TRANSLATES OF VAGUE β -ALGEBRA

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Abstract: In this paper we introduce translate operators on vague β – algebra, vague β – ideals and vague β – ideal extension, also studied their properties. These concepts are used in the development of some important results and theorems about translate of vague β – algebra and vague β – ideal extension. Also some of their important properties have been investigated.

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1. Introduction

BCK and BCI algebras are two classes of logical algebras. They were introduced by Imai and Iseki [5, 15] and have been extensively investigated by many Researchers. BCI algebras are generalization of BCK algebras. In 2002, J.

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Neggras and H. S. Kim introduced the notation of β -algebras (see [9]) which is another generalization of BCK-algebras, see [5]. A β -algebra $\mathfrak{F}$ is a non-empty set with constant 0 and two binary operations $+$ and $-$ satisfying the following axioms:

- (1) $x - 0 = x$,
- (2) $(0 - x) + x = 0$, and
- (3) $(x - y) - z = x - (z + y)$ for all $x, y, z \in \mathfrak{F}$.

In 1965 The concept proposed by L.A. Zadeh (see [16]) defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function μ_A defined from X into $[0, 1]$ has revolutionized the theory of Mathematical modeling, Decision making etc. , in handling the imprecise real life situations mathematically.

Several generalizations of Zadeh's fuzzy set theory have been proposed, such as Interval valued fuzzy sets, intuitionistic fuzzy sets by Atanassov [1], vague sets [4] are mathematically equivalent. Any such set A of a given Universe X can be characterized by means of a pair of function (t_A, f_A) where t_A and f_A are function from X into $[0, 1]$ such that $0 \leq t_A(x) + f_A(x) \leq 1$ for all x in X. The set t_A is called the truth function and the set f_A is called false function or non membership function and $t_A(x)$ gives the evidence of how much $x \in A$ $f_A(x)$ gives the evidence of how much x does not $\in A$. Gau. W. L and Bueher D. J. [4] have initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague, than the theory of Fuzzy sets do. Ranjit Biswas [13] initiated the study of vague groups and Ramakrishna. N [12] T. Eswarlal (see [3]), are extended the study of vague algebra. The objective of this paper is to contribute further to the study of vague Algebra by introducing concepts of translate operators on vague β -algebra, vague β -ideals and vague β -ideal extension.

2. Preliminaries

We give here a review of some definitions and results which are in W.L. Gau and Buehrer, see [4], Ranjit Biswas [13], N. Ramakrishna [10], [11], [12].

Definition 1.1. (see [3]) Let S be a non empty set of a vague set A is said to be vague semi ring if:

- (1) $t_A(x + y) \geq \min\{t_A(x), t_A(y)\}$;
- (2) $f_A(x + y) \leq \max\{f_A(x), f_A(y)\}$;

$$(3) \ t_A(xy) \geq \min\{t_A(x), t_A(y)\};$$

$$(4) \ f_A(xy) \leq \max\{f_A(x), f_A(y)\} \text{ for all } x, y \in S.$$

Definition 1.2. (see [3]) Let S be vague semi ring of a vague set A is said to be vague left ideal if:

$$(1) \ t_A(x - y) \geq \min\{t_A(x), t_A(y)\};$$

$$(2) \ f_A(x - y) \leq \max\{f_A(x), f_A(y)\};$$

$$(3) \ t_A(xy) \geq t_A(y);$$

$$(4) \ f_A(xy) \leq f_A(y) \text{ for all } x, y \in S.$$

Definition 1.3. (see [3]) Let S be vague semiring of a vague set A is said to be vague right ideal if:

$$(1) \ t_A(x - y) \geq \min\{t_A(x), t_A(y)\};$$

$$(2) \ f_A(x - y) \leq \max\{f_A(x), f_A(y)\};$$

$$(3) \ t_A(xy) \geq t_A(x);$$

$$(4) \ f_A(xy) \leq f_A(x) \text{ for all } x, y \in S.$$

Definition 1.4. (see [3]) Let S be vague semiring of a vague set A is said to be vague ideal if:

$$(1) \ t_A(x - y) \geq \min\{t_A(x), t_A(y)\};$$

$$(2) \ f_A(x - y) \leq \max\{f_A(x), f_A(y)\};$$

$$(3) \ t_A(xy) \geq \max\{t_A(x), t_A(y)\};$$

$$(4) \ f_A(xy) \leq \min\{f_A(x), f_A(y)\}, \text{ for all } x, y \in S.$$

Definition 1.5. (see [5]) A β -algebra $\mathfrak{F}$ is a non-empty set with constant 0 and two binary operations $+$ and $-$ satisfying the following axioms:

$$(1) \ x - 0 = x;$$

$$(2) \ (0 - x) + x = 0; \text{ and}$$

$$(3) \ (x - y) - z = x - (z + y) \text{ for all } x, y, z \in \mathfrak{F}.$$

Example 1.6. Let $\mathfrak{F} = \{0, 1, 2, 3\}$ is a β -algebra with constant 0 and two binary operations $+$ and $-$ are defined on $\mathfrak{F}$ by the following Cayley's tables.

+	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

−	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Definition 1.7. (see [9]) A non-empty subset A of a β -algebra $(\mathfrak{F}, +, -, 0)$ is called a β -subalgebra of $\mathfrak{F}$, if:

- (1) $x + y \in A$, for all $x, y \in A$, and
- (2) $x - y \in A$, for all $x, y \in A$.

Example 1.8. $\mathfrak{F} = \{0, 1, 2, 3\}$ is a β -algebra with constant 0 and two binary operations $+$ and $-$ are defined on $\mathfrak{F}$ by the Cayley's tables (1.6).

