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NUMERICAL STUDY OF A CHARGED PARTICLE IN A GENERAL MAGNETIC FIELD

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Abstract: In this work we propose a new numerical scheme to simulate the motion of a nonrelativistic charge in a general magnetic field. The scheme is very accurate in the sense that it shows a discrete analogous of the underlying continuous energy conservation law, as well as an analogous of the angular momentum variation law.

Key Words: charged particles, magnetic field, angular momentum

1. Introduction

The study of the behaviour and trajectories description of charge particles has

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an important role in the understanding of the particle dynamics. Such importance lies in the need to understand in detail the interaction between the particles and different fields, for example magnetic and electric ones or the magnetospheres of the planets ([1], [2]). Charged particles like electrons, protons or alpha particles that are carried by the solar wind play an important role in the interaction with terrestrial magnetosphere causing luminous phenomena such as auroras. This phenomenon has been possible to visualize both on Earth ([3], [4], [5])and on other planets ([6]), such as Mars ([7], [8]). By this interaction among others, the most clear understanding of the behaviour of these particles is needed.

There exist many works that explains the interaction from a physical viewpoint ([9], [10]). In this paper we propose a viewpoint besides physicist, mathematician. For example, in previous studies we have presented a numerical scheme that describes a family of trajectories of charged particles under the action of a generic magnetic field in Cartesian coordinates ([11]). This numerical scheme explains the trajectories of these particles under the action of a dipolar magnetic field and is described in Ramírez-Nicolás et al, 2014. Here, besides using the numerical scheme mentioned above, we have focused on dipolar magnetic fields and their interaction with charged particles ([12]).

In this work we present a numerical study of the angular momentum for these charged particles, comparing the discrete and continuous analysis. Along this article the results of this study are presented. A review of the Strmer theory and the introduction of the numerical scheme are presented in Section 1. The detailed analysis of the angular momentum of a charged particle is described in Section 2, while numerical simulations showing the results are provided in Section 3 and Section 4. Finally, a summary of the main conclusions is given in Section 5.

2. Störmer Theory

The equation of a nonrelativistic particle with rest mass m and charge q moving in a magnetic field $\mathbf{B}$ is ([1], [13], [14])

$$\frac{d(m\mathbf{v})}{dt} = \frac{q}{c}(\mathbf{v} \times \mathbf{H}), \tag{1}$$

where m is the mass and $\mathbf{v}$ is the velocity of the particle, and c is the velocity of light. Multiplying eq (1) by $\mathbf{v}$ and using the vector products relations, the energy conservation

$$\frac{d(m\mathbf{v}^2)}{dt} = 0 \Rightarrow m\mathbf{v}^2 = \text{Const.} \tag{2}$$

Let us consider the well known Strmers approach of the motion of one particle in the magnetic field of the Earth. In this sense the magnetic field $\mathbf{B}$ is given by $\mathbf{H} = rot(\mathbf{A})$ being

$$A = \frac{\mathbf{M}_E \times \mathbf{r}}{r^3} \tag{3}$$

the potential-vector, and $\mathbf{M}_E$ the dipole moment of the Earth. Then, the magnetic potential-vector of the Earth, in cylindrical coordinates (ρ, z, ϕ) can be characterized by its azimuthal component following:

$$A_\phi = \frac{M_E \rho}{(\rho^2 + z^2)^{3/2}}. \tag{4}$$

Equation eq (1) can be derived from Lagrangian

$$\mathfrak{L} = \frac{m_0}{\sqrt{1 - v^2/c^2}} c^2 + \frac{q}{c} \mathbf{v} \cdot \mathbf{A}. \tag{5}$$

The nonrelativistic Lagrangian function of a particle moving in a dipolar magnetic field, can be written in Cylindrical coordinates by

$$\mathfrak{L} = m(\dot{\rho}^2 + \dot{z}^2 + \rho^2 \dot{\phi}^2) + \frac{q}{c} \rho \dot{\phi} A_\phi, \tag{6}$$

where A_ϕ corresponds to the ϕ component of the potential-vector. The axial symmetry of dipole field, as the one considered for the Earth, implies the non-dependence of the magnetic field from the azimuth ϕ component. For this reason it is obtained that $\partial \mathfrak{L}/\partial \phi = 0$, leading us to the conservation of the angular momentum p_ϕ

$$p_\phi = \frac{\partial \mathfrak{L}}{\partial \dot{\phi}} = m\rho^2 \dot{\phi} + \frac{q}{c} \rho A_\phi = Const. \tag{7}$$

