VAGUE MAGNIFIED TRANSLATION IN Γ-SEMIRINGS

Y. Bhargavi¹ §, T. Eswarlal²
¹,²Department of Mathematics
K.L. University
Guntur, India

Abstract: In this paper, we introduce and study the concept of vague magnified translation of a vague set in Γ-semiring and we characterized vague Γ-semiring, left(resp. right) vague ideal, vague bi-ideal, vague quasi ideal in terms of vague magnified translation.

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1. Introduction

The concept of vague set theory was introduced by Gau W.L and Buehrer D.J[4] in 1993, as a improvement of the theory of fuzzy sets by Zadeh L.A[9] in approximating the real life situations. The idea of fuzzy magnified translation has been introduced by Majumder S.K and Sardar S.K[7]. In 1995, M.K.Rao[6] introduced the notion of Γ-semiring as a generalization of Γ-ring as well as semiring and studied the concepts of Γ-semirings and its sub Γ-semirings with a left(resp. right) unity. Moreover the concept of Γ-semiring not only generalizes the concepts of semiring and Γ-ring but also the notion of ternary semiring. In this paper we introduce and study the concept of vague magnified translation of a
vague set in $\Gamma$-semiring with membership and non membership functions taking values in unit interval of real numbers and established some of the properties. Further we prove that, if $A$ is a left(resp. right) vague ideal of a $\Gamma$-semiring $R$ then the vague magnified translation $A^c_{\beta\alpha}$ of $A$ is a vague bi-ideal of $R$ and if $A$ is a left(resp. right) vague ideal of a left(resp. right) zero $\Gamma$-semiring $R$, then $A^c_{\beta\alpha}$ is a constant vague set.

Throughout this paper, $R$ stands for $\Gamma$-semiring. That is for two additive commutative semigroups $R$ and $\Gamma$ and there exists a mapping $R \times \Gamma \times R \to R$ image to be denoted by $aab$ for $a, b \in R$ and $\alpha, \beta \in \Gamma$ satisfying the following conditions.

1. $a(b + c) = aab + aac$
2. $(a + b)ac = aac + bac$

1 Correspondence author
3. $a(\alpha + \beta)c = aac + a\beta c$
4. $aa(b\beta c) = (aab)\beta c, \forall a, b, c \in R; \alpha, \beta \in \Gamma$.

Also, $\delta$ stands for the characteristic set of $R$.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

**Definition 2.1**: A $\Gamma$-semiring $R$ is called left-zero(resp. right-zero) $\Gamma$-semiring if $x\gamma y = x (\text{resp. } x\gamma y = y), \forall x, y \in R; \gamma \in \Gamma$.

**Definition 2.2**: A $\Gamma$-semiring $R$ is said to be regular if for all $x \in R$, there exists $a \in R$ and $\alpha, \beta \in \Gamma$ such that $x = x\alpha a\beta x$.

**Definition 2.3**: A $\Gamma$-semiring $R$ is said to be intra-regular if for all $x \in R$, there exists $a, b \in R$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x = aax\beta x\gamma b$.

**Definition 2.4**: Let $\mu$ be a non-empty fuzzy subset of $X$ and $\alpha \in [0, 1 - \sup \{\mu(x) / x \in X\}]$ and $\beta \in [0, 1]$. A mapping $\mu^c_{\beta\alpha} : X \to [0, 1]$ is called a fuzzy magnified translation of $\mu$ if $\mu^c_{\beta\alpha}(x) = \beta \mu(x) + \alpha, \forall x \in X$.

**Definition 2.5**: A vague set $A$ in the universe of discourse $U$ is a pair $(t_A, f_A)$, where $t_A : U \to [0, 1]$ and $f_A : U \to [0, 1]$ are mappings such that $t_A(u) + f_A(u) \leq 1, \forall u \in U$. The functions $t_A$ and $f_A$ are called true membership function and false membership function respectively.

**Definition 2.6**: A vague set $A$ of a $\Gamma$-semiring $R$ is called a constant vague set if $V_A(x) = V_A(y), \forall x, y \in R$.