Example 1.9 Let $\mathfrak{F} = \{0, 1, 2, 3\}$ be a β -algebra with cayley's table. The subsets $\{0, 1\}$, $\{0, 2\}$ $\{0, 3\}$ are β -subalgebra of $\mathfrak{F}$. But the sub set $A = \{0, 1, 2\}$ is not a β -subalgebra of $\mathfrak{F}$. Since $1 + 2 = 3$ is not in A

3. Translate Operators on Vague β -algebra

We introduce the following definition.

Definition 2.1 Let $A = (t_A, f_A)$ be a vague set of β -algebra $\mathfrak{F}$. Then A is called a vague β -algebra of $\mathfrak{F}$ if:

- (a) $t_A(x + y) \geq \min\{t_A(x), t_A(y)\}$;
- (b) $f_A(x + y) \leq \max\{f_A(x), f_A(y)\}$;
- (c) $t_A(x - y) \geq \min\{t_A(x), t_A(y)\}$;
- (d) $f_A(x - y) \leq \max\{f_A(x), f_A(y)\}$ for all $x, y \in \mathfrak{F}$, where $0 \leq t_A(x) + f_A(x) \leq 1$.

Definition 2.2. Let $\mathfrak{F}$ be a β -algebra. Let and $A = (t_A, f_A)$ be a vague set of $\mathfrak{F}$ then the translation operators of vague β -algebra can be defined by:

- (a) $T_{\alpha+}(A) = (t_{AT_{\alpha+}}, f_{AT_{\alpha+}})$,
where $t_{AT_{\alpha+}}(x) = \min\{t_A(x) + \alpha, 1\}$ and $f_{AT_{\alpha+}}(x) = \max\{f_A(x) - \alpha, 0\}$.
- (b) $T_{\alpha-}(A) = (t_{AT_{\alpha-}}, f_{AT_{\alpha-}})$,
where $t_{AT_{\alpha-}}(x) = \max\{t_A(x) - \alpha, 0\}$ and $f_{AT_{\alpha-}}(x) = \min\{f_A(x) + \alpha, 1\}$.

In this case $T_{\alpha+}(A)$ is called translate of vague β -increasing operator and $T_{\alpha-}(A)$ is called translate of vague β -decreasing operator on vague β -algebra $\mathfrak{F}$ respectively.

Theorem 2.3. *If A, B are two vague sets of β -algebras of $\mathfrak{F}$. Then $T_{\alpha+}(A \cap B)$ and $T_{\alpha-}(A \cap B)$ are also translates of vague β -algebras of $\mathfrak{F}$.*

Proof. Let A and B are vague sets of a β -algebra of $\mathfrak{F}$, and Let x, y be any two elements in $\in \mathfrak{F}$ then

$$\begin{aligned} t_{(A \cap B)T_{\alpha+}}(x + y) &= \min\{t_{(A \cap B)}(x + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x + y) + \alpha, 1\}, \min\{t_B(x + y) + \alpha, 1\}, 1\} \\ &= \min\{\min\{t_A(x + y) + \alpha, 1\}, \min\{t_B(x + y) + \alpha, 1\}, 1\} \\ &= \min\{\min\{t_A(x), t_A(y)\}, \min\{t_B(x), t_B(y)\}\} \\ &= \min\{\min\{t_A(x), t_B(x)\}, \min\{t_A(y), t_B(y)\}\} \\ &= \min\{t_{A \cap B}(x), t_{A \cap B}(y)\} \text{ therefore} \end{aligned}$$

$$t_{(A \cap B)T_{\alpha+}}(x + y) \geq \min\{t_{A \cap B}(x), t_{A \cap B}(y)\} \dots (2.3.1)$$

$$\begin{aligned} t_{(A \cap B)T_{\alpha+}}(x - y) &= \min\{t_{(A \cap B)}(x - y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x - y) + \alpha, 1\}, \min\{t_B(x - y) + \alpha, 1\}, 1\} \\ &= \min\{\min\{t_A(x - y) + \alpha, 1\}, \min\{t_B(x - y) + \alpha, 1\}, 1\} \\ &= \min\{\min\{t_A(x), t_A(y)\}, \min\{t_B(x), t_B(y)\}\} \\ &= \min\{\min\{t_A(x), t_B(x)\}, \min\{t_A(y), t_B(y)\}\} \\ &= \min\{t_{A \cap B}(x), t_{A \cap B}(y)\} \text{ therefore} \end{aligned}$$

$$t_{(A \cap B)T_{\alpha+}}(x - y) \geq \max\{t_{A \cap B}(x), t_{A \cap B}(y)\} \dots (2.3.2)$$

$$\begin{aligned} f_{(A \cap B)T_{\alpha+}}(x + y) &= \max\{t_{(A \cap B)}(x + y) - \alpha, 0\} \\ &\leq \max\{\max\{t_A(x + y) - \alpha, 0\}, \max\{t_B(x + y) - \alpha, 0\}, 0\} \\ &= \max\{\max\{t_A(x + y) - \alpha, 0\}, \max\{t_B(x + y) - \alpha, 0\}, 0\} \\ &= \max\{\max\{t_A(x), t_A(y)\}, \max\{t_B(x), t_B(y)\}\} \\ &= \max\{\max\{t_A(x), t_B(x)\}, \max\{t_A(y), t_B(y)\}\} \\ &= \max\{t_{A \cap B}(x), t_{A \cap B}(y)\} \text{ therefore} \end{aligned}$$

$$t_{(A \cap B)T_{\alpha+}}(x + y) \leq \max\{t_{A \cap B}(x), t_{A \cap B}(y)\} \dots (2.3.3)$$

$$\begin{aligned} f_{(A \cap B)T_{\alpha+}}(x - y) &= \max\{f_{(A \cap B)}(x - y) - \alpha, 0\} \\ &\leq \max\{\max\{f_A(x - y) - \alpha, 0\}, \max\{f_B(x - y) - \alpha, 0\}, 0\} \\ &= \max\{\max\{f_A(x - y) - \alpha, 0\}, \max\{f_B(x - y) - \alpha, 0\}, 0\} \\ &= \max\{\max\{f_A(x), f_A(y)\}, \max\{f_B(x), f_B(y)\}\} \\ &= \max\{\max\{f_A(x), f_B(x)\}, \max\{f_A(y), f_B(y)\}\} \\ &= \max\{f_{A \cap B}(x), f_{A \cap B}(y)\} \text{ therefore} \end{aligned}$$

$$f_{(A \cap B)T_{\alpha+}}(x - y) \leq \max\{f_{A \cap B}(x), f_{A \cap B}(y)\} \dots (2.3.4)$$

