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AN MODIFIED METHOD FOR SOLVING BALANCED FUZZY TRANSPORTATION PROBLEM FOR MAXIMIZING THE PROFIT

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Abstract: The main objective of Fuzzy transportation is to find the maximum profit cost of some commodities through a capacitated network, when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. In this paper we are presenting a new technique for finding the maximum profit cost for fuzzy transportation problem by Using Yager's Ranking Method where fuzzy quantities are transformed in to crisp quantities. We have given a numerical illustration to check the validity of the proposal.

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Key Words: fuzzy balanced transportation problem, triangular fuzzy number (TFNs), Yager's ranking method, profit maximization

1. Introduction

Transportation models provide a powerful framework to meet the challenge of how to supply the commodities to the customers in more efficient ways. They ensure the efficient movement and timely availability of raw materials and finished goods. In 1941, Hitchcock [6] originally developed the basic transportation problem. In 1953, Charnes et al [3] developed the stepping stone method which

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provided an alternative way of determining the simplex method information. In 1963, Dantzig [4] used the simplex method to the transportation problems as the primal simplex transportation method. Till date, several researchers studied extensively to solve cost minimizing transportation problem in various ways. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh in 1965. Zimmermann[13] showed that solutions obtained by fuzzy linear programming are always efficient.

A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. In this paper the objective is to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number.

First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by Yager's ranking method which satisfies the properties of compensation, linearity and additivity, and then by using the classical algorithms, obtain the solution of the problem. This method is a systematic procedure, easy to apply and can be utilized for all types of transportation problems.

2. Basic Definitions

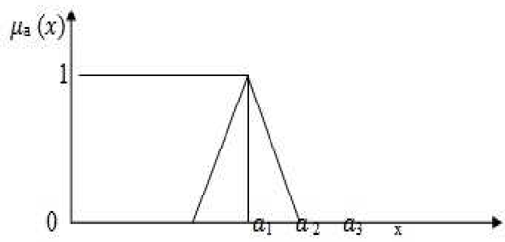
Definition 1. (**Fuzzy set**) A fuzzy set of a base set (or reference set) A is specified by its membership function μ, where $\mu : A \to [0, 1]$ assigning to each x in A the degree or grade to which $x \in A$.

Definition 2. (**Triangular Fuzzy Numbers**) The fuzzy number $a = a_1, a_2, a_3$ is a triangular fuzzy numbers, denoted by $a_{1,2}, a_3$ its membership function μ_a is given below the figure.

Definition 3. (**Operations of TFNs**) Let $a = [a_1, a_2, a_3,]$ and $b = [b_1, b_2, b_3]$ be two triangular fuzzy numbers then the arithmetic operations on a and b as follows.

Addition: $a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction: $a - b = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$



Multiplication:

$$a \cdot b = \frac{a_1}{3}(b_1 + b_2 + b_3), \frac{a_2}{3}(b_1 + b_2 + b_3), \frac{a_3}{3}(b_1 + b_2 + b_3), \quad \text{if} \quad R(a) > 0,$$

$$a \cdot b = \frac{a_3}{3}(b_1 + b_2 + b_3), \frac{a_2}{3}(b_1 + b_2 + b_3), \frac{a_1}{3}(b_1 + b_2 + b_3), \quad \text{if} \quad R(a) < 0,$$

Definition 4. (**Defuzzification**) Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here use Yager's ranking to defuzzify the TFNs because of its simplicity and accuracy.

Definition 5. (**Yager's Ranking Technique**) Yager's ranking technique [6] which satisfies compensation, linarity, additivity properties and provides results which consists of human intuition.F(R) represents the set of all TFNs. It R be any ranking function, then,

$$F(R) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha,$$

where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, c - (c-b)\alpha\}$.

Definition 6. (**Fuzzy balanced Transportation problem**) The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as LPP as follows

$$\text{Minimize} \quad \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} \star x_{ij}$$

subject to

$$\sum_{j=1}^{q} x_{ij} = a_i, \quad i = 1, 2, \ldots, p,$$

$$\sum_{i=1}^{p} x_{ij} = b_j, \quad j = 1, 2, \ldots, q,$$
$$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j.$$

Here x_{ij} is a non- negative trapezoidal fuzzy number, where

$p =$ total number of sources

$q =$ total number of destinations

$a_i =$ the fuzzy availability of the product at ith source

$b_j =$ the fuzzy demand of the product at jth destination

$c_{ij} =$ the fuzzy transportation cost for unit quantity of the product from ith source to jth destination

$x_{ij} =$ the fuzzy quantity of the product that should be transported from ith source to jth destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{p} a_i =$ total fuzzy availability of the product,

$\sum_{j=1}^{q} b_j =$ total fuzzy demand of the product

$\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} \star x_{ij} =$ total fuzzy transportation cost.