This conserved quantity makes possible to introduce a new Lagrangian form (Rauss function)

$$\mathfrak{L}_R = \mathfrak{L} - \frac{\partial \mathfrak{L}}{\partial \dot{\phi}} \dot{\phi} = \frac{m}{2} (\dot{\rho}^2 + \dot{z}^2) - U, \tag{8}$$

where the potential is given by the following equation

$$U = \frac{1}{2m} \left(\frac{p_\phi}{\rho} - \frac{q}{c} A_\phi \right)^2 \tag{9}$$

This symmetry of the field makes possible to reduce a 3D problem into a 2D, considering a particle moving in the plane (ρ, z) with potential U. Following Vázquez and Jiménez. (1988) and working in the appropriate dimensionless units, it is possible to redefine the variables (t, ρ and z) and the corresponding equations of motion as

$$\frac{d^2\rho}{dt^2} = -\frac{\partial U}{\partial \rho}, \tag{10}$$

$$\frac{d^2 z}{dt^2} = -\frac{\partial U}{\partial z} \tag{11}$$

being

$$U = \frac{1}{2}\left(\frac{1}{\rho} - \frac{\rho}{(\rho^2+z^2)^{3/2}}.\right)^2 \tag{12}$$

Following the structure of the numerical scheme described in previous studies ([12], [15], [16]) a conservative scheme is presented and used to solve numerically equations eq (1) and eq (2).

$$\frac{\rho^{n+2} - 2\rho^{n+1} + \rho^n}{\Delta t^2} = -\frac{U(\rho^{n+2}, z^n) - U(\rho^n, z^n)}{\rho^{n+2} - \rho^n} \tag{13}$$

$$\frac{z^{n+1} - 2z^{n+1} - z^n}{\Delta t^2} = -\frac{U(\rho^{n+2}, z^{n+2}) - U(\rho^{n+2}, z^n)}{z^{n+2} - z^n} \tag{14}$$

This scheme preserves the discrete energy

$$E^n = \frac{1}{2}\left(\frac{\rho^{n+1} - \rho^n}{\Delta t}\right)^2 + \frac{1}{2}\left(\frac{z^{n+1} - z^n}{\Delta t}\right)^2 + \frac{1}{2}\left(U(\rho^{n+1}, z^{n+1}) + U(\rho^n, z^n)\right). \tag{15}$$

And trivially from the motion equations eq (1) and eq (2), the angular momentum p_ϕ is preserved.

Strmer's theory is widely used to approximate the trajectories of charged particles in dipolar fields but when the magnetic field does not present such symmetry, then, this theory is not valid. Another problem appears when trying to include other interactions such us the electromagnetical field created by other particles or the gravitational one. These other interactions do not has azimuthal symmetry, and we have to deal with the motion equation in three space dimenssion.

The aim of this work is to present a new family ([16], [17]) of numerical schemes to solve equation eq (1) in Cartesian coordinates for a general magnetic field. Such scheme show a discrete conserved energy and angular momentum which are preserved by the underluing continuous equation.

3. Conservation Laws of the Numerical Scheme

In a continuous system, the motion equation for a nonrelativistic charged particle with charge q and mass m, with a vector position $\mathbf{r}$ and a velocity $\mathbf{v}$ moving in a magnetic field $\mathbf{B}$, can be written as follows

$$m\frac{d\mathbf{v}}{dt} = \gamma(\mathbf{v} \times \mathbf{B})\frac{d\mathbf{r}}{dt} = \mathbf{v}, \tag{16}$$

where $\gamma = \frac{q}{c}$, q and c corresponds to the charge of the particle and to the velocity of the light repectively.

On the other hand, in a general way, the angular momentum equation can be written as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m \cdot \mathbf{v}. \tag{17}$$

The variation law of the angular momentum is described by eq (18)

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \frac{d\mathbf{v}}{dt} = \mathbf{r} \times \gamma(\mathbf{v} \times \mathbf{B}), \tag{18}$$

operating the triple vector product, we obtain eq (19)

$$\frac{d\mathbf{L}}{dt} = \gamma[\mathbf{v}(\mathbf{r}\cdot\mathbf{B}) - \mathbf{B}(\mathbf{r}\cdot\mathbf{v})] = \gamma\left[\mathbf{v}(\mathbf{r}\cdot\mathbf{B}) - \mathbf{B}\frac{d\mathbf{r}^2}{2dt}\right]. \tag{19}$$