Hence from (2.3.1), (2.3.2), (2.3.3) and (2.3.4) $T_{\alpha+}(A \cap B)$ is a translate of a vague β -algebra of $\mathfrak{F}$.

similarly, we have to prove that $t_{(A \cap B)T\alpha-}$ is a translate of a vague β -algebra of $\mathfrak{F}$.

Theorem 2.4. *If $T_{\alpha+}(A)$ is a translate of vague β -algebra on β -algebra $\mathfrak{F}$ then:*

$$(a) t_{AT\alpha+}(x) \leq t_{AT\alpha+}(0) \text{ and}$$

$$(b) f_{AT\alpha+}(x) \geq f_{AT\alpha+}(0).$$

Proof: (a) Let x, y be any elements, then we have

$$\begin{aligned} t_{AT\alpha+}(0) &= \min\{t_A(x - x) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x), t_A(x)\} + \alpha, 1\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}, 1\} \\ &= \min\{t_{AT\alpha+}(x), t_{AT\alpha+}(x)\} \geq t_{AT\alpha+}(x) \\ \text{therefore } t_{AT\alpha+}(x) &\leq t_{AT\alpha+}(0) \\ (b) f_{AT\alpha+}(0) &= \max\{f_A(x - x) - \alpha, 0\} \\ &\leq \max\{\max\{f_A(x), f_A(x)\} - \alpha, 0\} \\ &= \min\{\min\{f_A(x) + \alpha, 1\}, \min\{f_A(x) + \alpha, 1\}\} \\ &= \min\{\min\{f_A(x) + \alpha, 1\}, \min\{f_A(x) + \alpha, 1\}, 1\} \\ &= \min\{f_{AT\alpha+}(x), f_{AT\alpha+}(x)\} \leq f_{AT\alpha+}(x) \\ \text{therefore } f_{AT\alpha+}(x) &\geq f_{AT\alpha+}(0). \end{aligned}$$

Similarly we can get the following theorem.

Theorem 2.5. *If $T_{\alpha+}(A)$ is a translate of vague β -algebra on β -algebra $\mathfrak{F}$ then (a) $t_{AT\alpha+}(x) \leq t_{AT\alpha+}(x - 0)$ and (b) $f_{AT\alpha+}(x) \geq f_{AT\alpha+}(x - 0)$.*

Definition 2.6. Let $\phi : \mathfrak{F}_1 \rightarrow \mathfrak{F}_2$ be a function. Let A and B be two translates of vague β -algebras in $\mathfrak{F}_1$ and $\mathfrak{F}_2$ respectively. Then the inverse image of B under ϕ is defined by $T_{\alpha+}(\phi^{-1}(B)) = (\phi_{t_{BT\alpha+}}^{-1}(x), \phi_{f_{BT\alpha+}}^{-1}(x))$.

Theorem 2.7. *Let $\mathfrak{F}_1$ and $\mathfrak{F}_2$ be two β -algebras. Let A and B be two translates on vague β -algebras. Let $\phi : \mathfrak{F}_1 \rightarrow \mathfrak{F}_2$ be a homomorphism. If A is a translate on vague β -algebra of $\mathfrak{F}_2$. Then $\phi^{-1}(A)$ is a translates on β -algebra of $\mathfrak{F}_1$.*

Proof. Let A be translation on vague β -algebras of $\mathfrak{F}_1$ and $x, y \in \mathfrak{F}_2$. Then

$$\begin{aligned} \phi^{-1}(t_{AT\alpha+}(x + y)) &= \min\{\phi^{-1}(t_A)(x + y) + \alpha, 1\} \\ &\geq \min\{\min\{\phi^{-1}(t_A)(x), \phi^{-1}(t_A)(y)\} + \alpha, 1\} \\ &= \min\{\min\{\phi^{-1}(t_A)(x) + \alpha, 1\}, \min\{\phi^{-1}(t_A)(y) + \alpha, 1\} + \alpha, 1\} \\ &= \min\{\min\{\phi^{-1}(t_A)(x) + \alpha, 1\}, \min\{\phi^{-1}(t_A)(y) + \alpha, 1\}\} \\ &= \min\{\phi^{-1}(t_A)(x), \phi^{-1}(t_A)(y)\} \end{aligned}$$