In LPP minimize (Z) = -maximize $(-Z)$, ie to maximize the profit is equal to minimize the cost. If $\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$, then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.Consider transportation with m fuzzy origins (rows) and n fuzzy destinations (Columns). Let $C_{ij} = [C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}]$ be the cost of transporting one unit of the product from ith fuzzy origin to jth fuzzy destination, $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$ be the quantity of commodity available at fuzzy origin i and $b_i = [b_i^{(1)}, b_i^{(2)}, b_i^{(3)}]$ be the quantity of commodity requirement at fuzzy destination j. $X_{ij} = [X_{ij}^1, X_{ij}^2, X_{ij}^3]$ is quantity transported from ith fuzzy origin to jth fuzzy destination.

3. Algorithm for Vogel Approximation Method

Step 1: Find the crisp value of the given Fuzzy Transportation problem by using Yager's Ranking

Step 2: Balance the given fuzzy transportation problem if either (total Supply > total demand) or (total supply < total demand).

Step 3: Convert the given maximization problem into a minimization problem by multiplying the cost elements by -1.The modified minimization problem can be solved by following the remaining steps

Step 4: Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.

Step 5: Select the row or column with the highest penalty cost (breaking ties arbitrarily or choosing the lowest-cost cell).

Step 6: Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.

Step 7: Repeat steps 2, 3 and 4 until all requirements have been meet.

Step 8: Compute total transportation cost for the feasible allocations.

4. Numerical Example

Consider the fuzzy transportation problem for maximizing the profit. A firm owns facilities at seven places. It has manufacturing plants at places A, B and C with daily production of (13, 23, 33), (34, 44, 54), (23, 33, 43) units respectively. At point D, E, F and G it has four warehouses with daily demands of (13, 23, 33), (21, 31, 41) (6, 16, 26) and (20, 30, 40) units respectively. Per unit shipping costs are given in the following table. Find the maximum profit of the firm ?

Since the given problem is a maximization type, first convert this into a minimization problem by multiplying the cost elements by -1.The modified minimization problem can be solved in the usual manner

Since $\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j = 100$ there exist a basic feasible solution to this problem and is displayed in the following table by using VAM.

	D	E	F	G	Fuzzy Available
A	(10,15,20)	(41,51,61)	(32,42,52)	(23,33,43)	(13,23,33)
B	(70,80,90)	(32,42,52)	(16,26,36)	(71,81,91)	(34,44,54)
C	(80,90,100)	(30,40,50)	(56,66,76)	(50,60,70)	(23,33,43)
Fuzzy Requirement	(13,23,33)	(21,31,41)	(6,16,26)	(20,30,40)	

After converting the fuzzy numbers into crisp values.

	D	E	F	G	Fuzzy Available
A	15	51	42	33	23
B	80	42	26	81	44
C	90	40	66	60	33
Fuzzy Requirement	23	31	16	30	100

	D	E	F	G	Fuzzy Available
A	-15	-51	-42	-33	23
B	-80	-42	-26	-81	44
C	-90	-40	-66	-60	33
Fuzzy Requirement	23	31	16	30	100

Since the number of non negative allocations at independent positions is (m+n-1) = 6, we apply MODI method for optimal solution

	D	E	F	G	Fuzzy Available
A	-15	-51 [23]	-42	-33	23
B	-80 [6]	-42 [8]	-26	-81 [30]	44
C	-90 [17]	-40	-66 [16]	-60	33
Fuzzy Requirement	23	31	16	30	

	D	E	F	G	
A	-15 -89 74	-51 23	-42 -65 23	-33 -90 57	U_1= -9
B	-80 6	-42 8	-26 -56 30	-81 30	U_2 = 0
C	-90 17	-40 -52 12	-66 16	-60 -91 31	U_3= -10
	V_1= -80	V_2= -42	V_3= -56	V_4= -81	

5. Conclusion

In this paper, the transportation costs are considered as imprecise numbers by fuzzy numbers which are more realistic and general in nature. More over fuzzy transportation problem of triangular numbers has been transformed into crisp transportation problem using Yager's ranking indices. Numerical examples show that by this method we can have the fuzzy optimal solution (maximum profit).This technique can also be used for solving profit maximization of balanced Fuzzy Assignment problems.

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