International Journal of Pure and Applied Mathematics
Volume 106 No. 5 2016, 79-87
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i5.8

ijpam.eu

ANALYSIS OF $M^X/G(a,b)/1$ QUEUE WITH CLOSEDOWN TIME WITH CONTROLLABLE ARRIVALS DURING MULTIPLE ADAPTIVE VACATIONS

S. Jeyakumar[1], E. Rameshkumar[2]

[1]Department of Mathematics
Government Arts College
Coimbatore, INDIA

[2]Department of Mathematics
C.M.S. College of Science and Commerce
Coimbatore, INDIA

Abstract: In this paper a $M^x/G(a,b)/1$ queueing system with multiple adaptive vacation, closedown times with controllable arriving customers is considered. The server provides service to all customers with service time follows general distribution. On completion of a service, if the queue $'\xi'$ is less than 'a' then the server performs a closedown work. Following closedown the server leaves for M number of multiple adaptive vacations of random length, irrespective of queue length. After a vacation or some vacation, if the queue length is still empty, the server avails another M number of adaptive vacation otherwise, if the server finds atleast 'a' customer waiting for service say $'\xi'$, then the server serves a batch of size min $('\xi', b)$ customers, where $b \geq a$. During, multiple adaptive vacations if the customers of size min (a,b) entering with probability then the server completes the current vacation and enter the system for giving the service. Using the supplementary variable techniques, various characteristics of the queueing systems are obtained.

AMS Subject Classification: 60A10
Key Words: bulk arrival, general bulk service, controllable arriving customers, dormant period, Closedown time, Multiple vacations

Received: January 23, 2016
Published: February 7, 2016

1. Introduction

Queueing models play a very important role in real life situations. Hence, the study of this model is very important for the growth and development of real life systems such as computer systems, industrial assembly lines, road traffic flow, arrival of aircraft passengers, manufacturing systems, communication systems etc. In particular, queues with batch arrivals and bulk service are quite common. Server vacation models are useful for the systems in which the server wants to utilize the idle time for different purposes. Application of server vacation model may be in the designing of local area networks process such as file management, process management, memory management, shared resources maintenance, communication system, manufacturing systems and Chemical processing etc. Arumuganathan and Jeyakumar (1) have mainly focused the results of closedown times in steady state analysis of the bulk queue with multiple vacation. Steady state analysis of a bulk queue with multiple vacations, setup times with N-policy and closedown times is analysed by Arumuganathan and Jeyakumar (2). They derived the expected number of customers in the queue at an arbitrary time epoch and obtained other measures too, the motivation of this paper is to study the behavior of the bulk arrival general service queueing system with controllable arriving customers during multiple adaptive vacation and closedown time. Queueing systems with general bulk service and vacations have been studied, extensively by many authors. Lee (8) has developed a procedure to calculate the system size probabilities for a bulk queueing model. Krishna Reddy et al. (7) discussed the analysis of a bulk queue with N policy multiple vacations and setup times. Ke et al. (6) has studied the operating characteristics of a batch arrival queues under vacation policies with server breakdowns and startup and closedown times. His system there is two vacation policies one is multiple vacation and single vacation and if a customer arrives during a closedown times, the server is immediately started without a startup time. Choudhury, (4) obtained the characteristics of an $M^X/G/1$ queueing system with a setup period and a vacation period. Cox, D.R. (5) obtained the non-Markovian stochastic processes by the inclusion of Supplementary variables. Steady state analysis of two $M^X/M(a,b)/1$ queue models with random breakdowns studied by Madan et al. (9). They considered that the repair time is exponential for one model and deterministic for another one. Chunanyi Lou and Xisheng Yu (3) analyzed a single server markovian bulk arrival general service queue with multiple adaptive vacations in transient state. It may be remarked that this paper addresses the following points 'p' entering service is introduced in a bulk queuing model with closedown time. Variable size batch service is considered here. Probability generating function (PGF) of queue length distribution at an arbitrary time epoch and different completion epoch are obtained by using supplementary variable technique. The paper is organized as follows: firstly, the steady state equations are developed by using supplementary variables. Secondly, the probability generating function of queue size at an arbitrary time epoch is obtained. Next through this some important performance measures such as expected queue length, expected busy period, expected waiting time are derived.

