ON SOFT CONTRA-$\pi$gb-CONTINUOUS FUNCTIONS
IN SOFT TOPOLOGICAL SPACES

C. Janaki$^1$, D. Sreeja$^2$

$^1$Department of Mathematics
L.R.G. Govt. Arts College for Women
Tirupur, 641604, Tamil Nadu, INDIA
$^2$Department of Mathematics
C.M.S. College of Science and Commerce
Coimbatore, 641049, Tamil Nadu, INDIA

Abstract: The aim of this paper is to define and study the concepts of soft contra $\pi$gb-continuous function and soft almost contra $\pi$gb-continuous function in soft topological spaces.

AMS Subject Classification: 06D72, 54A40
Key Words: soft contra $\pi$gb-continuous, soft contra- $\pi$gb-irresolute, soft contra $\pi$gb-closed, soft almost contra $\pi$gb-continuous function

1. Introduction and Preliminaries

Molodtsov [12] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Soft systems provide a general framework with the involvement of parameters. Soft set theory has a wider application and its progress is very rapid in different fields. Levine

In this paper, the concept of soft contra \( \pi \)gb-continuous function and soft almost contra \( \pi \)gb-continuous function on soft topological spaces are discussed and some characterizations of these mappings are obtained.

2. Preliminaries

Let \( U \) be an initial universe set and \( E \) be a collection of all possible parameters with respect to \( U \), where parameters are the characteristics or properties of objects in \( U \). Let \( P(U) \) denote the power set of \( U \), and let \( A \subset E \).

**Definition 1.** [12] A pair \( (F,A) \) is called a soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow P(U) \). In other words, a soft set over \( U \) is a parametrized family of subsets of the universe \( U \). For a particular \( e \in A \), \( F(e) \) may be considered the set of \( e \)-approximate elements of the soft set \( (F,A) \).

**Definition 2.** [4] For two soft sets \( (F,A) \) and \( (G,B) \) over a common universe \( U \), we say that \( (F,A) \) is a soft subset of \( (G,B) \) if (i) \( A \subset B \), and (ii) \( \forall e \in A, F(e) \subset G(e) \). We write \( (F,A) \subset (G,B) \). \( (F,A) \) is said to be a soft super set of \( (G,B) \), if \( (G,B) \) is a soft subset of \( (F,A) \).

**Definition 3.** [11] A soft set \( (F,A) \) over \( U \) is said to be:

(i) null soft set denoted by \( \phi \) if \( \forall e \in A, F(e) = \phi \).

(ii) absolute soft set denoted by \( A \), if \( \forall e \in A, F(e) = U \).

**Definition 4.** [14] Let \( \tau \) be the collection of soft sets over \( X \), then \( \tau \) is called a soft topology on \( X \) if \( \tau \) satisfies the following axioms:

1) \( \phi, X \) belong to \( \tau \)
2) The union of any number of soft sets in $\tau$ belongs to $\tau$.

3) The intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, \tau, E)$ is called a soft topological space over $X$. For simplicity, we can take the soft topological space $(X, \tau, E)$ as $X$ throughout the work.

**Definition 5.** A soft subset $(A, E)$ of $X$ is called:
(i) a soft b-open[6] if $(A, E) \subseteq \text{Cl} (\text{Int} (A, E)) \cap \text{Int} (\text{Cl} (A, E))$;
(ii) a soft generalized closed (soft g-closed)[8] if $\text{Cl} (A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and $(U, E)$ is soft open in $X$;
(iii) a soft $\pi gb$-closed[6] in $X$ if $\text{sbcl} (A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and $(U, E)$ is soft $\pi$-open in $X$;
(iv) a soft generalized $\beta$ closed (Soft $g\beta$-closed)[1] in a soft topological space $(X, \tau, E)$ if $\beta \text{Cl} (A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and $(U, E)$ is soft open in $X$;
(v) a soft $gs\beta$ closed[1] if $\beta \text{Cl} (A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and $(U, E)$ is soft semi open in $X$.

