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SOFT L-FUZZY QUASI-UNIFORMITIES INDUCED BY SOFT L-NEIGHBORHOOD SYSTEMS

Jung Mi Ko[1], Yong Chan Kim[2 §]
[1,2]Department of Mathematics
Gangneung-Wonju University
Gangneung, Gangwondo, 210-702, KOREA

Abstract: In this paper, we obtain soft L-fuzzy quasi-uniformities induced by soft L-neighborhood systems in complete residuated lattices. Moreover, every N-continuous surjective soft maps are uniformly continuous soft maps. We give their examples.

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1. Introduction

Molodtsov [13] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,3,4,9.10,16,17,19,20]. Pawlak's rough set [14,15] can be viewed as a special case of soft rough sets [4].

Kim [9,10] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice [2,5,6]. He introduced soft L-fuzzy interior and closure operators, quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

In this paper, we obtain soft L-fuzzy quasi-uniformities induced by soft L-neighborhood systems in complete residuated lattices. Moreover, every N-continuous surjective soft maps are uniformly continuous soft maps. We give

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§Correspondence author

their examples.

2. Preliminaries

Definition 2.1. [2,5,6] An algebra $(L, \wedge, \vee, \odot, \rightarrow, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L, \leq, \vee, \wedge, 1, 0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \rightarrow z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \rightarrow)$ is a complete residuated lattice and we denote $L_0 = L - \{0\}$.

Lemma 2.2. [2,5,6] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \rightarrow x = x$, $0 \odot x = 0$,
(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \rightarrow y \leq x \rightarrow z$ and $z \rightarrow x \leq y \rightarrow x$,
(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,
(4) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,
(5) $x \rightarrow (\bigwedge_i y_i) = \bigwedge_i (x \rightarrow y_i)$,
(6) $(\bigvee_i x_i) \rightarrow y = \bigwedge_i (x_i \rightarrow y)$,
(7) $x \rightarrow (\bigvee_i y_i) \geq \bigvee_i (x \rightarrow y_i)$,
(8) $(\bigwedge_i x_i) \rightarrow y \geq \bigvee_i (x_i \rightarrow y)$,
(9) $(x \odot y) \rightarrow z = x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,
(10) $x \odot (x \rightarrow y) \leq y$ and $x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$,
(11) $(x \rightarrow y) \odot (z \rightarrow w) \leq (x \odot z) \rightarrow (y \odot w)$,
(12) $x \rightarrow y \leq (x \odot z) \rightarrow (y \odot z)$ and $(x \rightarrow y) \odot (y \rightarrow z) \leq x \rightarrow z$.

Definition 2.3. [9,10] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F, A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \rightarrow L^X$ is a mapping. We denote $S(X, A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 2.4.[9,10] Let (F, A) and (G, A) be two fuzzy soft sets over a common universe X.

(1) (F, A) is a fuzzy soft subset of (G, A), denoted by $(F, A) \leq (G, A)$ if $F(a) \leq G(a)$, for each $a \in A$.

(2) $(F,A)\wedge(G,A)=(F\wedge G,A)$ if $(F\wedge G)(a)=F(a)\wedge G(a)$ for each $a\in A$.
(3) $(F,A)\vee(G,A)=(F\vee G,A)$ if $(F\vee G)(a)=F(a)\vee G(a)$ for each $a\in A$.
(4) $(F,A)\odot(G,A)=(F\odot G,A)$ if $(F\odot G)(a)=F(a)\odot G(a)$ for each $a\in A$.
(6) $\alpha\odot(F,A)=(\alpha\odot F,A)$ for each $\alpha\in L$.

Definition 2.5. [9,10] Let $S(X,A)$ and $S(Y,B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X,A)\to S(Y,B)$ is a soft mapping where $f : X\to Y$ and $\phi : A\to B$ are mappings.