therefore $\phi^{-1}(t_{AT_{\alpha+}})(x+y) \geq \min\{\phi^{-1}(t_A)(x), \phi^{-1}(t_A)(y)\} \dots (2.7.1)$
 $\phi^{-1}(t_{AT_{\alpha+}})(x-y) = \min\{(\phi^{-1}t_A)(x-y) + \alpha, 1\}$
 $\geq \min\{\min\{(\phi^{-1}t_A)(x), \phi^{-1}(t_A)(y)\} + \alpha, 1\}$
 $= \min\{\min\{\phi^{-1}(t_A)(x) + \alpha, 1\}, \min\{\phi^{-1}(t_A)(y) + \alpha, 1\} + \alpha, 1\}$
 $= \min\{\min\{\phi^{-1}(t_A)(x) + \alpha, 1\}, \min\{\phi^{-1}(t_A)(y) + \alpha, 1\}\}$
 $= \min\{\phi^{-1}(t_A)(x), \phi^{-1}(t_A)(y)\}$
 therefore $\phi^{-1}(t_{AT_{\alpha+}})(x-y) \leq \{\min\{\phi^{-1}(t_A)(x), \phi^{-1}(t_A)(y)\} \dots (2.7.2)$
 $\phi^{-1}(f_{AT_{\alpha+}})(x+y) = \max\{\phi^{-1}(f_A)(x+y) - \alpha, 0\}$
 $\leq \max\{\max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\} - \alpha, 0\}$
 $= \max\{\max\{\phi^{-1}(f_A)(x) - \alpha, 0\}, \max\{\phi^{-1}(f_A)(y) - \alpha, 0\} - \alpha, 0\}$
 $= \max\{\max\{\phi^{-1}(f_A)(x) - \alpha, 0\}, \min\{\phi^{-1}(f_A)(y) - \alpha, 0\}\}$
 $= \max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\}$
 therefore $\phi^{-1}(f_{T_{\alpha+}})(x+y) \leq \max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\} \dots (2.7.3)$
 $\phi^{-1}(f_{AT_{\alpha+}})(x-y) = \max\{\phi^{-1}(f_A)(x-y) - \alpha, 0\}$
 $\leq \max\{\max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\} - \alpha, 0\}$
 $= \max\{\max\{\phi^{-1}(f_A)(x) - \alpha, 0\}, \max\{\phi^{-1}(f_A)(y) - \alpha, 0\} - \alpha, 0\}$
 $= \max\{\max\{\phi^{-1}(f_A)(x) - \alpha, 0\}, \min\{\phi^{-1}(f_A)(y) - \alpha, 0\}\}$
 $= \max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\}$
 therefore $\phi^{-1}(f_{T_{\alpha+}})(x-y)$
 $\leq \max\{\phi^{-1}(f_A)(x), \phi^{-1}(f_A)(y)\} \dots (2.7.4)$
 Hence from (2.7.1), (2.7.2), (2.7.3) and (2.7.4) $\phi^{-1}(A)$
 is a translates of vague β -algebra of $\mathfrak{F}_1$.

Theorem 2.8. Let $\mathfrak{F}_1$ and $\mathfrak{F}_2$ be two β -algebras. Let A and B be two translates on vague β -algebras. Let $\phi : \mathfrak{F}_1 \rightarrow \mathfrak{F}_2$ be a epimorphism. If A is translate on vague β -algebra of β -algebra $\mathfrak{F}_1$. Then $\phi(A)$ is vague translates on β -algebra of $\mathfrak{F}_2$.

Proof. Let A be translate on vague β -algebras of $\mathfrak{F}_2$ and $x, y \in \mathfrak{F}_2$. Then $\phi(t_{AT_{\alpha+}}(x+y)) = \min\{\phi(t_A)(x+y) + \alpha, 1\}$
 $\geq \min\{\min\{\phi(t_A)(x), \phi(t_A)(y)\} + \alpha, 1\}$
 $= \min\{\min\{\phi(t_A)(x) + \alpha, 1\}, \min\{\phi(t_A)(y) + \alpha, 1\} + \alpha, 1\}$
 $= \min\{\min\{\phi(t_A)(x) + \alpha, 1\}, \min\{\phi(t_A)(y) + \alpha, 1\}\}$
 $= \{\min\{\phi(t_A)(x), \phi(t_A)(y)\} \text{ therefore } \phi(t_{AT_{\alpha+}})(x+y)$
 $\geq \min\{\phi(t_A)(x), \phi(t_A)(y)\} \dots (2.8.1)$
 $\phi(t_{AT_{\alpha+}})(x-y) = \min\{(\phi)t_A(x-y) + \alpha, 1\}$
 $\geq \min\{\min\{(\phi)t_A(x), \phi(t_A)(y)\} + \alpha, 1\}$
 $= \min\{\min\{\phi(t_A)(x) + \alpha, 1\}, \min\{\phi(t_A)(y) + \alpha, 1\} + \alpha, 1\}$
 $= \min\{\min\{\phi(t_A)(x) + \alpha, 1\}, \min\{\phi(t_A)(y) + \alpha, 1\}\}$
 $= \min\{\phi(t_A)(x), \phi(t_A)(y)\}$
 therefore $\phi(t_{AT_{\alpha+}})(x-y) \leq \{\min\{\phi(t_A)(x), \phi(t_A)(y)\} \dots (2.8.2)$

$$\begin{aligned}
\phi(f_{AT_{\alpha+}})(x+y) &= \max\{\phi(f_A)(x+y) - \alpha, 0\} \\
&\leq \max\{\max\{\phi(f_A)(x), \phi(f_A)(y)\} - \alpha, 0\} \\
&= \max\{\max\{\phi(f_A)(x) - \alpha, 0\}, \max\{\phi(f_A)(y) - \alpha, 0\} - \alpha, 0\} \\
&= \max\{\max\{\phi(f_A)(x) - \alpha, 0\}, \min\{\phi(f_A)(y) - \alpha, 0\}\} \\
&\quad \{\max\{\phi(f_A)(x), \phi(f_A)(y)\} \\
\text{therefore } \phi(f_{T_{\alpha+}})(x+y) &\leq \max\{\phi(f_A)(x), \phi(f_A)(y)\} \dots (2.8.3) \\
\phi(f_{AT_{\alpha+}})(x-y) &= \max\{\phi(f_A)(x-y) - \alpha, 0\} \\
&\leq \max\{\max\{\phi(f_A)(x), \phi(f_A)(y)\} - \alpha, 0\} \\
&= \max\{\max\{\phi(f_A)(x) - \alpha, 0\}, \max\{\phi(f_A)(y) - \alpha, 0\} - \alpha, 0\} \\
&= \max\{\max\{\phi^{-1}(f_A)(x) - \alpha, 0\}, \min\{\phi^{-1}(f_A)(y) - \alpha, 0\}\} \\
&= \max\{\phi(f_A)(x), \phi(f_A)(y)\} \\
\text{therefore } \phi(f_{T_{\alpha+}})(x-y) & \\
&\leq \max\{\phi(f_A)(x), \phi(f_A)(y)\} \dots (2.8.4)
\end{aligned}$$

Hence from (2.8.1), (2.8.2), (2.8.3) and (2.8.4) $\phi(A)$ is translate of β -algebra of $\mathfrak{F}_2$.