**Definition 2.7[1]**: A vague set $A = (t_A, f_A)$ on $R$ is said to be vague $\Gamma$-semiring
if the following conditions are true:

For all \( x, y \in R; \gamma \in \Gamma \),
\[
V_A(x + y) \geq \min\{V_A(x), V_A(y)\} \quad \text{and} \quad V_A(x\gamma y) \geq \min\{V_A(x), V_A(y)\}
\]
i.e.,

(i). \( t_A(x + y) \geq \min\{t_A(x), t_A(y)\} \),
(ii) \( t_A(x\gamma y) \geq \min\{t_A(x), t_A(y)\} \).
\[
1 - f_A(x + y) \geq \min\{1 - f_A(x), 1 - f_A(y)\} \quad \text{and} \quad 1 - f_A(x\gamma y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}.
\]

**Definition 2.8[2]**: A vague set \( A = (t_A, f_A) \) on \( R \) is said to be left(resp. right) vague ideal of \( R \) if the following conditions are true:

For all \( x, y \in R; \gamma \in \Gamma \),
\[
V_A(x + y) \geq \min\{V_A(x), V_A(y)\} \quad \text{and} \quad V_A(x\gamma y) \geq V_A(y) \quad \text{(resp. } V_A(x\gamma y) \geq V_A(x)\text{)}
\]
i.e.,

(i). \( t_A(x + y) \geq \min\{t_A(x), t_A(y)\} \),
(ii) \( t_A(x\gamma y) \geq t_A(y)(t_A(x\gamma y) \geq t_A(x)) \),
\[
1 - f_A(x + y) \geq \min\{1 - f_A(x), 1 - f_A(y)\} \quad \text{and} \quad 1 - f_A(x\gamma y) \geq 1 - f_A(y) \quad \text{(resp. } 1 - f_A(x\gamma y) \geq 1 - f_A(x)\text{)}.
\]

**Definition 2.9[3]**: A vague \( \Gamma \)-semiring \( A = (t_A, f_A) \) of \( R \) is said to be vague bi-ideal of \( R \) if for all \( x, y, z \in R; \alpha, \beta \in \Gamma \),
\[
V_A(x\alpha y\beta z) \geq \min\{V_A(x), V_A(z)\}
\]
i.e.,
\[
t_A(x\alpha y\beta z) \geq \min\{t_A(x), t_A(z)\},
\]
\[
1 - f_A(x\alpha y\beta z) \geq \min\{1 - f_A(x), 1 - f_A(z)\}.
\]

**Definition 2.10[3]**: A vague set \( A = (t_A, f_A) \) of \( R \) is said to be vague quasi ideal of \( R \) if for all \( x, y \in R \),
1. \( V_A(x + y) \geq \min\{V_A(x), V_A(y)\} \)
2. \((A\Gamma\delta) \cap (\delta\Gamma A) \subseteq A\), where \( \delta \) is a vague characteristic set of \( R \).

3. Vague Magnified Translation of a Vague set

We introduce the concept of vague magnified translation of a vague set in \( \Gamma \)-semiring. We prove that, if \( A \) is a left(resp. right) vague ideal of a \( \Gamma \)-semiring \( R \) then the vague magnified translation \( A_{\delta\alpha}^\gamma \) of \( A \) is a vague bi-ideal of \( R \) and if \( A \) is a left(resp. right) vague ideal of a left(resp. right) zero \( \Gamma \)-semiring \( R \), then \( A_{\delta\alpha}^\gamma \) is a constant vague set.
We begin with the following.

**Definition 3.1:** Let $A$ be a non-empty vague set of $R$ and $\alpha \in [0, 1 - \sup\{t_A(x) + f_A(x)/ x \in R\}]$ and $\beta \in [0, 1]$. The vague magnified translation of $A$, $A^c_{\beta \alpha}$ is a pair $(t_{A^c_{\beta \alpha}}, f_{A^c_{\beta \alpha}})$, where $t_{A^c_{\beta \alpha}} : R \rightarrow [0, 1]$ and $f_{A^c_{\beta \alpha}} : R \rightarrow [0, 1]$ are mappings such that $t_{A^c_{\beta \alpha}}(x) = \beta t_A(x) + \alpha$ and $f_{A^c_{\beta \alpha}}(x) = \beta f_A(x) - \alpha$, $\forall x \in R$.

**Verification 3.2:** Vague magnified translation is also a vague set.

Let $A = (t_A, f_A)$ be a vague set of $R$.

Let $\alpha \in [0, 1 - \sup \{t_A(x) + f_A(x)/ x \in R\}]$ and $\beta \in [0, 1]$.

The vague magnified translation of $A$ is $A^c_{\beta \alpha} = (t_{A^c_{\beta \alpha}}, f_{A^c_{\beta \alpha}})$.

Let $x \in R$.