(1) The image of $(F,A)\in S(X,A)$ under the mapping f_ϕ is denoted by $f_\phi((F,A))=(f_\phi(F),B)$ where

$$f_\phi(F)(b)(y)=\begin{cases}\bigvee_{a\in\phi^{-1}(\{b\})} f^{\rightarrow}(F(a))(y), & \text{if } \phi^{-1}(\{b\})\neq\emptyset,\\ 0, & \text{otherwise.}\end{cases}$$

(2) The inverse image of $(G,B)\in S(Y,B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G,B))=(f_\phi^{-1}(G),A)$ where

$$f_\phi^{-1}(G)(a)(x)=f^{\leftarrow}(G(\phi(a)))(x),\ \forall a\in A, x\in X.$$

(3) The soft mapping $f_\phi : S(X,A)\to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 2.6. [9,10] Let $f_\phi : S(X,A)\to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A),(F_i,A)\in S(X,A)$ and $(G,B),(G_i,B)\in S(Y,B)$,

(1) $(G,B)\geq f_\phi(f_\phi^{-1}((G,B)))$ with equality if f is surjective,
(2) $(F,A)\leq f_\phi^{-1}(f_\phi((F,A)))$ with equality if f is injective,
(3) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B))=\bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,
(4) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B))=\bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,
(5) $f_\phi(\bigvee_{i\in I}(F_i,A))=\bigvee_{i\in I} f_\phi((F_i,A))$,
(6) $f_\phi(\bigwedge_{i\in I}(F_i,A))\leq\bigwedge_{i\in I} f_\phi((F_i,A))$ with equality if f is injective,
(7) $f_\phi^{-1}((G_1,B)\odot(G_2,B))=f_\phi^{-1}((G_1,B))\odot f_\phi^{-1}((G_2,B))$,
(8) $f_\phi((F_1,A)\odot(F_2,A))\leq f_\phi((F_1,A))\odot f_\phi((F_2,A))$ with equality if f is injective.

Definition 2.7. [9] A map $N : X\to(L^A)^{S(X,A)}$ is called a soft L-neighborhood system on X if $N=\{N_x=N(x)\mid x\in X\}$ satisfies the following conditions

(SN1) $N_x((1_X,A))=(1_X,A)(x)=1_A$ and $N_x((1_X,A))=(0_X,A)(x)=0_A$, (SN2) $N_x((F,A)\odot(G,A)) \geq N_x((F,A))\odot N_x((G,A))$ for each $(F,A),(G,A) \in S(X,A)$,

(SN3) If $(F,A) \leq (G,A)$, then $N_x((F,A)) \leq N_x((G,A))$,

(SN4) $N_x((F,A)) \leq (F,A)(x)$ for all $(F,A) \in S(X,A)$ where $(F,A)(x) = F(-)(x)$.

A soft L-neighborhood system is called stratified if

(S) $N_x(\alpha\odot(F,A)) \geq \alpha\odot N_x((F,A))$ for all $(F,A) \in S(X,A)$ and $\alpha \in L$.

The triple (X,A,N) is called a soft L-neighborhood space.

Let (X,A,N) and (Y,B,M) be soft L-neighborhood spaces. A mapping $f_\phi : (X,A,N) \to (Y,B,M)$ is said to be an N-continuous soft map iff

$$M_{f(x)}((G,B))(\phi(a)) \leq N_x(f_\phi^{-1}((G,B)))(a)$$

for each $x \in X, a \in A, (G,B) \in S(Y,B)$.

Definition 2.8. [10] A mapping $\mathcal{U} : S(X \times X, A) \to L$ is called a soft L-fuzzy quasi-uniformity on X iff it satisfies the properties.

(SU1) There exists $(U,A) \in S(X \times X, A)$ such that $\mathcal{U}((U,A)) = 1$,

(SU2) If $(V,A) \leq (U,A)$, then $\mathcal{U}((V,A)) \leq \mathcal{U}((U,A))$,

(SU3) For every $(U,A),(V,A) \in S(X \times X, A)$,

$$\mathcal{U}((U,A) \odot (V,A)) \geq \mathcal{U}((U,A)) \odot \mathcal{U}((V,A))$$

(SU4) If $\mathcal{U}((U,A)) \neq 0$, then $(1_\triangle, A) \leq (U,A)$, where

$$1_\Delta(a)(x,y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y, \end{cases}$$

(SU5) $\bigvee\{\mathcal{U}((V,A)) \mid (V,A) \circ (V,A) \leq (U,A)\} \geq \mathcal{U}((U,A))$

$$\begin{aligned} &((V,A) \circ (V,A))(a)(x,y) = (V(a) \circ V(a))(x,y) \\ &= \textstyle\bigvee_{z \in X}(V(a)(z,x) \odot V(a)(x,y)), \quad \forall\, x,y \in X, a \in A. \end{aligned}$$

The triple $(X,A,\mathcal{U})$ is called a soft L-fuzzy quasi-uniform space.