2. Notations and Assumptions

Let X be the group size random variable of the arrival, g_k be the probability that 'k' customers arrive in a batch and $X(z)$ be its probability generating function. Let S(.), V (.) and C(.)be the cdf of service time, vacation time, closedown time. Let $\tilde{S}(\theta), \tilde{V}(\theta)$,and $\tilde{C}(\theta)$ denotes Laplace - Stieltje's transforms of S,V and C. S(x),V(x)and C(x) denotes Pdf of S,V and C. $S^0(t), V^0(t)$ and $C^0(t)$ denotes remaining service, vacation and closedown time of S,V and C at't '.
Let

$$Z(t) : j, \text{if the server is on jth vacation}$$
$$N_s(t) : \text{Number of customers in the service}$$
$$N_q(t) : \text{Number of customers in the queue}$$

Define the random variables as $\xi(t) = (0), [1], \{2\}, < 3 > :$
if the customers in the(service),[vacation],$\{idle\}, < closedownjob >$.

The probabilities for the number of customers in the queue and service are defined as follows:

$$P_{ij}(x,t)dt$$
$$= p\{N_s(t) = i, N_q(t) = j, X \leq S^0(t) \leq X + dt, \xi(t) = 0\}, j \geq 0$$
$$Q_{jn}(x,t)dt$$
$$= p\{N_q(t) = j, X \leq V^0(t) \leq X + dt, \xi(t) = 1, Z(t) = j, n \geq 0\},$$
$$T_0(t)dt = p\{N_q(t) = 0, \xi(t) = 2\}, n \geq 0$$
$$C_n(x,t)dt$$
$$= p\{N_q(t) = n, X \leq C^0(t) \leq X + dt, \xi(t) = 3, Z(t) = j, n \geq 0\}.$$

3. Model Description

In this paper we study the steady state behavior of a single server, general bulk service queue with restricted number of vacations. A batch of customers arrive according to Poisson with arrival rate λ, after completing a bulk service, if the queue length is empty then the server leaves for a vacation of random length. When he returns from a vacation, if the number of customer in the queue is still zero, then the server avails another vacation and so on until he completes M number of vacations, in successions or he finds at least 'a' customer waiting for service. During the vacation time customers entering in to the system with probability 'p'. The Customer bulk arrival is a time homogenous Poisson process, the intervals between bulk customers, are independent identically distributed random variables generated by distribution function $x(z) = 1 - e^{-\lambda z}$, $z \geq 0$, With finite mean and PGF P(z). The service order for customers in different bulk arrivals is under the rule of First Come First Serve and the order in one bulk arrival is arbitrary. The service times $\mu \geq 1$ are independent identically distributed random variables each with distribution s(x). The server takes adaptive multiple vacations when the system becomes empty. Let T be the times of vacation generating by distribution function with PGF Q(z). Assume that the length of each vacation Q_n are independent identically distributed random variables each with general distribution V(z) and finite mean E[v]. The arriving customers directly enter the system when the server does not take vacations. However, the customers who arrive during server vacations enter the system with probability 'p'$(0 \leq p \leq 1)$. The arrival, service, vacation, closedown time and idle time are identically independent of each other.

4. Steady State Analysis

In this section, the probability generating function of the queue size at an arbitrary time epoch is derived the PGF by supplementary variable techniques, will be useful in deriving the important performance measures.