Discretizing the motion equation described in equation (16) we obtain:

$$m\left(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t}\right) = \gamma\frac{\mathbf{v}^{n+1} + \mathbf{v}^n}{2} \times \mathbf{B}, \tag{20}$$

$$\frac{\mathbf{r}^{n+1} - \mathbf{r}^n}{\Delta t} = \frac{\mathbf{v}^{n+1} + \mathbf{v}^n}{2}. \tag{21}$$

Now, if we multiply scalarly eq (20) by $\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2}$ we obtain

$$\frac{\mathbf{r}^{n+1} + \mathbf{r}^n}{2} \cdot m\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} = 0, \tag{22}$$

from where

$$\frac{1}{2}m(\mathbf{v}^{n+1})^2 = \frac{1}{2}m(\mathbf{v}^n)^2. \tag{23}$$

Equation (23) express the discrete conservation law of the kinetic energy. Otherwise, if we consider the vectorial product eq (20) by $\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2}$ we get

$$\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} \times m\frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta t} = \gamma\left(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} \times \frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2} \times \mathbf{B}\right). \tag{24}$$

Operating left hand term of equation (24), and analyzing this in detail we obtain:

$$\begin{aligned} m\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} &\times \frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta t} \\ &= m\frac{\mathbf{r}^{n+1}\times\mathbf{v}^{n+1}-\mathbf{r}^n\times\mathbf{v}^n}{2\Delta t} + m\frac{-\mathbf{r}^{n+1}\times\mathbf{v}^n+\mathbf{r}^n\times\mathbf{v}^{n+1}}{2\Delta t}. \end{aligned} \tag{25}$$

Adding and subtracting $m\frac{\mathbf{r}^{n+1}\times\mathbf{v}^{n+1}}{2\Delta t}$, $m\frac{\mathbf{r}^n\times\mathbf{v}^n}{2\Delta t}$ to the right hand term of eq (25) equation and rearranging terms we obtain

$$\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} \times m\frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta t} = m\frac{\mathbf{r}^{n+1}\times\mathbf{v}^{n+1}-\mathbf{r}^n\times\mathbf{v}^n}{\Delta t}. \tag{26}$$

That, in terms of the angular momentum can be written as follows:

$$\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} \times \frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta t} = \frac{\mathbf{L}^{n+1}-\mathbf{L}^n}{\Delta t}. \tag{27}$$

Now focussing on the right term of eq (24) and operating the triple product it is obtained

$$\begin{aligned} \gamma\Big(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} &\times \frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2} \times \mathbf{B}\Big) \\ &= \gamma\left[\frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2}\left(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2}\cdot\mathbf{B}\right) - \mathbf{B}\left(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2}\cdot\frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2}\right)\right]. \end{aligned} \tag{28}$$

Taking into account the discretization of the position vector of the particle (eq (21)), expression ((27)) can be written as:

$$\begin{aligned} \gamma\Big(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2} &\times \frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2} \times \mathbf{B}\Big) \\ &= \gamma\left[\frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2}\left(\frac{\mathbf{r}^{n+1}+\mathbf{r}^n}{2}\cdot\mathbf{B}\right) - \mathbf{B}\left(\frac{(\frac{r^{n+1})^2}{2} - \frac{(\mathbf{r}^n)^2}{2}}{\Delta t}\right)\right]. \end{aligned} \tag{29}$$

As it has been shown, the proposed scheme ((20)) satisfies a discrete analogous of the underlying continuous angular momentum variation law. Joining all these results, equation (1) can be written as:

$$\frac{\mathbf{L}^{n+1}-\mathbf{L}^{n}}{\Delta t}=\gamma\left[\frac{\mathbf{v}^{n+1}+\mathbf{v}^{n}}{2}\left(\frac{\mathbf{r}^{n+1}+\mathbf{r}^{n}}{2}\right)\cdot\mathbf{B}-\mathbf{B}\left(\frac{\frac{(\mathbf{r}^{n+1})^2}{2}-\frac{(\mathbf{r}^{n})^2}{2}}{\Delta t}\right)\right]. \quad (30)$$

4. Numerical Simulations

In addition to aabove analysis, here are presented the numerical results obtained.

