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MHD FORCED CONVECTION FLOW OF METALLIC AND NON-METALLIC NANOFLUIDS PAST A STRETCHING SURFACE WITH HEAT TRANSFER AND NONLINEAR THERMAL RADIATION EFFECTS

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Abstract: Nonlinear hydromagnetic two dimensional steady, laminar, hydromagnetic boundary layer flows of a viscous, incompressible, electrically conducting, radiating metallic and non-metallic nanofluids, with nonlinear radiation effects past a plate stretching with power law velocity is analyzed in the presence of a variable magnetic field. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. Then the resulting nonlinear ordinary differential equations are solved numerically using Fourth-Order Runge Kutta shooting method along with the Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions. Velocity and temperature profiles are obtained for different values of velocity exponent parameter, magnetic interaction parameter, nanoparticle volume fraction, surface temperature parameter, radiation parameter and are displayed graphically. Values for skin friction coefficient and non-dimensional rate of heat transfer are also derived and displayed graphically.

AMS Subject Classification: 76Wxx, 80A20
Key Words: metallic and non-metallic nanofluids, non-linear radiation effects, Nachtsheim-Swigert iteration scheme, stretching sheet, power law velocity

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1. Introduction

Sakiadis [1] studied theoretically the boundary layer on a continuous semi-infinite sheet moving steadily through an otherwise quiescent fluid environment. The boundary layer solutions of Sakisdis [1] resulted in a skin friction of about thirty percent higher than that of Blasius [2] for the flow past stretching flat plate. Many excellent theoretical models have been developed for radiative-convection flows and radiative-conductive transport. Plumb et al.[3] analyzed the effect of horizontal cross-flow and radiation on natural convection from vertical heated surface in saturated porous media. Mukhopadhyay and Layek [4] investigated the effects of thermal radiation and variable fluid viscosity on free convection flow and heat transfer past a porous stretching surface. In most of the above mentioned studies, the radiation term appears in linear form. Elbashbeshy[5] analyzed the radiation effect on heat transfer over a stretching surface by taking into account of the full form of radiation term. Swati Mukhopadhyay [6] investigated the effect of boundary layer flow and heat transfer over a porous moving plate in the presence of thermal radiation, taking into account of the full form of radiation term. A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of ordinary heat transfer fluids is not adequate to meet todays cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. Nanofluids are suspensions of submicronic solid particles (nanoparticles) in common fluids. The term was coined by Choi [7]. Recently Vishnu Ganesh et al. [8] discussed the Buoyancy effect on MHD flow of Nanofluid over a stretching sheet in the presence of thermal radiation.

The nonlinear thermal radiation effect is taken into account which holds good for small and large temperature differences between the plate and the ambient fluid. Thus the nonlinear thermal radiation effects gives a more generalized picture about the effect of radiation when compared to linear assumption which just result in rescaling of the Prandtl number by a factor involving the radiation parameter (Asterios Pantokratoras and Eugen Magyari [9]) and hence the present work is carried out due to its immense applications.

2. Formulation of the Problem

We consider a steady, incompressible, laminar, two-dimensional boundary layer flow of a viscous nanofluid (metallic / non-metallic) past a flat sheet coinciding with the plane $y = 0$ and the flow being confined to $y > 0$. The flow is generated due to nonlinear stretching of the sheet caused by the simultaneous application of two equal and opposite forces along the x-axis. Keeping the origin fixed, the sheet is then stretched with a velocity $u_w(x) = Cx^n$, where C is a constant, n is a velocity exponent parameter, and x is the coordinate measured along the stretching surface, varying nonlinearly with the distance from the slit. A variable magnetic field is applied normal to the horizontal physical model and coordinate system.

The pressure gradient and external forces are neglected. The basic steady conservation of mass, momentum, and thermal energy equations for nanofluids by using usual boundary-layer approximations in the presence of radiation and viscous dissipation can be written in Cartesian coordinates x and y as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{nf}\frac{\partial^2 u}{\partial y^2} - (\frac{\sigma B^2(x)}{\rho})u \tag{2}$$

where $B(x) = B_o x^{\frac{n-1}{2}}$.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}\frac{1}{(\rho C_p)_{nf}} \tag{3}$$

With the associated boundary conditions $u = u_w(x) = Cx^n$, $v = 0$; $T = T_w$, at $y = 0$.

$$u \longrightarrow 0; T \longrightarrow T_\infty \textbf{ as } y \longrightarrow \infty \tag{4}$$

where x and y denote the Cartesian coordinates along the sheet and normal to it, and u and v are the velocity components of the nanofluid in the x- and y-directions, respectively. C is a dimensional constant, B_o is a constant magnetic field. The temperature on the wall is T_w , and the ambient fluid is held at constant temperature T_∞ . ρ_{nf} and μ_{nf} are the density and effective viscosity of the nanofluid, α_{nf} and ν_{nf} are the thermal diffusivity and the kinematic viscosity, respectively, which are defined as

$$\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{5}$$

Here, ϕ is the nanoparticle volume fraction, whereμ_f is the viscosity of the basic fluid, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, respectively.