4.1. Queue Size Distribution

In the steady state, let us define for $x > 0$,dividing the equations by Δt and letting the $lim_{\Delta t \to 0}$, the steady state queue size equations are obtained.

$$P_{1,j}(x) = lim_{t \to \infty} P_{1,j}(x,t)$$

$$P_{1,j}(0) = lim_{t\to\infty} P_{1,j}(0,t), \qquad for\, j \geq 0$$
$$Q_{i,j}(x) = lim_{t\to\infty} Q_{i,j}(x,t)$$
$$Q_{i,j}(0) = lim_{t\to\infty} Q_{i,j}(0,t), \qquad for\, 1 \leq j \leq M$$

The Steady state equations are obtained as difference - differential equations of the above are given as

$$-\frac{d}{dx}P_{i0}(x) = -\lambda P_{i0}(x) + \sum_{m=a}^{b} P_{11}(0,t)s(x)\Delta t$$
$$+\sum_{l=1}^{M} Q_{1j}(0,t)s(x)\Delta t + \lambda T_0(0)g_1 s(x) \tag{1}$$
$$-\frac{d}{dx}P_{ij}(x) = -\lambda P_{ij}(x) + \sum_{k=1}^{j} P_{i,j-k}(x,t)\lambda g_k \Delta t \tag{2}$$
$$-\frac{d}{dx}P_{bj}(x) = -\lambda P_{bj}(x) + \sum_{m=a}^{b} P_{m,b+j}(0,t)s(x)\Delta t$$
$$+\sum_{l=1}^{M} Q_{b,j+1}(0,t)s(x)\Delta t + \lambda T_0(t)g_{b+j}s(x)\Delta t$$
$$+\sum_{k=1}^{j} p_{b,j-k}\lambda g_k \Delta t \quad j \geq 1, \tag{3}$$
$$-\frac{d}{dx}C_n(x) = -\lambda C_n(x) + \sum_{m=a}^{b} P_{m,n}(0,t)C(x)\Delta t$$
$$+p\sum_{k=1}^{n} C_{n-k}(x,t)\lambda g_k \Delta t,\ n \leq a-1 \tag{4}$$
$$-\frac{d}{dx}C_n(x) = -\lambda C_n(x) + p\sum_{k=1}^{n} C_{n-k}(x,t)\lambda g_k \Delta t,\ n \geq a \tag{5}$$
$$-\frac{d}{dx}Q_{10}(x) = -\lambda Q_{10}(x) + C_0(0,t)V(x)\Delta t \tag{6}$$
$$-\frac{d}{dx}Q_{1n}(x) = -\lambda Q_{1n}(x) + p\sum_{k=1}^{n} Q_{1,n-k}(x)\lambda g_k + C_n(0)v(x) \tag{7}$$
$$-\frac{d}{dx}Q_{j0}(x) = -\lambda Q_{j0}(x) + Q_{j-1,0}(0)v(x), \qquad 2 \leq j \leq M \tag{8}$$

$$-\frac{d}{dx}Q_{jn}(x) = -\lambda Q_{jn}(x) + p\sum_{k=1}^{n} Q_{j,n-k}(x)\lambda g_k, \qquad 2 \le j \le M \tag{9}$$

$$0 = -\lambda T_0 + \sum_{l=1}^{m} Q_{l,0}(0) \tag{10}$$

The Laplace streusels' transforms of $P_{ij}(x), Q_{jn}(x)$ and $C_n(x)$ are defined as follows

$$\tilde{P}_{1n}(\theta) = \int_0^\infty e^{-\theta x} P_{1n}(x)dx$$

$$\tilde{Q}_{jn}(\theta) = \int_0^\infty e^{-\theta x} Q_{jn}(x)dx$$

and

$$\tilde{C}_n(\theta) = \int_0^\infty e^{-\theta x} C_n(x)dx$$

Taking Laplace streusels' transforms on both sides of the above equations we get,

$$\theta\tilde{P}_{i0}(\theta) - P_{i0}(0) = \lambda\tilde{P}_{i0}(\theta) - \sum_{m=a}^{b} P_{m,i}(0)\tilde{S}(\theta)$$