4. Translate Operators on Vague β -Ideals of β -algebras

Definition 3. 1 [3] A non-empty sub- set I of a β - algebra $(\mathfrak{F}, +, -, 0)$ is called a β - Ideal of $\mathfrak{F}$, if

1. $0 \in I$
2. $x + y \in I$
3. if $x - y$ and $y \in I$ then $x \in I$, for all $x, y \in \mathfrak{F}$.

We introduce the following:

Definition 3. 2 Let $A = (t_A, f_A)$ be a vague set in a β - algebra $\mathfrak{F}$. Then A is called vague β - Ideal of $\mathfrak{F}$, if

1. $t_A(0) \geq t_A(x)$ for all $x \in \mathfrak{F}$
2. $t_A(x + y) \geq \min\{t_A(x), t_A(y)\}$ for all $x \in \mathfrak{F}$ and
3. $t_A(x) \geq \min\{t_A(x - y), t_A(y)\}$ for all $x, y \in \mathfrak{F}$.

Example 3. 3 Consider the β -algebra $(\mathfrak{F}, +, -, 0)$ Where $\mathfrak{F} = \{0, 1, 2, 3\}$ with cayley's table (1. 6) and the vague set $A = \{(0, 0.7, 0.1), (1, 0.4, 0.3), (2, 0.5, 0.4), (3, 0.2, 0.1)\}$ is a vague β - ideal of $\mathfrak{F}$.

Definition 3. 4 Let A and B are two vague sets of a β -algebra $\mathfrak{F}$. If $t_A(x) \leq t_B(x)$ and $f_A(x) \geq f_B(x)$ for all $x \in \mathfrak{F}$. Then B is a vague extension of A.

Example 3. 5 Consider the β -algebra $(\mathfrak{F}, +, -, 0)$ Where $\mathfrak{F} = \{0, 1, 2, 3\}$ with cayley's table (1. 6) and the vague set $A = \{(0, 0.7, 0.1), (1, 0.4, 0.3), (2, 0.5, 0.3), (3, 0.2, 0.4)\}$ is a vague β -ideal of $\mathfrak{F}$. $B = \{(0, 0.8, 0.01), (1, 0.5, 0.2), (2, 0.6, 0.2), (3, 0.3, 0.2)\}$ is a vague β -ideal of $\mathfrak{F}$. Then B is a vague extension of A.

Theorem 3.6. Let A be a vague β -ideal of a β -algebra $\mathfrak{F}$. then $T_{\alpha+}(A)$ is also vague β -ideal of β -algebra $\mathfrak{F}$.

Proof. Assume that Let A be a vague β -ideal β -algebra $\mathfrak{F}$ and $x, y \in \mathfrak{F}$.

(a) Let x, y be any elements , then we have

$$\begin{aligned} t_{AT_{\alpha+}}(0) &= \min\{t_A(x - x) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x), t_A(x)\} + \alpha, 1\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha\}, 1\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}, 1\} \\ &= \min\{t_{AT_{\alpha+}}(x), t_{AT_{\alpha+}}(x)\} \geq t_{AT_{\alpha+}}(x) \\ &= t_{AT_{\alpha+}}(x) \leq t_{AT_{\alpha+}}(0) \end{aligned}$$

$$\text{therefore } t_{AT_{\alpha+}}(0) \geq t_{AT_{\alpha+}}(x) \dots (3.6.1)$$

$$\begin{aligned} (b) t_{AT_{\alpha+}}(x + y) &= \min\{t_A(x + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x), t_A(y)\} + \alpha, 1\} \\ &= \min\{\{t_A(x) + \alpha\}, \{t_A(y) + \alpha\}\} \\ &= \min\{\{t_A(x)\}, \{t_A(y)\}\} \end{aligned}$$

$$\text{Therefore } t_{AT_{\alpha+}}(x + y) \geq \min\{t_A(x), t_A(y)\} \dots (3.6.2)$$

$$\begin{aligned} (c) t_{AT_{\alpha+}}(x) &= \min\{t_A(x - y + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x - y), t_A(y)\} + \alpha, 1\} \\ &= \min\{\{t_A(x - y) + \alpha, 1\}, \{t_A(y) + \alpha, 1\}\} \\ &= t_{AT_{\alpha+}}(x - y), t_{AT_{\alpha+}}(y) \end{aligned}$$

$$\text{Therefore } t_{AT_{\alpha+}}(x) \geq \min\{t_{AT_{\alpha+}}(x - y), t_{AT_{\alpha+}}(y)\} \dots (3.6.3)$$

Hence from (3.6.1), (3.6.2) and (3.6.3)

$T_{\alpha+}(A)$ is a vague β -ideal of β -algebra $\mathfrak{F}$.

Remark 3. 7 In general for any vague set A of $\mathfrak{F}$, the translate of vague β -ideal $T_{\alpha+}(A)$ of A need not be a vague β -ideal of $\mathfrak{F}$ for all $\alpha \in [0, 1]$, as shown in the following example.