3. Flow Analysis

The equation of continuity is satisfied if we choose a stream function such that $\psi(x, y)$ such that

$$u = \frac{\partial\psi}{\partial y} \textbf{ and } v = -\frac{\partial\psi}{\partial x} \tag{6}$$

Introduction the usual similarity transformation [M.E.Ali [10]]

$$\eta(x, y) = y\sqrt{\frac{(n+1)Cx^{n-1}}{2\nu_f}}, \psi(x, y) = \sqrt{\frac{2\nu_f Cx^{n+1}}{n+1}} f(\eta) \tag{7}$$

Equation (2) can be written as

$$f''' + (1-\phi)^{2.5}(1-\phi+\phi\frac{\rho_s}{\rho_f})[ff'' - \frac{2n}{n+1}f'^2 - M^2 f'] = 0 \tag{8}$$

where $M = \sqrt{\frac{2\sigma B_o^2}{(n+1)\rho C}}$ is the magnetic interaction parameter The boundary conditions for the velocity are given by

$$f(0) = 0, f'(0) = 1 \textbf{ and } f'(\infty) \longrightarrow 0 \tag{9}$$

4. Heat Transfer Analysis

The radiative heat flux term is simplified by using the Rosseland approximation.In literature, most of the problems with radiation effects have considered the Rosseland approximation in a linear form which is valid only for small temperature difference between the plate and the ambient fluid. In the present work we consider the nonlinear Rosseland thermal radiation effects which holds good for small and large temperature difference between the plate and the ambient fluid. This assumption leads to a new physical parameter θ_w, namely the surface temperature parameter. Thus the radiative heat flux is given by

$$q_r = -\frac{16\sigma^* T^3}{3k^*}\frac{\partial T}{\partial y} \tag{10}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. The non-dimensional temperature and surface temperature are defined as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta_w = \frac{T_w}{T_\infty} \tag{11}$$

Equation (3) can be written as

$$[1 + \frac{4}{3R^*}(1 + \theta(\theta_w - 1))^3]\theta'' + \frac{4}{R^*}(1 + \theta(\theta_w - 1))^2(\theta_w - 1)\theta'^2 + Prf\theta' = 0 \tag{12}$$

where $R^* = \frac{k_{nf}k^*}{[4\sigma^* T_\infty^3]}$ is the radiation parameter and $Pr = \frac{\nu_f}{\alpha_{nf}}$ is the Prandtl number with boundary conditions

$$\theta(0) = 1 \textbf{ and } \theta(\infty) \longrightarrow 0 \tag{13}$$

5. Solution of the Problem

Equation (8) and (12) are nonlinear differential equations which constitute the nonlinear boundary value problem, it has to be reduced to an initial value problem. This is done by using shooting method.

Equations (8) and (12) are solved numerically subject to equations (9) and (13) using Fourth-Order Runge-Kutta shooting method. The crux of the problem is that we have to make an initial guess for the values of $f''(0)$ and $\theta'(0)$ to initiate the shooting process. The success of the procedure depends very much on how good this guess is. For different values of ϕ,θ_w,R^*,M and Pr different initial guesses were made into account of the convergence. Numerical results are obtained for several values of the physical parameter ϕ,θ_w,R^*,M and Pr.

6. Results and Discussions

Numerical values are depicted graphically by means of figures for dimensionless velocity $f'(\eta)$ and temperature distribution $\theta(\eta)$ for several set of values of magnetic interaction parameter M ,nanoparticle volume fraction ϕ , velocity exponent parameter n , thermal radiation parameter R^* and surface temperature parameter θ_w for different kinds of nanofluids.

To validate the numerical results, comparison of the present results with those of Vishnu Ganesh et al [8] in the absence of nonlinear radiation is carried

			$f''(0)$ Vishnu Ganesh et al(2014)		$f''(0)$ (Present Result)	
R^*	M	ϕ	Metallic (Cu)	Non-metallic (Al_2O_3)	Metallic (Cu)	Non-metallic (Al_2O_3)
10^9	0	0.05	-1.1089207	-1.0053797	-1.1089145	-1.0053695
		0.2	-1.2180440	-0.9559200	-1.2180345	-0.9559145
	2	0.05	-1.7288700	-1.6643600	-1.7288675	-1.6643590
		0.2	-1.6212641	-1.4347985	-1.6212585	-1.4347889