The finite union of soft regular open sets is called soft $\pi$-open set and its complement is soft $\pi$-closed set. The soft regular open set of $X$ is denoted by $\text{SRO}(X)$ or $\text{SRO}(X, \tau, E)$.

**Definition 6.** [8] A soft topological space $X$ is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in $X$.

**Definition 7.** Let $(X, \tau, E)$ and $(Y, \tau', E)$ be two soft topological spaces. A function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be:
(i) soft semi-continuous[13] if $f^{-1}((G, E))$ is soft semi-open in $(X, \tau, E)$, for every soft open set $(G, E)$ of $(Y, \tau, E)$.
(ii) soft pre-continuous [15] if $f^{-1}((G, E))$ is soft pre-open in $(X, \tau, E)$, for every soft open set $(G, E)$ of $(Y, \tau', E)$.
(iii) soft $\pi gr$-continuous[6] if $f^{-1}((G, E))$ is soft $\pi gr$-closed in $(X, \tau, A)$ for every soft closed set $(G, E)$ of $(Y, \tau', E)$.

**Definition 8.** [7] Let $(X, \tau, A)$ and $(Y, \tau', B)$ be two soft topological spaces and $f : (X, \tau, A) \rightarrow (Y, \tau', B)$ be a function. Then the function $f$ is soft $\pi gb$- continuous if $f^{-1}(G,B)$ is soft $\pi gb$- closed in $(X, \tau, A)$ for every soft closed set $(G,B)$ of $(Y, \tau', B)$.

**Definition 9.** [7] Let $(X, \tau, A)$ and $(Y, \tau', B)$ be soft topological spaces and $f : (X, \tau, A)\rightarrow (Y, \tau', B)$ be a function. Then the function $f$ is soft $\pi gb$-open if $f (G,A)$ is soft $\pi gb$-open in $(Y, \tau', B)$ for every soft open set $(G,A)$ of $(X, \tau, A)$.
Definition 10. [7] A soft topological space X is called soft $\pi$gb-$T_{1/2}$ space if every soft $\pi$gb- closed set is soft b-closed.

3. Soft Contra-$\pi$gb-Continuous Functions

In this section, we study the notion of soft contra $\pi$gb-continuous functions.

Definition 11. Let $(X, \tau, A)$ and $(Y, \tau', B)$ be two soft topological spaces and $f : (X, \tau, A) \rightarrow (Y, \tau', B)$ be a function. Then the function $f$ is:

(i) soft contra continuous if $f^{-1}(G,B)$ is soft closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

(ii) soft contra b-continuous if $f^{-1}(G,B)$ is soft b-closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

(iii) soft contra g-continuous if $f^{-1}(G,B)$ is soft g-closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

(iv) soft contra $\pi$g-continuous if $f^{-1}(G,B)$ is soft $\pi$g-closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

(v) soft contra $\pi$gr-continuous if $f^{-1}(G,B)$ is soft $\pi$gr-closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

Definition 12. Let $(X, \tau, A)$ and $(Y, \tau', B)$ be two soft topological spaces and $f : (X, \tau, A) \rightarrow (Y, \tau', B)$ be a function. Then the function $f$ is soft contra $\pi$gb- continuous if $f^{-1}(G,B)$ is soft $\pi$gb-closed in $(X, \tau, A)$ for every soft open set $(G,B)$ of $(Y, \tau', B)$.

Theorem 13. (i) Every soft contra continuous function is soft contra-$\pi$gb- continuous.

(ii) Every soft contra-b-continuous function is soft contra-$\pi$gb- continuous.

(iii) Every soft contra-g-continuous function is soft contra-$\pi$gb- continuous.

(iv) Every soft contra-$\pi$g-continuous function is soft contra-$\pi$gb- continuous.

(v) Every soft contra-$\pi$gr-continuous function is soft contra-$\pi$gb- continuous.