4.1. Trapped Trajectories

Following the theory and the equations above described in Section 2 and Section 3, here we present the main results of the work: the comparisonbetween the trajectories obtained from different conditions applied to the mentioned numerical schemes and the angular momentum analysis with the corresponding results. The study was developed considering an electron with charge ,q , moving in a general dipolar magnetic field (eq (29)).

$$\mathbf{B}=\nabla\times\mathbf{A}, \quad (31)$$

where A is a vector potential for $\mathbf{B}$, whose relation is described by equation (31). In this case, the vector-potential is described by the following expression

$$\mathbf{A}=A(y,x,0), \quad (32)$$

$$A=\frac{\mu}{(x^2+y^2+z^2)^{1/2}}, \quad (33)$$

where μ is the permeability of free space and (x, y, z) are the corresponding Cartesian coordinates for the charged particle (electron), in SI system units.

To describe the family of trajectories of the electron, the numerical scheme shown bellow ([11]), describes the movement of this particle.

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^{n}}{h}=\frac{q}{2mc}(\mathbf{v}^{n+1}+\mathbf{v}^{n})\times\mathbf{B}^{n}\mathbf{r}^{n+1}=\mathbf{r}^{n}+\mathbf{v}^{n}\Delta t, \quad (34)$$

where $h=\Delta t$, q is the charge of the corresponding charged particle, m the mass of the particle and c is the speed of light in vacuum (in SI system units).

This numerical scheme possesses the same energy conservation law that the continuous model has (see eq (2)). This can be shown multiplying (scalar product) by $\mathbf{v}^{n+1} + \mathbf{v}^n$ in both sides of eq (1). Recalling the above mentioned, right hand side vanishes since the resulting vector of the vectorial product is perpendicular to $\mathbf{v}^{n+1} + \mathbf{v}^n$. From this mathematical analysis and to see the results graphically, the results obtained from eq (13)-(14) equations have been compared with the ones from equation (1). For this purpose, Figure 1 shows the path described by an electron as a result from the (eq (13), eq (14)) equations (on the left) and the one described by equation (32) in the $\rho - z$ plane.

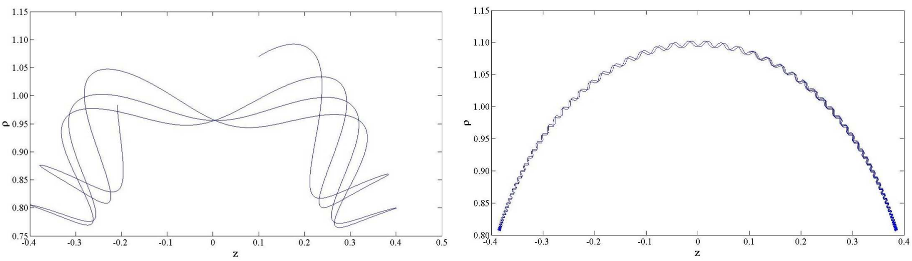

Figure 1: Left, trajectory for an electron in the $\rho - z$ plane calculated from ((13), (14)) equations. Right, trajectory for an electron in the $\rho - z$ plane calculated from eq (34) equation

The calculations performed for the right graphic of Figure 1 are in Cartesian coordinates, but has beed reduced to a two-dimensional problem $\rho - z$ by using $\rho = (x^2 + y^2)^{1/2}$ for a better comparison. In both cases the same initial conditions for the electron have been selected and are summarized in Table 1 and Table 2.

In view of the results of Figure 1, there exist two ways to detect trapped orbits for an electron under a dipolar magnetic field. The obvious differences between left and right graphics lies in the multiple combinations that exist for x and y to obtain the corresponding ρ value. But if we focus on the obtained values, it is possible to check that in both cases the reached values are the same, strenghening the viability of using the scheme in Cartesian coordinates, getting rid of mathematics limitations compared to using the scheme in Cylindrical coordinates. These results are in perfect agreement with those described in Ramírez-Nicolás et al., 2014.

Table 1: Summary of the main imput parameters to solve equations (13) and (14) in cylindrical coordinates. All the values are assumed in SI system units.

Parameter	Value
(ρ, z)	$(1.07, 0.10)$
(v_ρ, v_z)	$(0.05, 0.10)$
N	5000
h	1×10^{-2}

Table 2: Summary of the main imput parameters to solve equation (32) in cartesian coordinates. All the values are assumed in SI system units.

