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COMPLEX FUZZY HYPERGROUPS BASED ON COMPLEX FUZZY SPACES

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Abstract: The aim of this study is to define and elaborate the concept of a complex fuzzy hypergroup, which is based on the concept of a complex fuzzy space.

Key Words: complex fuzzy space, complex fuzzy hyperoperation, complex fuzzy hyperstructure, complex fuzzy hypergroup

1. Introduction

Fuzzy algebraic structures started in early 70's when Rosenfeld (1971) introduced the concept of fuzzy subgroup of a group. Dib (1994) stated the absence of the fuzzy universal set and explained some problems in Rosenfeld's approach. The absence of the fuzzy universal set has direct effect on the mentioned structure of fuzzy theory. A new approach of how to define and study fuzzy groups and fuzzy subgroups is derived in Dib (1994).

Davvaz et al (2013) defined the notion of fuzzy hyperstructure and introduced a correspondence relation between these fuzzy notion and the classical ordinary notion. After introducing fuzzy hyperstructure, they defined the notion of a fuzzy hypergroup (fuzzy H_v-group) and fuzzy sub-hypergroup (fuzzy H_v-subgroup).

Buckley (1989) incorporated the concepts of fuzzy numbers and complex numbers under the name fuzzy complex numbers. This concept has become a

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famous research topic and a goal for many researchers (Buckley 1989; Ramot et al 2002, 2003). However, the concept given by Buckley has different range compared to the range of Ramot's et al definition for complex fuzzy set (CFS). Buckley's range goes to the interval $[0, 1]$, while Ramot's et al range extends to the unit circle in the complex plane.

The purpose of this study is to use the notion of complex fuzzy space to define a complex fuzzy hyperstructure and complex fuzzy hyperoperation as a generalization of fuzzy hypergroup in the sense of Davvaz et al (2013) and to introduce and discuss the concept of complex fuzzy hypergroup.

2. Preliminaries

In this section, we summarize the preliminary definitions and results of Marty (1934) Vougiouklis (1994)required in the sequel.

Let H be a non-empty set and let $\mathcal{P}^*(H)$ be the set of all non-empty subsets of H . A *hyperoperation* on H is a map $\circ : H \times H \to \mathcal{P}^*(H)$ and the couple $(H, \circ)$ is called a *hypergroupoid.*

If A and B are non-empty subsets of H then we denote

$$A \circ B = \bigcup_{a \in A,\ b \in B} \{a\} \circ \{b\}, \quad x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if for all x, y, z of H we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

We say that a semihypergroup $(H, \circ)$ is a *hypergroup* if for all $x \in H$, we have $x \circ H = H \circ x = H$.

A *subhypergroup* $(K, \circ)$ of $(H, \circ)$ is a non-empty set K, such that for all $k \in K$, we have $k \circ K = K \circ k = K$.

The *hypergroupoid* $(H, \circ)$ is called an H_v *group*, if for all $x, y, z \in H$ the following conditions hold:

(1) $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \phi$,

(2) $x \circ H = H \circ x = H$.

An H_v-subgroup $(K, \circ)$ of $(H, \circ)$ is a non-empty set K, such that for all $k \in K$, we have $k \circ K = K \circ k = K$.

3. Complex Fuzzy Spaces

In this section, we present some basic definitions of the complex fuzzy space, complex fuzzy subspace and complex fuzzy binary operation.

Let E^2 be the unit disc. Then $E^2 \square E^2$ is the Cartesian product $E^2 \times E^2$ with partial order defined by:

(i) $(r_1 e^{i\theta_{r1}}, r_2 e^{i\theta_{r2}}) \leq (s_1 e^{i\theta_{s1}}, s_2 e^{i\theta_{s2}})$ iff $r_1 \leq s_1$ and $r_2 \leq s_2 \theta_{r_1} \leq \theta_{s_1}$, and $\theta_{r_2} \leq \theta_{s_2}$ whenever $s_1 \neq 0$ and $s_2 \neq 0$ for all $r_1, s_1, \theta_{r_1}, \theta_{S_1} \in E^2$ and $r_2, s_2, \theta_{r_2}, \theta_{S_2} \in E^2$.

(ii) $(0e^{i\theta_1}, 0e^{i\theta_2}) = (s_1 e^{i\theta_{s1}}, s_2 e^{i\theta_{s2}})$ whenever $s_1 = 0$ or $s_2 = 0$ and $\theta_1 = 0$ or $\theta_2 = 0$ for every $s_1, \theta_1 \in E^2$ and $s_2, \theta_2 \in E^2$.