Remark 14. Converse of the above statements is not true as shown in the following example.

Example 15. Let $X=Y=\{a,b,c,d\}$, $E=\{e_1,e_2\}$. Let $F_1, F_2, .., F_6$ are functions from $E$ to $P(X)$ and are defined as follows:

$F_1(e_1) = \{c\}$, $F_1(e_2) = \{a\}$

$F_2(e_1) = \{d\}$, $F_2(e_2) = \{b\}$
ON SOFT CONTRA-$\pi_{gb}$-CONTINUOUS FUNCTIONS... 127

\[ F_3(e_1) = \{c, d\}, F_3(e_2) = \{a, b\} \]
\[ F_4(e_1) = \{a, d\}, F_4(e_2) = \{b, d\} \]
\[ F_5(e_1) = \{b, c, d\}, F_5(e_2) = \{a, b, c\} \]
\[ F_6(e_1) = \{a, c, d\}, F_6(e_2) = \{a, b, d\} \]

Then \( \tau_1 = \{\phi, X, (F_1, E), \ldots, (F_6, E)\} \) is a soft topology and elements in \( \tau \) are soft open sets.

Let \( G_1, G_2, G_3, G_4 \) are functions from \( E \) to \( P(Y) \) and are defined as follows:
\[ G_1(e_1) = \{a\}, G_1(e_2) = \{d\} \]
\[ G_2(e_1) = \{b\}, G_2(e_2) = \{c\} \]
\[ G_3(e_1) = \{a, b\}, G_3(e_2) = \{c, d\} \]
\[ G_4(e_1) = \{b, c, d\}, G_4(e_2) = \{a, b, c\} \]

Then \( \tau_2 = \{\phi, X, (G_1, E), \ldots, (G_4, E)\} \) be a soft topology on \( Y \). Let \( f:X \rightarrow Y \) be an identity map. Here the inverse image of the soft open set
\[ (A, E) = \{\{b, c, d\}, \{a, b, c\}\} \]
in \( Y \) is not soft closed, soft \( b \)-closed in \( X \). Hence not soft contra continuous, soft contra \( b \)-continuous. Also soft open set \( (B, E) = \{\{a\}, \{d\}\} \) in \( Y \) is not soft \( \pi_{gb} \)-closed, soft \( \pi_{gr} \)-closed in \( X \). Hence not soft contra-$\pi_{gb}$-continuous, soft contra \( \pi_{gr} \)-continuous. Soft open set \( (C, E) = \{\{b, c, d\}, \{a, b, c\}\} \) in \( Y \) is not soft \( g \)-closed in \( X \). Hence not soft contra \( g \)-continuous.

**Definition 16.** A soft topological space \( X \) is called:

(i) soft \( \pi_{gb} \)-locally indiscrete if every soft \( \pi_{gb} \)-open set is soft closed.

(ii) a soft \( T_{\pi_{gb}} \)-space if every soft \( \pi_{gb} \)-closed set is soft \( \pi_{gb} \)-closed.

**Theorem 17.** \((X, \tau, A)\) and \((Y, \tau', B)\) be two soft topological spaces and \( f : (X, \tau, A) \rightarrow (Y, \tau', B) \) be a function.

(i) If a function \( f \) is soft \( \pi_{gb} \)-continuous and \((X, \tau, A)\) is soft \( \pi_{gb} \)-locally indiscrete, then \( f \) is soft contra-continuous.

(ii) If a function \( f \) is soft contra-$\pi_{gb}$-continuous and \((X, \tau, A)\) is a soft \( \pi_{gb} \)-\( T_{1/2} \) space, then \( f \) is soft contra-b-continuous.

(iii) If a function \( f \) is soft contra-$\pi_{gb}$-continuous and \((X, \tau, A)\) is a soft \( \pi_{gb} \)-space, then \( f \) is soft contra-continuous.