$$-\sum_{l=1}^{M} Q_{1j}(0)\tilde{S}(\theta) - \lambda T_0(0)g_i\tilde{S}(\theta) \tag{11}$$

$$\theta\tilde{P}_{ij}(\theta) - P_{ij}(0) = \lambda\tilde{P}_{ij}(\theta) - \sum_{k=1}^{j} \tilde{P}_{i,j-k}(\theta)\lambda g_k \tag{12}$$

$$\theta\tilde{P}_{bj}(\theta) - P_{bj}(0) = \lambda\tilde{P}_{bj}(\theta) - \sum_{m=a}^{b} P_{m,j+1}(0)\tilde{S}(\theta)$$

$$-\sum_{l=1}^{m} Q_{l,b+j}(0)\tilde{S}(\theta) - \lambda T_0(0)g_{b+j}\tilde{S}(\theta) + \sum_{k=1}^{j} \tilde{P}_{b,j-k}(\theta)\lambda g_k \quad j \ge 1 \tag{13}$$

$$\theta\tilde{C}_n(\theta) - C_n(0) = \lambda\tilde{C}_n(\theta) - \sum_{m=a}^{b} P_{m,n}(0)\tilde{C}(\theta) - p\sum_{k=1}^{n-a} \tilde{C}_{n-k}(\theta)\lambda g_k \tag{14}$$

$$\theta\tilde{C}_n(\theta) - C_n(0) = \lambda\tilde{C}_n(\theta) - P\sum_{k=1}^{n-a} \tilde{C}_{n-k}(\theta)\lambda g_k \tag{15}$$

$$\theta\tilde{Q}_{10}(\theta) - Q_{10}(0) = \lambda\tilde{Q}_{10}(\theta) - C_0(0)\tilde{V}(\theta) \tag{16}$$

$$\theta\tilde{Q}_{1n}(\theta) - Q_{1n}(0) = \lambda\tilde{Q}_{1n}(\theta) - P\sum_{k=1}^{n}\tilde{Q}_{1n-k}(\theta)\lambda g_k$$
$$- C_0(0)\tilde{V}(\theta)\theta\tilde{Q}_{1n}(\theta) - Q_{1n}(\theta) \quad (17)$$
$$\theta\tilde{Q}_{j0}(\theta) - Q_{j0}(0) = \lambda\tilde{Q}_{j0}(\theta) - Q_{j-1,0}(0)\tilde{V}(\theta) \qquad 2 \le j \le M \quad (18)$$
$$\theta\tilde{Q}_{jn}(\theta) - Q_{jn}(0) = \lambda\tilde{Q}_{jn}(\theta) - Q_{j-1,0}(0)\tilde{V}(\theta)$$
$$- p\sum_{k=1}^{n}\tilde{Q}_{j,n-k}(\theta)\lambda g_k \qquad 2 \le j \le M \quad (19)$$
$$\theta\tilde{Q}_{jn}(\theta) - Q_{jn}(0) = \lambda\tilde{Q}_{jn}(\theta) - p\sum_{k=1}^{n}\tilde{Q}_{j,n-k}(\theta)\lambda g_k \qquad 2 \le j \le M \quad (20)$$
$$0 = -\lambda T_0 + \sum Q_{l,0}(0) \quad (21)$$