Parameter	Value
(x, y, z)	$(0.72, 0.81, 0.10)$
(v_x, v_y, v_z)	$(0.04, 0.03, 0.10)$
N	5000
h	1×10^{-2}

4.2. Angular Momentum

Once it has been proven that the numerical scheme reproduces the desired results, next step has been work with the angular momentum. One of the goals of this work it is to proof that the conservation of the angular momentum is maintained for a continuous system and for the discretization of it. For this purpose two different cases have been studied. In the first case, we have considered a particle with mass m, charge q in a magnetic field $\mathbf{B}$ that follows $\mathbf{B} = (0, 0, B_z)$, and whose motion equation is described by equation (16). This particle moving in the perpendicular plane to the magnetic field with a velocity $\mathbf{v} = (v_x, v_y, 0)$. By considering these motion equations for an electron, tha analytical (equations (36) and (37)) and numerical (equation (34)) solutions have been plotted in Figure 2.

$$\begin{aligned} v_x &= a \cdot \sin(wt + \phi), \\ v_y &= a \cdot \cos(wt + \phi), \end{aligned} \tag{35}$$

$$x = -\frac{a}{w}\cos(wt+\phi),$$
$$y = -\frac{a}{w}\sin(wt+\phi), \tag{36}$$

where a and ϕ depends on the initial velocity and w is the cyclotron frequency of the charge, which is decribed by

$$w = \frac{qB}{mc}. \tag{37}$$

From these equations and considering an electron, in Figure 2, have been plotted the analytical solution (equation (37)) and the numerical one (equation (34)).

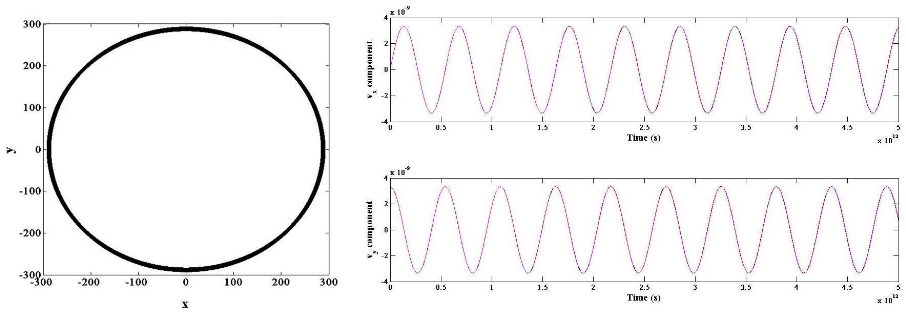

Figure 2: Left panel, trajectory of an electron in Cartesian coordinates, in black the numerical solution from equation (34) and in red the analytical solution from equation (37). Right panels, comparison between numerical (black) and analytical solution of x (up) and y (down) components for the velocty of the particle

In view of the obtained results, the numerical scheme reproduces the real solution for very large times. Therefore, the numerical scheme can be used with high accurancy for those configurations in which we don't know the analytical solution. This first case has been used as a check case for the numerical scheme since we know the analytical solution. in this sense, for the second case, the same particle has been selected but now in a dipolar magnetic field **B** (equations (38) to (40)) in Cartesian coordinates system

$$B_x = \frac{3}{4\pi}\frac{\mu xz}{(x^2+y^2+z^2)^{5/2}}, \tag{38}$$

$$B_y = \frac{3}{4\pi}\frac{\mu yz}{(x^2+y^2+z^2)^{5/2}}, \tag{39}$$

$$B_x = \frac{1}{4\pi} \frac{\mu(2z^2 - x^2 - y^2)}{(x^2 + y^2 + z^2)^{5/2}}. \tag{40}$$

For this case any restriction for position and velocity has not been taken into account. Under the assumptions and considerations described in this Section for an electron, the trajectories and the angular momentum have been analyzed from continuous and discrete points of view, and the trajectories of this particle have been plotted. So, Figure 3, represents the trajectory of an electron in a 3D Cartesian coordinate system (left) and the corresponding trajectory in (ρ, z) plane (right).

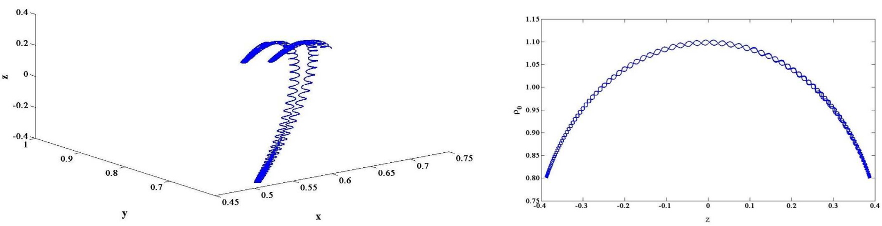

Figure 3: Left, trajectory of an electron in Cartesian coordinates and Gaussian units. Right, the corresponding trejectory in the (ρ, z) plane.