(iv) If a function \( f \) is contra-$\pi_{gb}$-continuous and \((X, \tau, A)\) is a soft \( T_{\pi_{gb}} \)-space, then \( f \) is soft contra-$\pi_{gb}$-continuous.

**Proof.** (i) Let \((B, E)\) be soft open in \((Y, \tau', B)\). By assumption inverse image of soft open set \((B, E)\) is soft \( \pi_{gb} \)-open in \( X \). Since \( X \) is soft \( \pi_{gb} \)-locally indiscrete, inverse image of soft open set \((B, E)\) is soft closed in \( X \). Hence \( f \) is soft contra-continuous.
(ii) Let \((B, E)\) be soft open in \((Y, \tau', B)\). By assumption, inverse image of soft open set \((B, E)\) is soft \(\pi gb\)-closed in \(X\). Since \(X\) is soft \(\tau gb-T_{1/2}\) space, inverse image of soft open set \((B, E)\) is soft b-closed in \(X\). Hence \(f\) is soft contra-b-continuous.

(iii) Let \((B, E)\) be soft open in \((Y, \tau', B)\). By assumption, inverse image of soft open set \((B, E)\) is soft \(\pi gb\)-closed in \(X\). Since \(X\) is soft \(\tau gb\)-space, inverse image of soft open set \((B, E)\) is closed in \(X\). Hence \(f\) is soft contra-continuous.

(iv) Let \((B, E)\) be soft open in \((Y, \tau', B)\). By assumption, inverse image of soft open set \((B, E)\) is soft \(\pi gb\)-closed in \(X\). Since \(X\) is soft \(T_{\pi gb}\)-space, inverse image of soft open set \((B, E)\) is soft \(\pi g\)-closed in \(X\). Hence \(f\) is soft contra-\(\pi g\)-continuous.

**Theorem 18.** Let \(A \subseteq Y \subseteq X\).

(i) If \(Y\) is soft open in \(X\), then \(A \in \pi GBO(X)\) implies \(A \in \pi GBO(Y)\).

(ii) If \(Y\) is soft regular open and soft \(\pi gb\)-closed in \(X\), then \(A \in \pi GBO(Y)\) implies \(A \in \pi GBO(X)\).

**Theorem 19.** Suppose \(\pi GBO(X)\) is soft closed under arbitrary union. Then the following are equivalent for a function \(f : (X, \tau, A) \to (Y, \tau', B)\).

(i) \(f\) is soft contra-\(\pi gb\)-continuous.

(ii) For every soft closed subset \((F, E)\) of \(Y\), \(f^{-1}((F, E)) \in \pi GBO(X)\).

(iii) For each \(x \in X\) and each \((F, E) \in SC(Y, f(x))\), there exists \((A, E) \in S\pi GBO(X, x)\) such that \(f((A, E)) \subseteq (F, E)\).

*Proof.* (i) \(\iff\) (ii) and (ii) \(\Rightarrow\) (iii) is obvious. (iii) \(\Rightarrow\) (ii): Let \((F, E)\) be any closed set of \(Y\) and \(x \in f^{-1}((F, E))\). Then \(f(x) \in (F, E)\) and there exists \((A, E)_x \in S\pi GBO(X)\) such that \(f((A, E)_x) \subseteq (F, E)\). Therefore we obtain

\[
f^{-1}((F, E)) = \bigcup \{(A, E) : x \in f^{-1}(F)\}
\]

and \(f^{-1}((F, E))\) is soft \(\pi gb\)-open.

**Theorem 20.** Suppose \(S\pi GBO(X)\) is soft closed under arbitrary intersections. If a function \(f : (X, \tau, A) \to (Y, \tau', B)\) is soft contra-\(\pi gb\)-continuous and \(U\) is soft open in \(X\), then \(f / U : (U, \tau, A) \to (Y, \tau', B)\) is soft contra-\(\pi gb\)-continuous.

