_w(G^{+--}) = \gamma(G)$. We have two cases depending on the number of $\gamma(G)$-dominating set for this proof.

Case 1. Let $\gamma(G)$-dominating set is unique. If an edge which incident any vertex of $\gamma(G)$-dominating set is deleted from the graph G^{+--}, we obtain $\gamma_w(G^{+--} - \{e\}) > \gamma_w(G^{+--}) = \gamma(G)$. Then, $b_w(G^{+--}) = 1$.

Case 2. Let $\gamma(G)$-dominating set is not unique. The edges which incident to any vertex with minimum degree must delete in the worst case. So, whole degree of vertices of set $N_G[u]$ decrease by 1. Even if $\gamma(G)$-dominating set includes this vertex, the weak domination number increases by 1. Hence, we obtain $b_w(G^{+--}) \le \delta(G)$.

By Cases 1 and 2 the proof is completed. □

Theorem 14. *Let G be a connected graph of order n and r-regular. If $n > 2r + 1$, then $b_w(G^{+--}) = b(G)$.*

Proof. The $\gamma_w(G^{+--})$-weak dominating set is the same as $\gamma(G)$-dominating set by the Theorem 3. Clearly, the set of edges formed by calculating the value of $b_w(G^{+--})$ are deleted from the graph G^{+--}, the $\gamma_w(G^{+--})$ increases with the increase in $\gamma(G)$. Hence, $b_w(G^{+--}) = b(G)$ is obtained.

The proof is completed. □

Theorem 15. *Let G be a connected graph of order n and size m. If G includes only one star subgraph and $d_G(u) + d_G(v) < m - n + 4$ for every $e_{uv} \in L(G)$, then $b(G^{++-}) \leq 3$ and $b_s(G^{++-}) \leq 3$.*

Proof. We have $\gamma(G^{++-}) = \gamma_s(G^{++-}) = 2$ by the Theorem 4. Let c be a vertex with $(n-1)$-degree in the graph G and e_{xy} be any vertex of set $V(L(G)) - \{e_{cN_G(c)}\}$. The edges of between vertices c and e_{xy}, e_{xy} and e_{cx}, e_{xy} and e_{cy} are called $E_1(t)$. When the edges of set $E_1(t)$ are deleted from the graph G^{++-}, we obtain $\gamma(G^{++-} - \{E_1(t)\}) = \gamma_s(G^{++-} - \{E_1(t)\}) = 3$. Due to there are a lot of edge sub set for computing the bondage number, 3 is an upper bound for the bondage and strong bondage number. As a result, $b(G^{++-}) \leq 3$ and $b_s(G^{++-}) \leq 3$.

The proof is completed. □

Theorem 16. *Let G be a connected graph of order n and r-regular. If $r > 2$, then $b_w(G^{++-}) \leq 2r - 2$.*

Proof. We have $\gamma_w(G^{++-}) = \gamma^{\square}(G)$ by the Theorem 5. It easy to see that $d_{G^{++-}}(u) > d_{G^{++-}}(e_{xy})$ for every $e_{xy} \in V(L(G))$ and $u \in V(G)$. Clearly, the graph $L(G)$ is $(n + 2r - 4)$-regular. To increase value of $\gamma_w(G^{++-})$, $(2r-2)$-vertices which are adjacent to any edge $e_{xy} \in V(L(G))$ must delete from the graph $L(G)$ in the worst case. Due to the vertex e_{xy} is not weak dominated, it must take $\gamma_w(G^{++-})$-weak dominating set. So, the weak domination number increases by 1. As a result, $2r-2$ is upper bound for the weak bondage number. Then, $b_w(G^{++-}) \leq 2r - 2$ is obtained.

The proof is completed. □

Theorem 17. *Let G be a connected graph of order n and size m. If G has more than one end vertices and includes at least one spanning star subgraph, then $b(G^{---}) \leq m - n + 1$.*

Proof. We have $\gamma(G^{---}) = 2$ by the Theorem 6. Let $d_G(c) = n-1$. Clearly, $d_{G^{---}}(c) = m - n + 1$. The proof follows directly from Theorem 16. Then, we obtain an upper bound that value is $m-n+1$. As a result, $b(G^{---}) \leq m-n+1$ is obtained.

The proof is completed. □

Theorem 18. *Let G be a connected graph of order n and r-regular. If $2\gamma'(G) < n < 2r+1$, then $b_s(G^{-++}) \leq b(K_{(n-2\gamma'(G))})$.*

Proof. We have $\gamma_s(G^{-++}) = \gamma'(G) + \gamma(K_{(n-2\gamma'(G))})$ by the Theorem 7. Clearly, $\gamma_s(G^{-++}) = \gamma'(G)+1$. Any set edges whose cardinality giving value of $b_s(G^{-++})$ is the same as any set edges whose cardinality giving value of bondage number for the graph $K_{n-2\gamma'(G)}$. Due to the proof of the strong bondage number of K_n, $\lceil\frac{n-2\gamma'(G)}{2}\rceil$-edges are deleted from the graph G^{-++}. Let T be an edge set that includes these $\lceil\frac{n-2\gamma'(G)}{2}\rceil$-edges. So, we obtain $\gamma_s(G^{-++} - T) = \gamma'(G)+2$. Clearly, the maximum value of $b_s(G^{-++})$ is $\gamma_s(K_{(n-2\gamma'(G))})$. As a result, we have $b_s(G^{-++}) \leq b(K_{(n-2\gamma'(G))})$.