Let $(X,A,\mathcal{U}_X)$ and $(Y,B,\mathcal{U}_Y)$ be soft L-fuzzy quasi-uniform spaces and $f_\phi : (X,A) \to (Y,B)$ be a soft map. Then $f_\phi : (X,A,\mathcal{U}_X) \to (Y,B,\mathcal{U}_Y)$ is called an uniformly continuous soft map if for all $(V,B) \in S(Y \times Y, B)$,

$$\mathcal{U}_Y((V,B)) \leq \mathcal{U}_X((f \times f)_\phi^{-1}((V,B))), \forall (V,B) \in S(Y \times Y, B).$$

3. Soft L-Fuzzy Quasi-Uniformities Induced by Soft L-Neighborhood Systems

Lemma 3.1. For every $(F,A),(G,A)\in S(X,A)$, we define $(U_F,A),(U_F^{-1},A)\in S(X\times X,A)$ by

$$U_F(a)(x,y)=F(a)(x)\rightarrow F(a)(y).$$

Then we have the following properties.

(1) $(1_{X\times X},A)=(U_{0_X},A)=(U_{1_X},A)$,

(2) $(1_\triangle,A)\le(U_F,A)$,

(3) $(U_F,A)\circ(U_F,A)=(U_F,A)$,

(4) $(U_F,A)\odot(U_G,A)\le(U_{F\odot G},A)$.

Proof. (1)

$$\begin{aligned}&1_{X\times X}(a)(x,y)=1=U_{0_X}(a)(x,y)\\&=0_X(a)(x)\rightarrow 0_X(a)(y)\\&=1_X(a)(x)\rightarrow 1_X(a)(y)=U_{1_X}(a)(x,y).\end{aligned}$$

(2) Since $U_F(a)(x,x)=F(a)(x)\rightarrow F(a)(x)=0$, $(1_\triangle,A)\le(U_F,A)$.

(3) $(U_F,A)\circ(U_F,A)\le(U_F,A)$ from

$$\begin{aligned}&(U_F(a)\circ U_F(a))(x,z)\\&=\bigvee_{y\in X}(U_F(a)(x,y)\circ U_F(a)(y,z))\\&=\bigvee_{y\in X}((F(a)(x)\rightarrow F(a)(y))\odot(F(a)(y)\rightarrow F(a)(z)))\\&\le F(a)(x)\rightarrow F(a)(z)=U_F(a)(x,z).\end{aligned}$$

$(U_F,A)\circ(U_F,A)\ge(U_F,A)$ from

$$\begin{aligned}&(U_F(a)\circ U_F(a))(x,z)\\&=\bigvee_{y\in X}(U_F(a)(x,y)\circ U_F(a)(y,z))\\&\ge((F(a)(x)\rightarrow F(a)(x))\odot(F(a)(x)\rightarrow F(a)(z)))\\&=U_F(a)(x,z).\end{aligned}$$

(4) By Lemma 2.2 (11),

$$\begin{aligned}&U_F(a)(x,y)\odot U_G(a)(x,y)\\&=(F(a)(x)\rightarrow F(a)(y))\odot(G(a)(x)\rightarrow G(a)(y))\\&\le(F(a)(x)\odot G(a)(x)\rightarrow F(a)(y)\odot G(a)(y)\\&=U_{F\odot G}(a)(x,y).\end{aligned}$$

Theorem 3.2. Let (X, A, N) be a soft L-neighborhood space. Define a map $\mathcal{U}_N : S(X \times X) \to L$ by

$$\mathcal{U}_N((U, A)) = \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X} \odot_{i=1}^{n} N_x((F_i, A))(a) \mid \odot_{i=1}^{n}(U_{F_i}, A) \leq (U, A)\}.$$

Then

(1) $(X, A, \mathcal{U}_N)$ is a soft L-fuzzy quasi-uniform space.