Now, $t_{A^c_{\beta \alpha}}(x) + f_{A^c_{\beta \alpha}}(x) = \beta t_A(x) + \alpha + \beta f_A(x) - \alpha$

$= \beta [t_A(x) + f_A(x)] \leq 1$.

Thus $A^c_{\beta \alpha}$ is a vague set.

**Example 3.3:** Let $R$ be the set of natural numbers including zero and $\Gamma$ be the set of positive even integers.

Define $a \gamma b = a \cdot \gamma \cdot b$, where \('\cdot\)' is the usual multiplication on $R$, for all $a, b \in R; \gamma \in \Gamma$.

Therefore $R$ is a $\Gamma$-semiring.

Let $A = (t_A, f_A)$, where $t_A : R \rightarrow [0, 1]$ and $f_A : R \rightarrow [0, 1]$ such that

$t_A(x) = \begin{cases} 
0.8 & \text{if } x = 0 \\
0.6 & \text{if } x \text{ is even} \\
0.4 & \text{if } x \text{ is odd}
\end{cases}$

and $f_A(x) = \begin{cases} 
0.2 & \text{if } x = 0 \\
0.3 & \text{if } x \text{ is even} \\
0.5 & \text{if } x \text{ is odd}
\end{cases}$

Therefore $A$ is a vague set.

Now, $A^c_{\beta \alpha} = (t_{A^c_{\beta \alpha}}, f_{A^c_{\beta \alpha}})$, where $\beta \in [0, 1]$ and $\alpha \in [0, 1 - \sup\{1, 0.9, 0.9\}] = [0, 1-1] = 0$.

put $\beta = 0.4$

Then

$t_{A^c_{\beta \alpha}}(x) = \begin{cases} 
0.32 & \text{if } x = 0 \\
0.24 & \text{if } x \text{ is even} \\
0.16 & \text{if } x \text{ is odd}
\end{cases}$

and $f_{A^c_{\beta \alpha}}(x) = \begin{cases} 
0.08 & \text{if } x = 0 \\
0.12 & \text{if } x \text{ is even} \\
0.2 & \text{if } x \text{ is odd}
\end{cases}$

Therefore $A^c_{\beta \alpha} = (t_{A^c_{\beta \alpha}}, f_{A^c_{\beta \alpha}})$ is a vague set.

**Theorem 3.4:** Let $A = (t_A, f_A)$ and $B = (t_B, f_B)$ be two vague sets of $R$. Then
2. proof of 2 follows from 1.

\[ f_{(A \cap B)}_{\beta \alpha} (x) = \beta f_{A \cap B}(x) - \alpha \]
\[ = \beta \max \{f_A(x), f_B(x)\} - \alpha \]
\[ = \max \{\beta f_A(x) - \alpha, \beta f_B(x) - \alpha\} \]
\[ = \max \{f_{A_{\beta \alpha}}(x), f_{B_{\beta \alpha}}(x)\} \]
\[ = f_{A_{\beta \alpha} \cap B_{\beta \alpha}}(x). \]
Hence \((A \cap B)_{\beta \alpha} = A_{\beta \alpha} \cap B_{\beta \alpha}.\)

2. proof of 2 follows from 1. \(\square\)

**Theorem 3.5:** Let \(A = (t_A, f_A)\) be a vague set of \(R\). Then \(A\) is a vague \(\Gamma\)-semiring of \(R\) if and only if the vague magnified translation of \(A, A_{\beta \alpha}^e\) is vague \(\Gamma\)-semiring of \(R\).