Table 1: Comparison of results for local skin friction coefficient $f''(0)$

				$\theta'(0)$ Vishnu Ganesh et al(2014)		$\theta'(0)$ (Present Result)	
R^*	θ_w	M	ϕ	Metallic (Cu)	Non-metallic (Al_2O_3)	Metallic (Cu)	Non-metallic (Al_2O_3)
10^9	0	0	0.05	-1.5500001	-1.5756510	-1.5500101	-1.5756630
			0.2	-1.0161489	-1.0845560	-1.0161495	-1.0845568
		2	0.05	-1.4148091	-1.4316959	-1.4148099	-1.4316969
			0.2	-0.9305950	-0.9809399	-0.9305959	-0.9809408

Table 2: Comparison of results for rate of heat transfer $\theta'(0)$ when $Pr = 7.0$ (Base Fluid)

out. It is found that the results are in good agreement which is shown in the table 1 and table 2.

We consider two different types of nanoparticles namely metallic (Cu) and non-metallic (Al_2O_3) with water as the base fluid. The Prandtl number of the base fluid (water) is kept constant at 7.0.

Figure 1 shows the effect of magnetic interaction parameter M on the velocity distribution $f'(\eta)$ for different types of nanoparticles with water as the base fluid. It is noted that $f'(\eta)$ decreases with the increase in the magnetic interaction parameter and the non-metallic nanoparticle have the highest value of velocity distribution than metallic nanoparticles.

The effect of thermal radiation parameter R^* on the temperature distribution for nanoparticle with water as the base fluid is shown in figure 2. It is observed that the dimensionless temperature $\theta(\eta)$ decreases with the increase

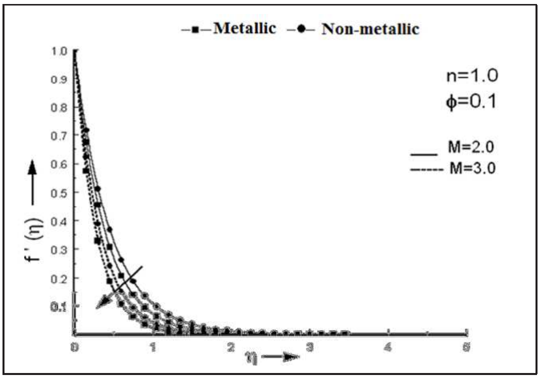

Figure 1: *Effects of M on velocity profile $f'(\eta)$ for different types of nanoparticles*

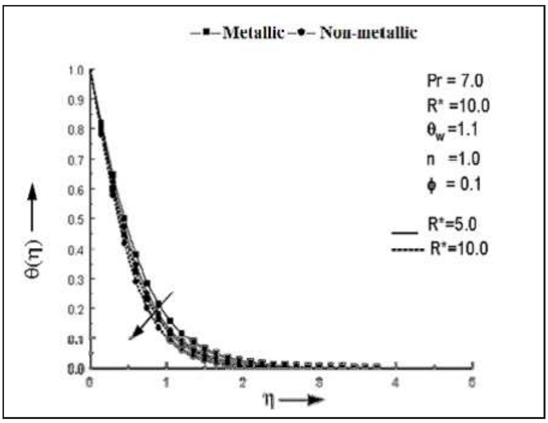

Figure 2: *Effects of thermal radiation parameter R^* on temperature profile $\theta(\eta)$for different types of nanoparticles*

in the thermal radiation parameter R^*. Further metallic nanoparticle has the highest value of temperature distribution than non-metallic nanoparticle.

The influence of surface temperature parameter θ_w for metallic and non-metallic nanoparticles is shown in figure 3. It is noted that as θ_w increases the dimensionless temperature $\theta(\eta)$ increases. It is also noted that θ_w effects is more for metallic nanoparticles when compare to non-metallic nanoparticles.

Figure 4 displays the variation of skin friction co-efficient $f''(0)$ against the magnetic interaction parameter M when $n = 1.0$, $\phi = 0.1$ for metallic

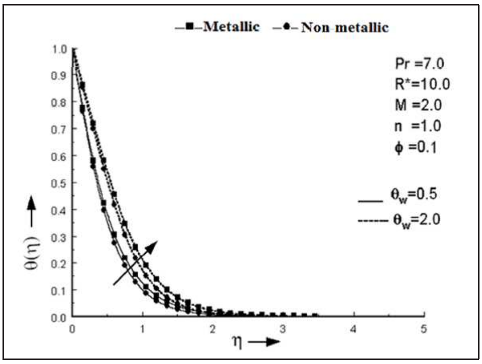

Figure 3: *Effects of surface temperature parameter* θ_{w} *on the temperature profile* $\theta(\eta)$ *for different types of nanoparticles*