*Proof.* Let \((B, E)\) be closed in \(Y\). Since \(f : (X, \tau, A) \to (Y, \tau', B)\) is soft contra-\(\pi gb\)-continuous, \(f^{-1}((B, E))\) is soft \(\pi gb\)-open in \((X, \tau, A)\). \((f / U)^{-1}(B, E)) = f^{-1}((B, E)) \cap U\) is soft \(\pi gb\)-open in \(X\). Hence \((f / U)^{-1}((B, E))\) is soft \(\pi gb\)-open in \(U\).
Theorem 21. Suppose $S\pi \text{GBO}(X)$ is soft closed under arbitrary unions. Let $(X, \tau, A) \to (Y, \tau', B)$ be a function and $\{U_i : i \in I\}$ be a soft cover of $X$ such that $U_i \in S\pi \text{GBC}(X)$ and soft regular open for each $i \in I$. If $f / U_i : (U_i, \tau / U_i) \to Y$ is soft contra-$\pi gb$-continuous for each $i \in I$, then $f$ is soft contra-$\pi gb$-continuous.

Theorem 22. Suppose $S\pi \text{GBO}(X)$ is soft closed under arbitrary unions. If $f : (X, \tau, A) \to (Y, \tau', B)$ is soft contra-$\pi gb$-continuous and $Y$ is soft regular open, then $f$ is soft $\pi gb$-continuous.

Definition 23. Let $(X, \tau, A)$ and $(Y, \tau', B)$ be soft topological spaces and $f : (X, \tau, A) \to (Y, \tau', B)$ be a function. Then the function $f$ is:

(i) soft contra $\pi gb$-open if $f(G, A)$ is soft $\pi gb$-open in $(Y, \tau', B)$ for every soft closed set $(G, A)$ of $(X, \tau, A)$.

(ii) soft contra $\pi gb$-closed if $f(G, A)$ is soft $\pi gb$-closed in $(Y, \tau', B)$ for every soft open set $(G, A)$ of $(X, \tau, A)$.

Theorem 24. If $f : (X, \tau, A) \to (Y, \tau', B)$ is a soft $\pi$-continuous and soft $b$-closed function, then $f((A,E))$ is soft $\pi gb$-closed in $Y$ for every soft $\pi gb$-closed set $(A,E)$ of $X$.

Proof. Let $(A,E)$ be soft $\pi gb$-closed set in $X$. Let $f((A,E)) \subset (F,E)$, where $(F,E)$ is a soft $\pi$-open set in $Y$. Since $f$ is soft $\pi$-continuous, $f^{-1}((F,E))$ is soft $\pi$-open in $X$ and $(A,E) \subset f^{-1}((F,E))$. Then we have $\text{sbcl}((A,E)) \subset f^{-1}((F,E))$ and so $f(\text{sbcl}((A,E))) \subset (F,E)$. Since $f$ is soft $b$-closed, $f(\text{sbcl}((A,E)))$ is soft $b$-closed in $Y$. Hence $\text{sbclf}(f((A,E))) \subset \text{sbcl}(\text{sbcl}((A,E))) = f(\text{sbcl}((A,E))) \subset (F,E)$. This shows that $f((A,E))$ is soft $\pi gb$-closed in $Y$.

Definition. A map $f : X \to Y$ is said to be soft contra-$\pi gb$-irresolute if $f^{-1}((F,E))$ is soft $\pi gb$-closed in $X$ for each $(F,E) \in S\pi \text{GBO}(Y)$.

Theorem 25. Let $f : (X, \tau, A) \to (Y, \tau', B)$ and $g : (Y, \tau', A) \to (Z, \tau'', E)$ be two maps in soft topological space such that $g \circ f : (X, \tau, E) \to (Z, \tau'', E)$.

(i) If $g$ is soft $\pi gb$-continuous and $f$ is soft contra-$\pi gb$-irresolute, then $g \circ f$ is soft contra-$\pi gb$-continuous.

(ii) If $g$ is soft $\pi gb$-irresolute and $f$ is soft contra-$\pi gb$-irresolute, then $g \circ f$ is soft contra-$\pi gb$-irresolute.