Example 3. 8 Consider a β -algebra $(\mathfrak{F}, +, -, 0)$ where $\mathfrak{F} = \{0, 1, 2, 3\}$ with

the cayley's table (1. 6) the vague set

$A = \{(0, 0.8, 0.1), (1, 0.5, 0.4), (2, 0.6, 0.3), (3, 0.7, 0.2)\}$ and $\alpha \in [0, 1]$

then $t_{AT_{\alpha+}}(x) = \min\{t_A(x) + \alpha, 1\}$ $f_{AT_{\alpha+}}(x) = \max\{f_A(x) - \alpha, 0\}$

$t_{AT_{\alpha+}}(0) = \min\{t_A(0) + 0.1, 1\} = \min\{0.8 + 0.1, 1\} = 0.9$

$f_{AT_{\alpha+}}(0) = \max\{f_A(0) - 0.1, 0\} = \max\{0.1 - 0.1, 0\} = 0$

$t_{AT_{\alpha+}}(1) = \min\{t_A(1) + 0.1, 1\} = \min\{0.5 + 0.1, 1\} = 0.6$

$f_{AT_{\alpha+}}(1) = \max\{f_A(1) - 0.1, 0\} = \max\{0.4 - 0.1, 0\} = 0.3$

$t_{AT_{\alpha+}}(2) = \min\{t_A(2) + 0.1, 1\} = \min\{0.6 + 0.1, 1\} = 0.7$

$f_{AT_{\alpha+}}(2) = \max\{f_A(2) - 0.1, 0\} = \max\{0.3 - 0.1, 0\} = 0.2$

$t_{AT_{\alpha+}}(3) = \min\{t_A(3) + 0.1, 1\} = \min\{0.7 + 0.1, 1\} = 0.8$

$f_{AT_{\alpha+}}(3) = \max\{f_A(3) - 0.1, 0\} = \max\{0.2 - 0.1, 0\} = 0.1$

$T_{\alpha+}(A) = (0, 0.9, 0)(1, 0.6, 0.1)(2, 0.7, 0.2), (3, 0.8, .1)$

Therefore $T_{\alpha+}(A)$ is not vague β - ideal of a vague β - algebra $\mathfrak{F}$ for all $\alpha \in [0, 1]$, as the definition of vague β - ideal of $\mathfrak{F}$.

Theorem 3.9. Let A be a vague set of $\mathfrak{F}$ such that $T_{\alpha+}(A)$ is translate of a vague β - ideal of a β - algebra $\mathfrak{F}$ for some $\alpha \in [0, 1]$ then A is vague β - ideal of β - algebra $\mathfrak{F}$.

Proof. Let A be a vague set of $\mathfrak{F}$ such that $T_{\alpha+}(A)$ is translate of a vague β - ideal of a β - algebra $\mathfrak{F}$ for some $\alpha \in [0, 1]$. (a) Let x, y be any elements, then we have

$$\begin{aligned} t_{AT_{\alpha+}}(0) &= \min\{t_A(x - x) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x), t_A(x)\} + \alpha, 1\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}\} \\ &= \min\{\min\{t_A(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}, 1\} \\ &= \min\{t_{AT_{\alpha+}}(x), t_{AT_{\alpha+}}(x)\} \geq t_{AT_{\alpha+}}(x) \\ &= t_{AT_{\alpha+}}(x) \leq t_{AT_{\alpha+}}(0) \end{aligned}$$

therefore $t_{AT_{\alpha+}}(0) \geq t_{AT_{\alpha+}}(x) \dots (3.9.1)$

$$\begin{aligned} (b) t_{AT_{\alpha+}}(x + y) &= \min\{t_A(x + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x), t_A(y)\} + \alpha, 1\} \\ &= \min\{\{t_A(x) + \alpha\}, \{t_A(y) + \alpha\}\} \\ &= \min\{\{t_A(x)\}, \{t_A(y)\}\} \end{aligned}$$

Therefore $t_{AT_{\alpha+}}(x + y) \geq \min\{\{t_A(x)\}, \{t_A(y)\}\} \dots (3.9.2)$

$$\begin{aligned} (c) t_{AT_{\alpha+}}(x) &= \min\{t_A(x - y + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_A(x - y), t_A(y)\} + \alpha, 1\} \\ &= \min\{\{t_A(x - y) + \alpha, 1\}, \{t_A(y) + \alpha, 1\}\} \\ &= t_{AT_{\alpha+}}\{(x - y), t_{AT_{\alpha+}}(y)\} \end{aligned}$$

Therefore $t_{AT_{\alpha+}}(x) \geq \min\{t_{AT_{\alpha+}}(x - y), t_{AT_{\alpha+}}(y)\} \dots (3.9.3)$

Hence from (3.9.1), (3.9.2) and (3.9.3)

$T_{\alpha+}(A)$ is a vague β -ideal of β -algebra $\mathfrak{F}$.

Theorem 3.10. *Intersection of translates of two vague β -ideals of a β -algebra $\mathfrak{F}$ is also vague β -ideal of β -algebra $\mathfrak{F}$.*

Proof. Let $t_{AT_{\alpha+}}$ and $t_{BT_{\alpha+}}$ be translates of a vague β -ideals of a β -algebra $\mathfrak{F}$, where $\alpha = 0.1, \in [0, 1]$. by known theorem (2.3) If A, B are two vague β -ideals of $\mathfrak{F}$. Then $T_{\alpha+}(A \cap B)$ and $T_{\alpha-}(A \cap B)$ are also translates of vague β -ideals of $\mathfrak{F}$.