**Proof.** Suppose \(A\) is a vague \(\Gamma\)-semiring of \(R\).
Let \(x, y \in R; \gamma \in \Gamma.\)
Now, \(t_{A_{\beta \alpha}}^e (x + y) = \beta t_A(x + y) + \alpha \geq \beta \min \{t_A(x), t_A(y)\} + \alpha = \min \{\beta t_A(x) + \alpha, \beta t_A(y) + \alpha\} \}
\[ = \min \{t_{A_{\beta \alpha}}^e (x), t_{A_{\beta \alpha}}^e (y)\} \]
and
\[ f_{A_{\beta \alpha}}^e (x + y) = \beta f_A(x + y) - \alpha \leq \beta \max \{f_A(x), f_A(y)\} - \alpha = \max \{\beta f_A(x) - \alpha, \beta f_A(y) - \alpha\} = \max \{f_{A_{\beta \alpha}}^e (x), f_{A_{\beta \alpha}}^e (y)\}. \]
Similarly, we can prove that \(t_{A_{\beta \alpha}}^e (x \gamma y) \geq \min \{t_{A_{\beta \alpha}}^e (x), t_{A_{\beta \alpha}}^e (y)\} \}
\[ = \min \{f_{A_{\beta \alpha}}^e (x), f_{A_{\beta \alpha}}^e (y)\}. \]
Hence \(A_{\beta \alpha}^e\) is a vague \(\Gamma\)-semiring of \(R.\)
Conversely suppose that \(A_{\beta \alpha}^e\) is a vague \(\Gamma\)-semiring of \(R.\)
Let \(x, y \in R; \gamma \in \Gamma.\)
Now, \(t_A(x + y) = \frac{1}{\beta}(t_{A_{\beta \alpha}}^e (x + y) - \alpha) \geq \frac{1}{\beta}(\min \{t_{A_{\beta \alpha}}^e (x), t_{A_{\beta \alpha}}^e (y)\} - \alpha) = \frac{1}{\beta}(\min \{t_{A_{\beta \alpha}}^e (x) - \alpha, t_{A_{\beta \alpha}}^e (y) - \alpha\} = \min \{\frac{1}{\beta}(t_{A_{\beta \alpha}}^e (x) - \alpha), \frac{1}{\beta}(t_{A_{\beta \alpha}}^e (y) - \alpha)\} = \min \{t_A(x), t_A(y)\} \}
and
\[ f_A(x+y) = \frac{1}{\beta}(f_{A_{\beta\alpha}}(x+y)+\alpha) \leq \frac{1}{\beta}(\max\{f_{A_{\beta\alpha}}(x), f_{A_{\beta\alpha}}(y)\} + \alpha) = \frac{1}{\beta}(\max\{f_{A_{\beta\alpha}}(x) + \alpha, f_{A_{\beta\alpha}}(y) + \alpha\}) = \max\{\frac{1}{\beta}(f_{A_{\beta\alpha}}(x) + \alpha), \frac{1}{\beta}(f_{A_{\beta\alpha}}(y) + \alpha)\} = \max\{f_A(x), f_A(y)\}. \]

Similarly we can prove that \( t_A(x\gamma y) \geq \min\{t_A(x), t_A(y)\} \) and \( f_A(x\gamma y) \leq \max\{f_A(x), f_A(y)\}. \)

Hence \( A \) is a vague \( \Gamma \)-semiring of \( R \).

The following two theorems follows theorem:3.5.

**Theorem 3.6:** Let \( A = (t_A, f_A) \) be a vague set of \( R \). Then \( A \) is a left(resp. right) vague ideal of \( R \) if and only if the vague magnified translation of \( A, A_{\beta\alpha}^c \) is left(right) vague ideal of \( R \).

**Theorem 3.7:** Let \( A = (t_A, f_A) \) be a vague set of \( R \). Then \( A \) is a vague bi-ideal of \( R \) if and only if the vague magnified translation of \( A, A_{\beta\alpha}^c \) is vague bi-ideal of \( R \).

**Theorem 3.8:** If \( A \) is a left(resp. right) vague ideal of \( R \), then \( A_{\beta\alpha}^c \) is a vague bi-ideal of \( R \).

**Proof.** : Let \( x, y, z \in R; \gamma, \eta \in \Gamma \).

1. \( t_{A_{\beta\alpha}^c}(x+y) = \beta t_A(x+y) + \alpha \geq \beta \min\{t_A(x), t_A(y)\} + \alpha = \min\{\beta t_A(x) + \alpha, \beta t_A(y) + \alpha\} = \min\{t_{A_{\beta\alpha}^c}(x), t_{A_{\beta\alpha}^c}(y)\}. \)
2. \( t_{A_{\beta\alpha}^c}(x\gamma y) = \beta t_A(x\gamma y) + \alpha \geq \beta t_A(y) + \alpha \) (resp. \( \beta t_A(x) + \alpha \)) = \( t_{A_{\beta\alpha}^c}(y) \) (resp. \( t_{A_{\beta\alpha}^c}(x) \)) \( \geq \min\{t_{A_{\beta\alpha}^c}(x), t_{A_{\beta\alpha}^c}(y)\}. \)
3. \( t_{A_{\beta\alpha}^c}(x\gamma yz) = \beta t_A(x\gamma yz) + \alpha \geq \beta t_A(z) + \alpha \) (resp. \( \beta t_A(x) + \alpha \)) = \( t_{A_{\beta\alpha}^c}(z) \) (resp. \( t_{A_{\beta\alpha}^c}(x) \)) \( = \min\{t_{A_{\beta\alpha}^c}(x), t_{A_{\beta\alpha}^c}(y)\}. \)