From this results now the three components of the angular momentum L_x, L_y, L_z for the dipolar case are shown in Figure 4.

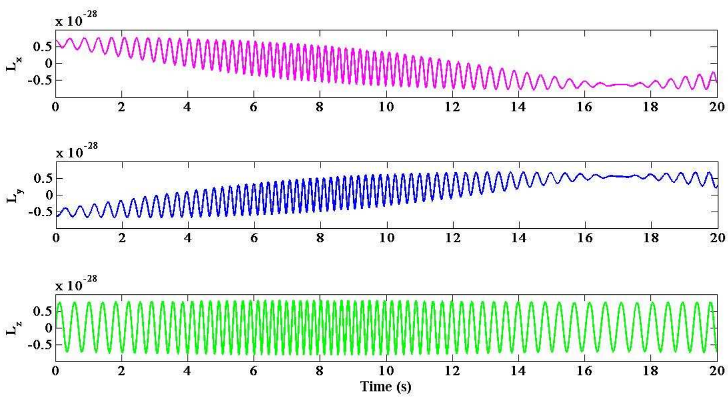

Figure 4: Three components of the angular momentum L_x (pink), L_y (blue), L_z (green) for an electron under the influence of a dipolar magnetic field.

By analyzing Figure 4 and the results of conservation described on Section 3, we can deduce that the angular momentum in a continuous and a discrete system is equivalent. For testing the validity of equation (30), in Figure 5 have been plotted the three components of the angular momentum for an electron in a dipolar magnetic field.

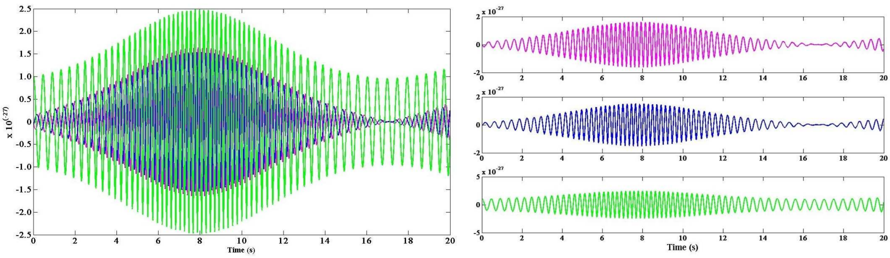

Figure 5: Comparison of the angular momentum in the SI units. On the left, it have been plotted the components of the right term from equations (26), (27) and (29). On the right, the temporal evolution of each component plotted individually. In both graphs each component x, y, z are presented in pink, blue and green respectively.

In view of the results of Figure 5, we can conclude that the temporal evolution of every component of the continuous angular momentum it is equivalent to the dicrete one. Also, from Figure 5 it is possible conclude that equation (30) it is satisfy in the case of an electrically charged particle in a dipolar magnetic field.

5. Conclusions

From the previous studies iniated by Störmer (Störmer, 1907) and the numerical scheme proposed by Ramírez-Nicolás et al., (2014), we have developed two studies related to the behaviour of a charged particle (en electron) under the effect of a dipolar magnetic field. First, we have checked (in Figure 1) that the trajectory described by this particle can be reproduced in Cylindrical coordinates by using the numerical scheme described by equations (13) and (14) as well as the Cartesian coordinates by using the numerical scheme described in equation (20). Although in a first view the paths may seen that are not the same, in detail we can observe the differences in the shape are related with the non-unique correlation between (x, y, z) and (ρ, z) coordinates. Secondly, we have developed a detailed study related to the angular momentum. In this sense, a complete mathematical analysis has been made to find the possible corelation between tha angular momentum of a continuous system (equation (16)) and the corresponding discrete one (equations (20), (21)). From this analysis a preliminary check has been made assuming the movement of a charged particle in a constant magnetic field. In view of the results, we prove that the analytical solution of the system proposed (equations (36), (37)) it is clearly reproduced by the numerical one. For the final analysis, a dipolar magnetic field has been selected. As the proposed scheme has a very good behaviour for long times the trajectory of the particle has been plotted in Cylindrical and Cartesian coordinates. Under the mentioned values the analysis of the angular momentum has been carried out. The obtained results led us to conclude that for an electron moving in a dipolar magnetic field the angular momentum it is equivalent from a continuous and a discrete point of view.

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