(a) Let x, y be any elements, then we have

$$\begin{aligned} t_{(A \cap B)T_{\alpha+}}(0) &= \min\{t_{(A \cap B)}(x - x) + \alpha, 1\} \\ &\geq \min\{\min\{t_{(A \cap B)}(x), t_{(A \cap B)}(x)\} + \alpha, 1\} \\ &= \min\{\min\{t_{(A \cap B)}(x) + \alpha, 1\}, \min\{t_A(x) + \alpha, 1\}\} \\ &= \min\{\min\{t_{(A \cap B)}(x) + \alpha, 1\}, \min\{t_{(A \cap B)}(x) + \alpha, 1\}, 1\} \\ &= \min\{t_{(A \cap B)T_{\alpha+}}(x), t_{(A \cap B)T_{\alpha+}}(x)\} \geq t_{AT_{\alpha+}}(x) \\ &= t_{(A \cap B)T_{\alpha+}}(x) \leq t_{(A \cap B)T_{\alpha+}}(0) \\ \text{therefore } t_{(A \cap B)T_{\alpha+}}(0) &\geq t_{(A \cap B)T_{\alpha+}}(x) \dots \dots \dots (3.10.1) \end{aligned}$$

$$\begin{aligned} (b) t_{(A \cap B)T_{\alpha+}}(x + y) &= \min\{t_{(A \cap B)}(x + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_{(A \cap B)}(x), t_{(A \cap B)}(y)\} + \alpha, 1\} \\ &= \min\{\{t_{(A \cap B)}(x) + \alpha\}, \{t_{(A \cap B)}(y) + \alpha\}\} \\ &= \min\{\{t_{(A \cap B)}(x), \{t_{(A \cap B)}(y)\}\} \end{aligned}$$

$$\text{Therefore } t_{(A \cap B)T_{\alpha+}}(x + y) \geq \min\{\{t_{(A \cap B)}(x), \{t_{(A \cap B)}(y)\}\} \dots \dots (3.10.2)$$

$$\begin{aligned} (c) t_{(A \cap B)T_{\alpha+}}(x) &= \min\{t_{(A \cap B)}(x - y + y) + \alpha, 1\} \\ &\geq \min\{\min\{t_{(A \cap B)}(x - y), t_{(A \cap B)}(y)\} + \alpha, 1\} \\ &= \min\{\{t_{(A \cap B)}(x - y) + \alpha, 1\}, \{t_{(A \cap B)}(y) + \alpha, 1\}\} \\ &= t_{(A \cap B)T_{\alpha+}}(x - y), t_{A(A \cap B)T_{\alpha+}}(y) \end{aligned}$$

$$\begin{aligned} \text{Therefore } t_{(A \cap B)T_{\alpha+}}(x) &\geq \\ \min\{t_{(A \cap B)T_{\alpha+}}(x - y), t_{(A \cap B)T_{\alpha+}}(y)\} &\dots (3.10.3) \end{aligned}$$

Hence from (3.10.1), (3.10.2) and (3.10.3)

$T_{\alpha+}(A)$ is a vague β -ideal of β -algebra $\mathfrak{F}$.

5. Translate Operators on Vague β -ideal extension of vague β -algebra

We introduce the following definition.

Definition 4. 1 Let A, B are two vague sets of a vague β -algebra $\mathfrak{F}$, such that B is a vague extension of A. If A is a vague β -ideal of $\mathfrak{F}$. implies that B is a vague β -ideal of $\mathfrak{F}$, then B is called a vague β -ideal extension of A.

Example 4. 2 Consider the vague β -algebra $(\mathfrak{F}, +, -, 0)$ where $\mathfrak{F} = \{0, 1, 2, 3\}$ with cayley's table (1. 6) and the vague sets of $\mathfrak{F}$ are $A = \{(0, 0.6, 0.1), (1, 0.4, 0.3), (2, 0.5, 0.2), (3, 0.5, 0.1)\}$, $B = \{(0, 0.7, 0.3), (1, 0.5, 0.4), (2, 0.6, 0.3), (3, 0.6, 0.2)\}$ and $\alpha = 0.2 \in [0, 1]$ are vague β -ideal of $\mathfrak{F}$.

then B is a vague β -ideal extension of A But B is not a translate of A.