Proof. (i) Let $(F,E)$ be soft closed set in $Z$. Then $g^{-1}((F,E))$ is soft $\pi gb$-closed in $Y$. Since $f$ is soft contra-$\pi gb$-irresolute, $f^{-1}(g^{-1}((F,E)))$ is soft $\pi gb$-open in $X$. Hence $g \circ f$ is soft contra $\pi gb$-continuous. (ii) Let $(F,E)$ be soft $\pi gb$-closed in $Z$. Then $g^{-1}((F,E))$ is soft $\pi gb$-closed in $Y$. Since $f$ is soft contra-$\pi gb$-irresolute, $f^{-1}(g^{-1}((F,E)))$ is soft $\pi gb$-open in $X$. Hence $g \circ f$ is soft contra-$\pi gb$-irresolute.
\[\pi_{gb}\text{- irresolute.}\]

4. Soft Almost Contra-\(\pi_{gb}\)-Continuous Functions

In this section, we study the concept of soft almost contra \(\pi_{gb}\)-continuous functions.

**Definition 26.** A function \(f : (X, \tau, A) \rightarrow (Y, \tau', B)\) is said to be soft almost contra- \(\pi_{gb}\)-continuous if \(f^{-1}((F,E)) \in S\pi_{GBC}(X)\) for each \((F,E) \in SRO(Y)\).

**Theorem 27.** Suppose \(S\pi_{GBO}(X)\) is soft closed under arbitrary unions. Then the following statements are equivalent for a function \(f : (X, \tau, A) \rightarrow (Y, \tau', B)\).

(i) \(f\) is soft almost contra- \(\pi_{gb}\)-continuous.

(ii) \(f^{-1}((F,E)) \in S\pi_{GBO}(X)\) for every \((F,E) \in SRC(Y)\).

(iii) For each \(x \in X\) and each soft regular closed set \((F,E)\) in \(Y\) containing \(f(x)\), there exists a soft \(\pi_{gb}\)-open set \((A,E)\) in \(X\) containing \(x\) such that \(f((A,E)) \supseteq (F,E)\).

(iv) For each \(x \in X\), and each soft regular open set \((B,E)\) in \(Y\) not containing \(f(x)\), there exists a soft \(\pi_{gb}\)-closed set \((G,E)\) in \(X\) not containing \(x\) such that \(f^{-1}((B,E)) \subseteq (G,E)\).

(v) \(f^{-1}(s\text{-int}(cl((G,E)))) \in S\pi_{GBC}(X)\) for every soft open subset \((G,E)\) of \(Y\).

(vi) \(f^{-1}(s\text{-int}(cl((F,E) F))) \in S\pi_{GBO}(X)\) for every soft closed subset \((F,E)\) of \(Y\).

**Proof.** (i) \(\Rightarrow\) (ii) Let \((F,E) \in SRC(Y)\). Then \(Y-(F,E) \in SRO(Y)\) by assumption. Hence \(f^{-1}((Y-(F,E)))=X-f^{-1}(((F,E)) \in S\pi_{GBC}(X)\). This implies

\[f^{-1}((F,E)) \in S\pi_{GBO}(X).\]

(ii) \(\Rightarrow\) (i). Let \((F,E) \in SRO(Y)\). Then by assumption \((Y-(F,E)) \in SRC(Y)\). Hence \(f^{-1}((Y-(F,E))) = X-f^{-1}(((F,E)) \in S\pi_{GBC}(X)\). This implies

\[f^{-1}((F,E)) \in S\pi_{GBO}(X).\]

(ii) \(\Rightarrow\) (iii) Let \((F,E)\) be any soft regular closed set in \(Y\) containing \(f(x)\). \(f^{-1}((F,E)) \in S\pi_{GBO}(X)\) and \(x \in f^{-1}((F,E))\) (by (ii)). Take \((A,E) = f^{-1}((F,E))\). Then \(f((A,E)) \supseteq (F,E)\).