Similarly we can prove \( f_{A_{\beta\alpha}^c}(x+y) \leq \max\{f_{A_{\beta\alpha}^c}(x), f_{A_{\beta\alpha}^c}(y)\} \), \( f_{A_{\beta\alpha}^c}(x\gamma y) \leq \max\{f_{A_{\beta\alpha}^c}(x), f_{A_{\beta\alpha}^c}(y)\} \) and \( f_{A_{\beta\alpha}^c}(x\gamma yz) \leq \max\{f_{A_{\beta\alpha}^c}(x), f_{A_{\beta\alpha}^c}(z)\}. \)

Hence \( A_{\beta\alpha}^c \) is a vague bi-ideal of \( R \).

**Theorem 3.9:** The vague magnified translation of the intersection of an arbitrary collection of vague bi-ideals of \( R \) is a vague bi-ideal of \( R \) if it is not empty.

**Proof.** : Let \( A \) be the intersection of arbitrary collection of vague bi-ideals of \( R \).

We have arbitrary collection of vague bi-ideals of \( R \) is a vague bi-ideal of \( R \).

Hence from theorem:3.7, \( A_{\beta\alpha}^c \) is a vague bi-ideal of \( R \).
Hence \( A = f \)

Similarly we can prove \( B \geq \inf = \inf \).

Again \( \min \geq \sup = \sup \).

Now, \( t_A(x_1a_y2p_3x) \geq \sup \{ t_A(x_1a_y2p_3x), t_B(x_5q_y1a_y2x) \} \)

\( = \sup \{ \beta t_A(x_1a_y2p_3x) + \alpha, \beta t_B(x_5q_y1a_y2x) + \alpha \} \)

\( = \sup \{ \min \{ t_A(x) + \alpha, \beta t_B(x) + \alpha \} \} \)

\( = t_A(\alpha) \cap \beta t_B(\alpha) \).

Again \( f_A(\alpha) \cap \beta f_B(x) \)

\( = \inf \{ \max \{ f_A(x_1a_y2p_3x), f_B(x_5q_y1a_y2x) \} \} \)

\( = \inf \{ \max \{ \beta f_A(x_1a_y2p_3x) - \alpha, \beta f_B(x_5q_y1a_y2x) - \alpha \} \} \)

\( = \max \{ f_A(x), f_B(x) \} \)

\( = f_A(\alpha) \cap \beta f_B(x) \).

Hence \( A \geq A \cap B \).

Similarly we can prove \( B \geq A \).

Combining these two, we get \( A(\cap B) \cap (B \cap A) \geq A \cap B \).

\[ \square \]

**Theorem 3.10:** Let \( R \) be a regular and intra regular \( \Gamma \)-semiring. Then
1. \( A \cap B \subseteq A \cap B \)
2. \( (A \cap B) \cap (B \cap A) \)

where \( A = (t_A, f_A), B = (t_B, f_B) \) are vague bi-ideals of \( R \).

**Proof.** : Let \( x \in R \).

Since \( R \) is regular and intra regular, we have

\[ x = x_1a_y2x \]

That implies \( x = x_1a_y2x \)

\[ = x_1a_y2(p_3xq_4x_5q_1a_y2x) \]

\[ = (x_1a_y2p_3x)q_4(x_5q_y1a_y2x). \]

Now, \( t_A(x_1a_y2p_3x) \geq \min \{ t_A(x), t_A(x) \} = t_A(x) \) and \( t_B(x_5q_y1a_y2x) \geq \min \{ t_B(x), t_B(x) \} = t_B(x) \).

Now, \( t_A(\alpha) \cap \beta t_B(\alpha) \) be a vague set of \( R \).

\[ = \inf \{ \max \{ f_A(x_1a_y2p_3x), f_B(x_5q_y1a_y2x) \} \} \]

\( = \inf \{ \max \{ \beta f_A(x_1a_y2p_3x) - \alpha, \beta f_B(x_5q_y1a_y2x) - \alpha \} \} \)

\( = \max \{ f_A(x), f_B(x) \} \)

\( = f_A(\alpha) \cap \beta f_B(x) \).

Hence \( A \geq A \).