$$\begin{aligned}
 t_{AT_{\alpha+}}(x) &= \min\{t_A(x) + \alpha, 1\} \quad f_{AT_{\alpha+}}(x) = \max\{f_A(1) - \alpha, 0\} \\
 t_{AT_{\alpha+}}(0) &= \min\{t_A(0) + 0.2, 1\} = \min\{0.6 + 0.2, 1\} = 0.8 \\
 f_{AT_{\alpha+}}(0) &= \max\{f_A(0) - 0.2, 0\} = \min\{0.1 - 0.2, 0\} = 0 \\
 t_{AT_{\alpha+}}(1) &= \min\{t_A(1) + 0.2, 1\} = \min\{0.4 + 0.2, 1\} = 0.6 \\
 f_{AT_{\alpha+}}(1) &= \max\{f_A(1) - 0.2, 0\} = \max\{0.3 - 0.2, 0\} = 0.1 \\
 t_{AT_{\alpha+}}(2) &= \min\{t_A(2) + 0.2, 1\} = \min\{0.5 + 0.2, 1\} = 0.7 \\
 f_{AT_{\alpha+}}(2) &= \max\{f_A(2) - 0.2, 0\} = \min\{0.2 - 0.2, 0\} = 0 \\
 t_{AT_{\alpha+}}(3) &= \min\{t_A(3) + 0.2, 1\} = \min\{0.5 + 0.2, 1\} = 0.7 \\
 f_{AT_{\alpha+}}(3) &= \max\{f_A(3) - 0.2, 0\} = \min\{0.1 - 0.2, 0\} = 0 \\
 T_{\alpha+}(A) &= (0, 0.8, 0)(1, 0.6, 0.1)(2, 0.7, 0), (3, 0.7, 0).....(4.1.1) \\
 t_{BT_{\alpha+}}(x) &= \min\{t_B(x) + \alpha, 1\} \quad f_{BT_{\alpha+}}(x) = \max\{f_B(1) - \alpha, 0\} \\
 t_{BT_{\alpha+}}(0) &= \min\{t_B(0) + 0.2, 1\} = \min\{0.8 + 0.2, 1\} = 1 \\
 f_{BT_{\alpha+}}(0) &= \max\{f_B(0) - 0.2, 0\} = \min\{0.3 - 0.2, 0\} = 0.1 \\
 t_{BT_{\alpha+}}(1) &= \min\{t_B(1) + 0.2, 1\} = \min\{0.5 + 0.2, 1\} = 0.7 \\
 f_{BT_{\alpha+}}(1) &= \max\{f_B(1) - 0.2, 0\} = \max\{0.4 - 0.2, 0\} = 0.2 \\
 t_{BT_{\alpha+}}(2) &= \min\{t_B(2) + 0.2, 1\} = \min\{0.6 + 0.2, 1\} = 0.8 \\
 f_{BT_{\alpha+}}(2) &= \max\{f_B(2) - 0.2, 0\} = \max\{0.3 - 0.2, 0\} = 0.1 \\
 t_{BT_{\alpha+}}(3) &= \min\{t_B(3) + 0.2, 1\} = \min\{0.6 + 0.2, 1\} = 0.8 \\
 f_{BT_{\alpha+}}(3) &= \max\{f_B(3) - 0.2, 0\} = \max\{0.2 - 0.2, 0\} = 0 \\
 T_{\alpha+}(B) &= (0, 1, 0.1)(1, 0.7, 0.2)(2, 0.8, 0.1), (3, 0.8, 0).....(4.1.2)
 \end{aligned}$$

From (4.1.1)and(4.1.2)B is a vague β -ideal extension of A.

But B is not a translate of A.

Theorem 4.3. *Intersection of any two vague β – ideal extensions of a vague β – ideal A of $\mathfrak{F}$ is a vague β – ideal extension of A .*

Proof. Let A_1 and A_2 be two vague β – ideal extensions of a vague β – ideal A of $\mathfrak{F}$.

Then $t_{A_1}(x) \geq t_A(x)$ and $t_{A_2}(x) \geq t_A(x)$ for all $x \in \mathfrak{F}$.

Now A is vague β – ideal of $\mathfrak{F}$. which implies $t_{A_1}(x)$ and $t_{A_2}(x)$ are vague β – ideals of $\mathfrak{F}$. Then by known theorem (3.10) $t_{A_1}(x) \cap t_{A_2}(x)$ is also vague β – ideals of $\mathfrak{F}$. Now $t_{A_1}(x) \cap t_{A_2}(x) = \min\{t_{A_1}(x), t_{A_2}(x)\} \geq (t_{A_1}(x) \cap t_{A_2}(x)) = t_A(x)$.

Hence $t_{A_1}(x) \cap t_{A_2}(x)$ is vague β – ideal extension of A .

Remark 4. 4 The following example shows that the Union of any two vague β – ideal extensions of a vague β – ideal A of a β – algebra $\mathfrak{F}$ need not be a vague β – ideal extension of A .

Example 4. 5 Consider the vague β – algebra $(\mathfrak{F}, +, -, 0)$ with cayley's table (1. 6) where

$\mathfrak{F} = \{0, 1, 2, 3\}$ and the vague sets of $\mathfrak{F}$ are

$A = \{(0, 0.7, 0), (1, 0.3, 0.3), (2, 0.5, 0.2), (3, 0.3, 0.1)\}$ and

$A_1 = \{(0, 0.8, 0.2), (1, 0.6, 0.2), (2, 0.6, 0.3), (3, 0.5, 0.2)\}$

$A_2 = \{(0, 0.9, 0.1), (1, 0.4, 0.3), (2, 0.6, 0.3), (3, 0.4, 0.2)\}$

and $\alpha = 0.2 \in [0, 1]$ clearly A, A_1, A_2 are vague β – ideal of $\mathfrak{F}$.

also A_1, A_2 are a vague β – ideal extension of A . But the set $(t_{A_1} \cup t_{A_2})(0) = 0.9$ and $(t_{A_1} \cup t_{A_2})(1) = 0.6$

$(t_{A_1} \cup t_{A_2})(2) = 0.6$ and $(t_{A_1} \cup t_{A_2})(3) = 0.5$ is not vague ideal extension.

since $(t_{A_1} \cup t_{A_2})(3) = 0.5$

$(t_{A_1} \cup t_{A_2})(3 - 2) = \min\{((t_{A_1} \cup t_{A_2})(3 - 2), (t_{A_1} \cup t_{A_2})(2))\}$

$\min\{0.6, 0.6\} = 0.6$ therefore $(t_{A_1} \cup t_{A_2})(3) = 0.5 < 0.6$

$= \min\{((t_{A_1} \cup t_{A_2})(3 - 2), (t_{A_1} \cup t_{A_2})(2))\}$

Theorem 4.6. *Let A be vague β – ideal of $\mathfrak{F}$. For $\forall \alpha \in [0, 1]$ the translate of vague β – ideal extension of A is itself.*

Proof. Let A be a vague β – ideal of $\mathfrak{F}$ then the translate of vague β – ideal of A is also vague β – ideal of $\mathfrak{F}$ $\forall \alpha \in [0, 1]$.

Also $t_{AT\alpha+} = \min\{t_A(x) + \alpha, 1\} \Rightarrow t_{AT\alpha+} = \min\{t_A(x) + \alpha, 1\} \geq t_A(x) \forall x \in \mathfrak{F}$.

Therefore $t_{AT\alpha+}(A)$ is vague β extension of A .

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