(iii) \(\Rightarrow\) (ii) Let \((F,E) \in SRC(Y)\) and \(x \in f^{-1}((F,E))\). From (iii), there exists a soft \(\pi_{gb}\)-open set \((A,E)_x\) in \(X\) containing \(x\) such that \((A,E)_x \supseteq f^{-1}((F,E))\).
We have \( f^{-1}((F, E)) = \bigcup \{(A, E)_x : x \in f^{-1}((F, E))\} \). Then \( f^{-1}((F, E)) \) is soft \( \pi \text{gb}-\)open.

(iii) \( \Rightarrow \) (iv) Let \((B, E)\) be any soft regular open set in \( Y \) containing \( f(x) \). Then \( Y-(B, E) \) is a soft regular closed set containing \( f(x) \). By (iii), there exists a soft \( \pi \text{gb}-\)open set \((A, E)\) in \( X \) containing \( x \) such that \( f((A, E)) \subset Y-(B, E) \). Hence \((A, E) \subset f^{-1}(Y-(B, E)) \subset X- f^{-1}((B, E)) \). Then \( f^{-1}((B, E)) \subset X-(A, E) \). Let us set \((G, E) = X-(A, E)\).

We obtain a soft \( \pi \text{gb}-\)closed set in \( X \) not containing \( x \) such that \( f^{-1}((B, E)) \subset (G, E) \).

(iv) \( \Rightarrow \) (iii). Let \((F, E)\) be soft regular closed set in \( Y \) containing \( f(x) \). Then \( Y-(F, E) \) is soft regular open set in \( Y \) containing \( f(x) \). By (iv), there exists a soft \( \pi \text{gb}-\)closed set \((G, E)\) in \( X \) not containing \( x \) such that \( f^{-1}(Y-(F, E)) \subset (G, E) \). Then \( X- f^{-1}((F, E)) \subset (G, E) \) implies \( X-(G, E) \subset f^{-1}((F, E)) \). Hence \( f(X-(G, E)) \subset (F, E) \). Take \((A, E) = X-(G, E) \). Then \( (A, E) \) is a soft \( \pi \text{gb}-\)open set in \( X \) containing \( x \) such that \( f((A, E)) \subset (G, E) \).

(i) \( \Rightarrow \) (v). Let \((G, E)\) be a soft open subset of \( Y \). Since \( s\text{-int}(\text{cl}((G, E))) \) is soft regular open, then by (i), \( f^{-1}(s\text{-int}(\text{cl}((G, E)))) \in S\pi GBO(X) \).

(v) \( \Rightarrow \) (i). Let \((G, E) \in SRO(Y) \). Then \((G, E)\) is soft open in \( Y \). By (v), \( f^{-1}(s\text{-int}(\text{cl}((G, E)))) \in S\pi GBO(X) \). This implies \( f^{-1}((G, E)) \in S\pi GBO(X) \).

(ii) \( \Leftrightarrow \) (vi) is similar as (i) \( \Leftrightarrow \) (v).

**Theorem 28.** If \( f : (X, \tau, A) \rightarrow (Y, \tau', B) \) is an soft almost contra- \( \pi \text{gb}-\)continuous function and \((A, E)\) is a soft open subset of \( X \), then the restriction \( f/ (A, E) : (A, E) \rightarrow Y \) is soft almost contra- \( \pi \text{gb}-\)continuous.

**Proof.** Let \((F, E) \in SRC(Y) \). Since \( f \) is soft almost contra- \( \pi \text{gb}-\)continuous, \( f^{-1}((F, E)) \in S\pi GBO(X) \). Since \((A, E)\) is soft open, it follows that

\[
(f/ (A, E))^{-1}((F, E)) = (A, E) \cap f^{-1}((F, E)) \in S\pi GBO((A, E)).
\]

Therefore \( f/ (A, E) \) is an soft almost contra- \( \pi \text{gb}-\)continuous.

**References**