To obtain the PGF of the queue size at an arbitrary time, the following probability generating functions are defined

$$\tilde{P}_1(z,\theta) = \sum_{j=0}^{\infty}\tilde{P}_{1j}(\theta)z^j$$
$$P_1(z,0) = \sum_{j=0}^{\infty}P_{1j}(0)z^j$$
$$\tilde{Q}_j(z,\theta) = \sum_{n=0}^{\infty}\tilde{Q}_{jn}(\theta)z^n$$
$$Q_j(z,0) = \sum_{n=0}^{\infty}Q_{jn}(0)z^n$$
$$\tilde{C}(z,\theta) = \sum_{n=0}^{\infty}\tilde{C}_n(\theta)z^n$$

and

$$C(z,0) = \sum_{n=0}^{\infty}C_n(0)z^n \quad (22)$$

Multiplying equations (4.15) by z^0 and (4.16) by $z^n (n \ge 1)$ and summing up from $n = 0$ to ∞ and using equation (4.22) we get after using LST

$$(\theta - \lambda + \lambda p x(z))\tilde{Q}_1(z,\theta) = Q_1(z,0) - C(z,0)\tilde{V}(\theta) \quad (23)$$

Multiplying equations (4.17) by z^0,(4.18) by z^n and (4.19) by z^n summing up from $n = 0\infty$ and using equation (4.22) we get after using LST

$$(\theta - \lambda + \lambda px(z))\tilde{Q}_j(z,\theta) = Q_j(z,0) - \sum_{n=0}^{a-1} Q_{j-1,0}(0)z^n\tilde{V}(\theta) \qquad J \geq 2 \tag{24}$$

Multiplying equations (4.14) by z^0,(4.15) by z^n and (4.19) by z^n summing up from $n = 0$ to ∞ and using equation (4.22) we get after using LST, for $J \geq 2$

$$(\theta - \lambda + \lambda px(z))\tilde{C}(z,\theta) = C(z,0) - \sum_{n=0}^{a-1} \tilde{C}(\theta) \sum_{m=a}^{b} P_{m,n}(0)z^n \tag{25}$$

Multiplying equations (4.11) by z^0 and (4.12),(4.13) by z^j and summing up from $j = 1$ to ∞ and using equation (4.22) we get after using LST

$$\begin{aligned} z(\theta - \lambda + \lambda x(z))\tilde{P}_i(z,\theta) = P_i(z,0) - \sum_{m=a}^{b} \tilde{S}(\theta)[\sum_{m=a}^{b} P_1(z,0)P_{10}(0) \\ + \sum_{m=a}^{b} Q_1(z,0) + g_i\lambda T_0 x(z)] \end{aligned} \tag{26}$$

$$\begin{aligned} &(\theta - \lambda + \lambda x(z))\tilde{P}_b(z,\theta) \\ &= P_b(z,0) - [\tilde{S}(\theta)/z^b][\sum_{m=a}^{b} P(z,0) \\ &+ P_b(z,0) - \sum_{m=a}^{b}\sum_{j=0} b - 1P_{m,j}(0)z^j \\ &+ \sum_{l=1}^{m} Q_1(z,0) + \sum_{m=a}^{b}\sum_{m=a}^{b-1} Q_{1,j}(z,0) \\ &+ \lambda T_0 x(z)] \end{aligned} \tag{27}$$

By Substituting $\theta = \lambda - \lambda px(z)$ in the Equations (4.23) and (4.24), we get

$$Q_1(z,0) = c(z,0)\tilde{V}(\lambda - \lambda px(z)) \tag{28}$$

$$Q_j(z,0) = \sum_{j=a}^{b-1} Q_{j-1,0}(0)\tilde{V}(\lambda - \lambda px(z))z^n \tag{29}$$

By Substituting $\theta = \lambda - \lambda px(z)$in the equation (4.24)

$$C(z,0) = \sum_{n=0}^{a-1} \tilde{C}(\lambda - \lambda px(z)) \sum_{m=a}^{b} P_{m,n}(0)z^n \tag{30}$$

By Substituting $\theta = \lambda - \lambda x(z)$in (4.26)we get,

$$z^b P_b(z,0) = \tilde{S}(\lambda - \lambda x(z))[\sum_{m=a}^{b-1} P_m(z,0) + P_b(z,0) - \sum_{j=0}^{b-1} P_{mj}(0)z^j$$

$$+ \sum_{l=1}^{m} Q_1(z,0) - \sum_{m=a}^{b} \sum_{j=0}^{b-1} Q_{1j}(z,0) + \lambda T_0 x(z)] \tag{31}$$