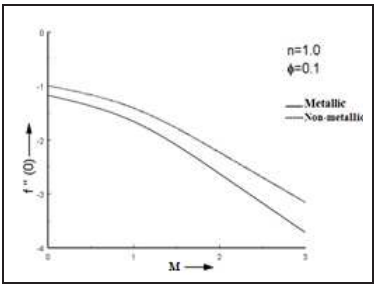

Figure 4: *Effects of parameter* M *on skin friction coefficient for different types of nanoparticles*

and non-metallic nanoparticles. It is noted that the skin friction co-efficient $f''(0)$ decreases for increasing magnetic interaction parameter M. It is observed that non-metallic nanoparticles have the highest skin friction than metallic nanoparticles.

The velocity exponent parameter n on skin friction co-efficient for different type of nanoparticle for $M = 2.0$ and $\phi = 0.1$ and is shown in figure 5. Skin friction co-efficient decreases with the increase of velocity exponent parameter n . It is interesting to observe that the alumina nanoparticle have the highest value of velocity distribution that metallic nanoparticles.

Figure 6 displays the dimensionless rate of heat transfer $\theta'(0)$ against the thermal radiation parameter R^* by fixing $Pr = 7.0$, $\theta_w = 1.1$, $M = 2.0$, $n = 1.0$ and $\phi = 0.1$ and for metallic nanoparticles with and non-metallic nanoparticles

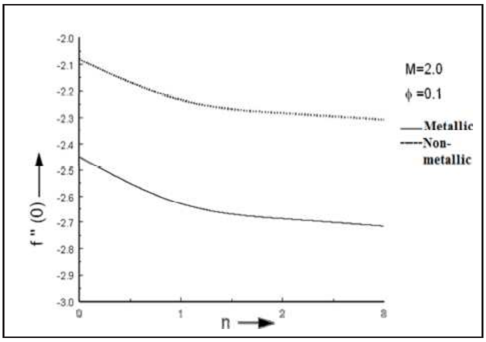

Figure 5: *Effects of parameter n on skin friction coefficient for different types of nanoparticles*

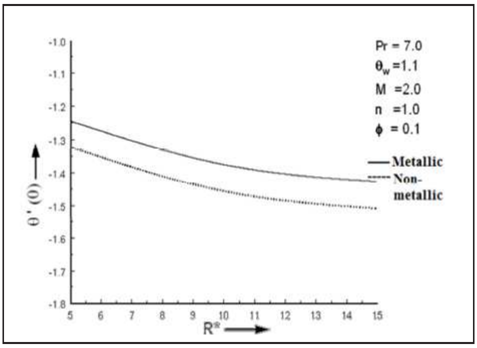

Figure 6: *Effects of thermal radiation parameter R^* on heat transfer rate for different types of nanoparticles*

with water as base fluid. It is obvious that the dimensionless rate of heat transfer decreases with the increase in thermal radiation parameter R^*. Further it is noted that metallic nanoparticles has the highest rate of heat transfer capacity when compare to non-metallic nanoparticles.

The surface temperature parameter θ_w effect on dimensionless rate of heat transfers $\theta'(0)$ when $Pr = 7.0$, $R^* = 10.0$, $M = 2.0$, $n = 1.0$ and $\phi = 0.1$ for metallic and non-metallic nanoparticles is shown in figure 7. It is noted that the rate of heat transfer increases with increase in surface temperature parameter θ_w. It is further observed that metallic nanoparticles possesses higher heat transfer capacity when compare to non-metallic nanoparticles

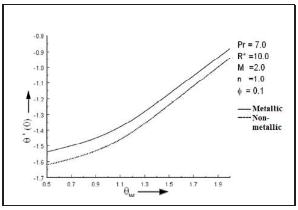

Figure 7: *Effects of surface temperature parameter* θ_w *on heat transfer rate for different types of nanoparticles*

7. Conclusions

Numerical solution for the problem of nonlinear MHD flow of metallic and non-metallic nanofluids with heat transfer and nonlinear radiation effects over a stretching surface has been obtained for various values of the physical parameters.

- It is seen that the dimensionless temperature decreases for increasing for metallic and non-metallic nanofluids. It is also interesting to note that non-metallic nanoparticles have the highest value of velocity distribution than metallic nanoparticles.

- It is observed that for increasing radiation parameter , metallic nanoparticles have the highest value of temperature distribution than non-metallic nanoparticles. Whereas there is significant different between metallic and non-metallic nanoparticles with respect to surface temperature parameter .

- The skin friction co-efficient of non-metallic nanoparticles is more than that of metallic nanoparticles for different and .

- It is found that metallic nanoparticles proved to have the highest cooling performance than non-metallic nanoparticles.

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