The proof is completed. □

Theorem 19. *Let G be a connected graph of order n and r-regular. If $n > 2\gamma'(G)$ and $n > 2r+1$, then $b_w(G^{-++}) \leq b(K_{(n-2\gamma'(G))})$*

Proof. The proof follows directly from Theorem 18. □

Theorem 20. *Let G be a connected graph of order n and size m. If G includes only one pendant vertex, the maximum vertex degree $\Delta(G) = n-1$ and $m > n+1$, then $b(G^{+-+}) = 1$.*

Proof. We have $\gamma(G^{+-+}) = 2$ by the Theorem 9. Let $d_G(c) = n-1$ and $d_G(u) = 1$. Clearly, $\gamma(G^{+-+})$-dominating set includes the vertices c and e_{cu}. The vertex c dominates vertices of $V(G) \cup (V(\overline{L(G)}) - \{e_{cN_G(c)}\})$. So, there are $(m-n+1)$-vertices which are not dominated remaining in the graph $\overline{L(G)}$. These vertices are adjacent to vertex e_{cu}. When $m > n+1$, these vertices are dominated by only the vertex e_{cu}. Let $\exists x \in N_{\overline{L(G)}}(e_{cu})$ and e be an edge between the vertices e_{cu} and x. When the edge e is deleted from the graph G^{+-+}, the vertex x is not dominated. Then, the domination number increases by 1. Since $\gamma(G^{+-+} - \{e\}) > \gamma(G^{+-+})$, we obtain $b(G^{+-+}) = 1$.

The proof is completed. □

Theorem 21. *Let $G = K_{1,n}$ be a star graph of order n+1. Then, $b(G^{+++}) = b_s(G^{+++}) = b_w(G^{+++}) = 1$.*

Proof. We have $\gamma(G^{+++}) = \gamma_s(G^{+++}) = 1$ by the Theorem 10. Let c be the center vertex of graph G. $\gamma(G^{+++})$-strong dominating set and $\gamma(G^{+++})$-dominating set must include the vertex c. It is easy to see that the vertex c is adjacent whole vertices of graph G^{+++}. Let e be any edge which is incident the vertex c. The edge e is deleted from the graph G^{+++}, we have $\gamma(G^{+++} - \{e\}) =$

$\gamma_s(G^{+++} - \{e\}) = 2$. As a result, $b(G^{+++}) = b_s(G^{+++}) = 1$. Therefore, we have $\gamma_w(G^{+++}) = n$ by the Theorem 10. The $\gamma_w(G^{+++})$-weak dominating set includes the vertices of $V(G) - \{c\}$. Let $u \in V(G) - \{c\}$ and e be an edge between the vertices c and e_{cu}. When the edge e is deleted from the graph G^{+++}, the vertex e_{cu} is not weak dominated. So, it must take into $\gamma_w(G^{+++})$-weak dominating set. Then, we have $\gamma_w(G^{+++} - \{e\}) > \gamma_w(G^{+++})$. As a result, $b_w(G^{+++}) = 1$ is obtained.

The proof is completed. □

Theorem 22. *Let $G = C_n$ be a cycle graph of order n. If $n \geq 6$, then*

$$b_w(G^{+++}) = \begin{cases} 1 & , n \equiv 0, 2, 4 (mod\, 5); \\ 2 & , otherwise. \end{cases}$$

Proof. The graph G^{+++} is 4-regular and $\gamma_w(G^{+++}) = \lceil \frac{2n}{5} \rceil$ by the Theorem 11. We have two cases depending on the number of vertices of graph G for this proof.

Case 1. Let $n \equiv 0, 2, 4 (mod\, 5)$. Let u and v be two vertices that are adjacent. If the edge e_{uv} is deleted from the graph G^{+++}, these vertices must take into $\gamma_w(G^{+++})$-weak dominating set by the definition of weak domination number. These vertices weak dominate totaly seven vertices with by them. Then there are $(2n - 7)$-vertices which are not weak dominated remaining in the graph G^{+++}. By the Theorem 11, we obtain $\gamma(G^{+++} - \{e\}) = 2 + \lceil \frac{2n-7}{5} \rceil = \lceil \frac{2n+3}{5} \rceil$. Because of $n \equiv 0, 2, 4 (mod\, 5)$; then $\lceil \frac{2n+3}{5} \rceil > \lceil \frac{2n}{5} \rceil$. As a result, $b_w(G^{+++}) = 1$.

Case 2. Let $n \equiv 1, 3 (mod\, 5)$. When any edge is deleted from the graph G^{+++}, the weak domination number is not change by the structure of graph G^{+++}. Let P_3 be any sub path graph and it's vertices labeled u, v and w in the graph G^{+++}. If the edges e_{uv} and e_{vw} are deleted from the graph G^{+++}, the vertices u, v and w must take into $\gamma_w(G^{+++})$- weak dominating set by the definition of weak domination number. These vertices weak dominate totaly nine vertices with by them. Then, there are $(2n - 9)$-vertices which are not weak dominated remaining in the graph G^{+++}. By the Theorem 11, we obtain $\gamma(G^{+++} - \{e_1, e_2\}) = 2 + \lceil \frac{2n-9}{5} \rceil = \lceil \frac{2n+1}{5} \rceil$. Because of $n \equiv 1, 3 (mod\, 5)$; then $\lceil \frac{2n+1}{5} \rceil > \lceil \frac{2n}{5} \rceil$. As a result, $b_w(G^{+++}) = 2$.

By Cases 1 and 2 the proof is completed. □

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