(2) If N is stratified, then $\mathcal{U}_N$ is stratified.

Proof. (SU1) Since $(U_{1_X}, A) = (1_{X\times X}, A)$, we have

$$\mathcal{U}_N((1_{X\times X}, A)) \geq \bigvee_{x\in X, a\in A} N_x((1_X, A))(a) = 1.$$

(SU2) If $(U_1, A) \leq (U_2, A)$, $(U_1, A), (U_2, A) \in S(X \times X, A)$, then

$$\begin{aligned}&\mathcal{U}_N((U_1, A)) = \bigvee\{\textstyle\bigvee_{x\in X, a\in A} N_x((F, A)) \mid (U_F, A) \leq (U_1, A)\}\\ &\leq \bigvee\{\textstyle\bigvee_{x\in X, a\in A} N_x((F, A)) \mid (U_F, A) \leq (U_2, A)\} = \mathcal{U}_N((U_2, A)).\end{aligned}$$

(SU3)

$$\begin{aligned}&\mathcal{U}_N((U, A)) \odot \mathcal{U}_N((V, A))\\ &\bigvee\{\textstyle\bigwedge_{a\in A}\bigvee_{x\in X} \odot_{i=1}^{n} N_x((F_i, A))(a) \mid \odot_{i=1}^{n}(U_{F_i}, A) \leq (U, A)\}\\ &\odot \bigvee\{\textstyle\bigwedge_{a\in A}\bigvee_{x\in X} \odot_{i=1}^{n} N_x((G_j, A))(a) \mid \odot_{j=1}^{m}(U_{G_j}, A) \leq (V, A)\}\\ &\leq \bigvee\{\textstyle\bigwedge_{a\in A}\bigvee_{x\in X} N_x(\odot_{i=1}^{n} N_x((F_i, A))(a) \odot (\odot_{i=1}^{n} N_x((F_i, A))(a)\\ &\mid \odot_{i=1}^{n}(U_{F_i}, A) \odot \odot_{i=1}^{n}(U_{F_i}, A) \leq (U, A) \odot (V, A)\}\\ &\leq \mathcal{U}_N((U, A) \odot (V, A)).\end{aligned}$$

(SU4) If $\mathcal{U}_N((U, A)) \neq \perp$, then there exist $(F_i, A) \in S(X, A)$ with $\odot_{i=1}^{n}(U_{F_i}, A) \leq (U, A)$ such that

$$\bigwedge_{a\in A}\bigvee_{x\in X} \odot_{i=1}^{n} N_x((F_i, A))(a) \neq 0$$

For all $a \in A$ and for some $x \in X$,

$$\odot_{i=1}^{n} N_x((F_i, A))(a) \neq 0.$$

By Lemma 3.1, $1_\triangle(a) \leq U_{F_i}(a)$, Hence $(1_\triangle, A) \leq \odot_{i=1}^{n}(U_{F_i}, A) \leq (U, A)$.

(SU5) Suppose there exists $(U, A) \in S(X \times X, A)$ such that

$$\bigvee\{\mathcal{U}_N((V, A)) \mid (V, A) \circ (V, A) \leq (U, A)\} \not\geq \mathcal{U}_N((U, A)).$$

Then there exist $(F_i, A) \in S(X, A)$ with $\odot_{i=1}^n (U_{F_i}, A) \leq (U, A)$ such that

$$\bigvee\{\mathcal{U}_N((V,A)) \mid (V,A)\circ(V,A) \leq (U,A)\} \not\geq \bigwedge_{a\in A}\bigvee_{x\in X} \odot_{i=1}^n N_x((F_i,A))(a).$$