Solving for $P_1(z,0)$ we get

$$P_1(z,0) = \frac{\tilde{S}(\theta)[-P_{10}(0) + Q_1(z,0) - Q_{10}(0) + \lambda T_0 x(z)]}{z - \tilde{S}(\lambda - \lambda x(z))} \tag{32}$$

By Substituting $Q_1(z,0)$ and $Q_j(z,0)$ in (4.16)and(4.17) we get

$$\tilde{P}_b(z,0) = \frac{[\tilde{S}(\lambda - \lambda x(z)) - \tilde{S}(\theta)]}{(\theta - \lambda + \lambda x(z))[z^b - \tilde{S}(\lambda - \lambda x(z))]} f(z) \tag{33}$$

where $f(z) = \sum_{i=a}^{b-1} P_i(z,0) - \sum_{i=a}^{b} \sum_{j=0}^{b-1} P_{ij}(0)z^j$
$+ \sum_{l=1}^{M} [Q_1(z,0) - \sum_{j=0}^{b-1} Q_{lj}(0)z^j] + \lambda T_0 x(z)$. Let P(z) be the PGF of the queue size at an arbitrary time epoch is the sum of PGF of queue size at service completion epoch, vacation completion epoch and idle time, then

$$P(z) = \sum_{m=a}^{b-1} \tilde{P}_m(z,0) + \tilde{P}_b(z,0) + \sum_{j=1}^{M} \tilde{Q}_j(z,0) + \tilde{C}(z,0) + T_0 \tag{34}$$

Let

$$P_i = \sum_{m=a}^{b} P_{mi}(0)$$

and

$$q_j = \sum_{l=1}^{M} Q_{ij}(0)$$

Using the equations (23),(24),(25) and (22) we get the PGF of queue size $P(z)$.

References

[1] Arumuganathan, R. and Jeyakumar, S. (2004) 'Analysis of a bulk queue with multiple vacations and closedown times', International journal of Information and Management Sciences, Vol. 15, No. 1, pp.45-60.

[2] Arumuganathan, R. and Jeyakumar, S. (2005) 'Steady state analysis of a bulk queue with multiple vacations, setup times with N-policy and closedown times', Applied Mathematical Modelling,Vol. 29, No. 10, pp.972-986.

[3] Chunanyi Lou, Xisheng Yu. 'Analysis of $M^{x}/G/1$ queue with p- entering discipline during adaptive multistage vacations', Mathematical and computer modeling, Vol.51, 2010, pp. 361-368

[4] Choudhury, G. (2000) 'An $MX/G/1$ queueing system with a setup period and a vacation period', Queueing Systems, Vol. 36, Nos. 1-3, pp.23-38.

[5] Cox, D.R. (1965) 'The analysis of non-Markovian stochastic processes by the inclusion of Supplementary variables', Proceedings of Computer Philosophical Society, Vol. 51, pp.433-441.

[6] Ke, J-C. (2003b) 'The optimal control of an M/G/1 queueing system with server startup and two vacation types', Applied Mathematical Modeling, Vol. 27, No. 6, pp.437-450.

[7] Krishna Reddy, G.V., Nadarajan, R. and Arumuganathan, R. (1998) 'Analysis of a bulk queue with N policy multiple vacations and setup times', Computers & Operations Research, Vol. 25, No. 11, pp.957-967.

[8] Lee, H.S.(1991) 'Steady state probabilities for the server vacation model with group arrivals and under control operation policy', Journal of Korean OR/ MS Society, Vol. 16, No. 2, pp.36-48.

[9] Takagi, H. (1991) Queueing Analysis: A Foundation of Performance Evaluation, Vol. I: Vacations and Priority Systems. Part-1, Elsevier Science Publishers, Amsterdam.