Similarly we can prove \( B \geq A \).

Combining these two, we get \( A(\cap B) \cap (B \cap A) \geq A \cap B \).

\[ \square \]

**Theorem 3.11:** Let \( A = (t_A, f_A) \) be a vague set of \( R \). Then \( A \) is a vague quasi ideal of \( R \) if and only if the vague magnified translation of \( A, A \) is vague quasi ideal of \( R \).

**Proof.** : Suppose \( A \) is a vague quasi ideal of \( R \).

Let \( x, y \in R \).

Now, \( t_A(x + y) \geq \min \{ t_A(x), t_A(y) \} \) and \( f_A(x + y) \leq \max \{ f_A(x), f_A(y) \} \)
Now, \( t_{(A_{\beta\alpha}^c, \Gamma \delta) \cap (\delta \Gamma A_{\beta\alpha}^c})(x) = \min \{t_{A_{\beta\alpha}^c, \Gamma \delta}(x), t_{\delta \Gamma A_{\beta\alpha}^c}(x)\} \)
\[ = \min \{\sup \{\min \{t_{A_{\beta\alpha}^c}(a), t_{\delta}(b)\}\}, \sup \{\min \{t_{\delta}(a), t_{A_{\beta\alpha}^c}(b)\}\}/x = a\gamma b\} \]
\[ = \min \{\beta t_A(a) + \alpha, \beta t_A(b) + \alpha\} \]
\[ = \beta \min \{t_A(a), t_A(b)\} + \alpha \]
\[ = \beta t_{(\Lambda \Gamma \delta) \cap (\delta \Gamma A)}(x) + \alpha \]
\[ \leq \beta t_A(x) + \alpha \]
\[ = t_{A_{\beta\alpha}^c}(x). \]
Therefore \( (A_{\beta\alpha}^c, \Gamma \delta) \cap (\delta \Gamma A_{\beta\alpha}^c) \subseteq A_{\beta\alpha}^c. \)
Hence \( A_{\beta\alpha}^c \) is a vague quasi ideal of \( R. \)

Conversely suppose that \( A_{\beta\alpha}^c \) is a vague quasi ideal of \( R. \)
Let \( x, y \in R. \)
Then \( t_A(x + y) \geq \min \{t_A(x), t_A(y)\} \) and \( f_A(x + y) \leq \max \{f_A(x), f_A(y)\}. \)
Now, \( t_{(\Lambda \Gamma \delta) \cap (\delta \Gamma A)}(x) = \min \{t_{\Lambda \Gamma \delta}(x), t_{\delta \Gamma A}(x)\} \)
\[ = \min \{\sup \{\min \{t_A(a), t_{\delta}(b)\}\}, \sup \{\min \{t_{\delta}(a), t_{A_{\beta\alpha}^c}(b)\}\}/x = a\gamma b\} \]
\[ = \min \{\beta t_A(a), t_A(b)\} \]
\[ = \beta t_{A_{\beta\alpha}^c}(x) - \alpha \]
\[ = t_A(x). \]
Therefore \( (\Lambda \Gamma \delta) \cap (\delta \Gamma A) \subseteq A. \)
Hence \( A \) is a vague quasi ideal of \( R. \)

\[ \Box \]

**Theorem 3.12:** Let \( A \) be a left(resp. right) vague ideal of a left(right) zero \( \Gamma \)-semiring \( R. \)Then \( A_{\beta\alpha}^c \) is a constant vague set.

**Proof.** : Let \( x, y \in R; \gamma \in \Gamma. \)
Since \( R \) is a left zero \( \Gamma \)-semiring, we have \( x\gamma y = x \) and \( y\gamma x = y. \)
Now, \( t_{A_{\beta\alpha}^c}(x) = \beta t_A(x) + \alpha = \beta t_A(x\gamma y) + \alpha \geq \beta t_A(y) + \alpha \)
Again \( t_{A_{\beta\alpha}^c}(y) = \beta t_A(y) + \alpha = \beta t_A(y\gamma x) + \alpha \geq \beta t_A(x) + \alpha \)
Therefore \( t_{A_{\beta\alpha}^c}(x) = t_{A_{\beta\alpha}^c}(y) \)
Similarly, \( f_{A_{\beta\alpha}^c}(x) = f_{A_{\beta\alpha}^c}(y) \)
Thus \( A_{\beta\alpha}^c \) is a constant vague set.
Similarly we can prove other case also.

\[ \Box \]
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