Put $(W, A) = \odot_{i=1}^n (U_{F_i}, A)$. Then

$$\begin{aligned}&(W,A)\circ(W,A) = (\odot_{i=1}^n (U_{F_i},A))\circ(\odot_{i=1}^n (U_{F_i},A))\\ &= \odot_{i=1}^n ((U_{F_i},A)\circ(U_{F_i},A)) = \odot_{i=1}^n (U_{F_i},A) \leq (U,A).\end{aligned}$$

$$\begin{aligned}&\bigvee\{\mathcal{U}_N((V,A)) \mid (V,A)\circ(V,A)\leq(U,A)\}\\ &\geq \mathcal{U}_N((W,A)) \geq \bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x((F_i,A))(a).\end{aligned}$$

It is a contradiction.

(2) Let $(U, A) \in S(X \times X, A), (F, A) \in S(X, A)$ and $\alpha \in L$, we have

$$\begin{aligned}&\mathcal{U}_N(\alpha\odot(U,A))\\ &= \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x((F_i,A))(a) \mid \odot_{i=1}^n (U_{F_i},A) \leq \alpha\odot(U,A)\}\\ &\geq \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x((\alpha\odot F_i,A))(a) \mid \odot_{i=1}^n (U_{\alpha\odot F_i},A) \leq \alpha\odot(U,A)\}\\ &\geq \bigvee\{\alpha\odot\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x((F_i,A))(a) \mid \odot_{i=1}^n (U_{\alpha\odot F_i},A) \leq \alpha\odot(U,A)\}\\ &\geq \bigvee\{\alpha\odot\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x((F_i,A))(a) \mid \odot_{i=1}^n (U_{F_i},A) \leq (U,A)\}\\ &\geq \alpha\odot\mathcal{U}_N((U,A)).\end{aligned}$$

Theorem 3.3. Let (X, A, N) and (Y, B, M) be soft L-neighborhood spaces. Let $f_\phi : (X, A, N) \to (Y, B, M)$ be N-continuous surjective soft map. Then $f_\phi : (X, A, \mathcal{U}_N) \to (Y, B, \mathcal{V}_M)$ is an uniformly continuous soft map.

Proof.

$$\begin{aligned}&(f\times f)_\phi^{-1}(U_G)(a)(x_1,x_2) = U_G(\phi(a))(f(x_1),f(x_2))\\ &= G(\phi(a))(f(x_1)) \to G(\phi(a))(f(x_2))\\ &= f_\phi^{-1}(G)(a)(x_1) \to f_\phi^{-1}(G)(a)(x_2)\\ &= U_{f_\phi^{-1}(G)}(a)(x_1,x_2).\end{aligned}$$

Thus,

$$\begin{aligned}&\mathcal{V}_M((V,A))\\ &= \bigvee\{\bigwedge_{b\in B}\bigvee_{y\in Y}\odot_{i=1}^n M_y((G_i,B))(b) \mid \odot_{i=1}^n (U_{G_i},A) \leq (V,B)\}\\ &(\ f_\phi \text{ is surjective})\\ &= \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n M_{f(x)}((G_i,B))(\phi(a)) \mid\\ &\odot_{i=1}^n (f\times f)_\phi^{-1}((U_{G_i},A)) \leq (f\times f)_\phi^{-1}((V,B))\}\\ &(\ f_\phi \text{ is } N\text{-continuous})\\ &= \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X}\odot_{i=1}^n N_x(f_\phi^{-1}(G_i,B))(a) \mid\\ &\odot_{i=1}^n (U_{f_\phi^{-1}(G_i)},A)) \leq (f\times f)_\phi^{-1}((V,B))\}\\ &\leq \mathcal{U}_N((f\times f)_\phi^{-1}(V,B))).\end{aligned}$$

Example 3.4. Let $U = \{h_i \mid i = \{1, ..., 6\}\}$ with h_i=house and $E = \{e, b, w, c, i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\},\ x \to y = \min\{1 - x + y, 1\}$$

$$x \oplus y = \min\{1, x + y\},\ x^* = 1 - x$$

Then $([0, 1], \wedge, \to, 0, 1)$ is a complete residuated lattice (ref.[2,7,21]). Let $A = \{b, c\} \subset E$ and $X = \{h_1, h_4, h_5\}$. Put (H, A) be a fuzzy soft set as follow:

(H,A)	h_1	h_4	h_5
b	0.5	0.6	0.2
c	0.4	0.5	0.6

$(H,A) \odot (H,A)$	h_1	h_4	h_5
b	0.0	0.2	0.0
c	0.0	0.0	0.2

Define a soft L-neighborhood system $N : X \to (L^A)^{S(X,A)}$ as follows

$$N_{h_1}((F, A)) = \begin{cases} (1, 1), & \text{if } (F, A) = (\overline{1}, A) \\ (0.5, 0.4), & \text{if } (H, A) \leq (F, A), \\ (0, 0), & \text{otherwise,} \end{cases}$$

$$N_{h_4}((F, A)) = \begin{cases} (1, 1), & \text{if } (F, A) = (\overline{1}, A) \\ (0.6, 0.5), & \text{if } (H, A) \leq (F, A), \\ (0.2, 0.0), & \text{if } (H, A) \odot (H, A) \leq (F, A), \\ (0, 0), & \text{otherwise,} \end{cases}$$

$$N_{h_5}((F, A)) = \begin{cases} (1, 1), & \text{if } (F, A) = (\overline{1}, A) \\ (0.2, 0.6), & \text{if } (H, A) \leq (F, A), \\ (0.0, 0.2), & \text{if } (H, A) \odot (H, A) \leq (F, A), \\ (0, 0), & \text{otherwise,} \end{cases}$$

We obtain $(U_H, A), (U_{H\odot H}, A) \in S(X \times X, A)$ such that, for $a \in A$, $U_H(a) \in L^{X\times X}$ with $U_H(a)(x, y) = H(a)(x) \to H(a)(y)$,

$$U_H(b) = \begin{pmatrix} 1 & 1 & 0.7 \\ 0.9 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix},\quad U_H(c) = \begin{pmatrix} 1 & 1 & 1 \\ 0.9 & 1 & 1 \\ 0.8 & 0.9 & 1 \end{pmatrix}$$

$$U_{H\odot H}(b) = \begin{pmatrix} 1 & 1 & 1 \\ 0.8 & 1 & 0.8 \\ 1 & 1 & 1 \end{pmatrix}, \quad U_{H\odot H}(c) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0.8 & 0.8 & 1 \end{pmatrix}$$

By Theorem 3.2, for $(U_H, A) \leq (U, A) \neq (1_{X\times X}, A)$, we have

$$\begin{aligned} &\mathcal{U}_N((U,A)) \\ &= \bigvee\{\bigwedge_{a\in A}\bigvee_{x\in X} N_x((H,A))(a) \mid (U_H,A) \leq (U,A)\} \\ &= N_{h_1}((H,A))(a) \vee N_{h_4}((H,A))(a) \vee N_{h_5}((H,A))(a) \\ &\vee N_{h_1}((H,A))(b) \vee N_{h_4}((H,A))(b) \vee N_{h_5}((H,A))(b) \\ &= 0.5 \vee 0.6 \vee 0.2 \vee 0.4 \vee 0.5 \vee 0.6 = 0.6. \end{aligned}$$

By a similar method, we obtain $\mathcal{U}_N : S(X \times X, A) \to L$ as follows

$$\begin{aligned} &\mathcal{U}_N((U,A)) \\ &= \begin{cases} 1, & \text{if } (U,A) = (1_{X\times X}, A), \\ 0.6, & \text{if } (U_H,A) \leq (U,A) \neq (1_{X\times X}, A), \\ 0.2, & \text{if } (U_{H\odot H},A) \leq (U,A) \not\geq (U_H,A), \\ 0.2, & \text{if } (U_H,A) \odot (U_H,A) \leq (U,A) \not\geq (U_H,A), \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Since $U_H(b) \circ U_H(x) = U_H(x)$ and $U_{H\odot H}(x) \circ U_{H\odot H}(x) = U_{H\odot H}(x)$ for $x \in \{a, b\}$, $\mathcal{U}_N$ is an soft L-fuzzy quasi-uniformity.

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