Definition 3.1. (see Al-Husban, Salleh 2016) A *complex fuzzy space*, denoted by (X, E^2), where E^2 is the unit disc, is set of all ordered pairs (x, E^2), $\quad x \in X$, i.e. $(X, E^2) = \{(x, E^2) : x \in X\}$. We can write

$$(x, E^2) = \{(x, re^{i\theta}) : re^{i\theta} \in E^2\},$$

where $i = \sqrt{-1}$, $r \in [0,1]$, and $\theta \in [0, 2\pi]$. The ordered pair (x, E^2) is called a complex fuzzy element in the complex fuzzy space (X, E^2).

Therefore, the complex fuzzy space is an (ordinary) set of ordered pairs. In each pair the first component indicates the (ordinary) element and the second component indicates a set of possible complex membership values ($re^{i\theta}$, where r represents an amplitude term and θ represents a phase term).

Definition 3.2. (Al-Husban, Salleh 2016) The *complex fuzzy subspace* U of the complex fuzzy space (X, E^2) is the collection of all ordered pairs $(x, r_x e^{i\theta_x})$, where $x \in U_0$ for some $U_0 \subset X$ and $r_x e^{i\theta_x}$ is a subset of E^2, which contains at least one element beside the zero element. If it happens that $x \notin U_0$, then $r_x = 0$ and $\theta_x = 0$. The complex fuzzy subspace U is denoted by $U = \{(x, r_x e^{i\theta_x}) : x \in U_0\}, U_0$ is called the support of U, that is $U_0 = \{ x \in U : r > 0 \text{and} \theta > 0\}$. and denoted by $SU = U_0$. Any empty complex fuzzy subspace is defined as

$$\{(x, \phi_x = 0_x e^{i0_x}) : x \in \emptyset\},$$

i.e. $S\emptyset = \emptyset$.

Definition 3.3. (Al-Husban, Salleh 2016) A complex fuzzy binary operation $\underline{F} = (F, f_{xy})$ on the complex fuzzy space (X, E^2) is a complex fuzzy function from $\underline{F} : (X, E^2) \times (X, E^2)$, to (X, E^2) with comembership functions f_{xy} satisfying:

(i) $f_{xy}(re^{i\theta_r}, se^{i\theta_s}) \neq 0$ if $r \neq 0$, $s \neq 0$, $\theta_s \neq 0$ and $\theta_r \neq 0$

(ii) f_{xy} are onto, i.e. $f_{xy}(E^2 \Box E^2) = E^2$, $x, y \in X$.

The complex fuzzy binary operation $\underline{F} = (F, f_{xy})$ on a set X is a complex fuzzy function from $X \times X$ to X and is said to be uniform if $\underline{F}$ is a uniform complex fuzzy function.

4. Complex Fuzzy Hypergroup

In this section, we use the notion of complex fuzzy space to define complex fuzzy hyperstructure, complex fuzzy hyperoperation and complex fuzzy hypergroup.

Definition 4.1. Let (H, E^2) be a non-empty complex fuzzy space. A *complex fuzzy hyperstructure* (hypergroupoid), denoted by $((H, E^2); \diamondsuit)$ is a complex fuzzy space together with a complex fuzzy function having onto co-membership functions (referred as a complex fuzzy hyperoperation)

$$\diamondsuit : (H, E^2) \times (H, E^2) \to \mathcal{P}^*((H, E^2)),$$

where $\mathcal{P}^*\left((H, E^2)\right)$ denotes the set of all non-empty complex fuzzy subspaces of (H, E^2) and $\diamondsuit = (\Delta, \nabla_{xy})$ with $\Delta : H \times H \to \mathcal{P}^*(H)$ and $\Delta_{xy} : E^2 \times E^2 \to E^2$.

A complex fuzzy hyperoperation $\diamondsuit = (\Delta, \nabla_{xy})$ on (H, E^2) is said to be *uniform* if the associated co-membership functions ∇_{xy} are identical, i.e. $\nabla_{xy} = \nabla$ for all $x, y \in H$.

A uniform complex fuzzy hyperstructure $\left\langle (H, E^2); \diamondsuit \right\rangle$ is a complex fuzzy hyperstructure $\left\langle (H, E^2); \diamondsuit \right\rangle$ with uniform complex fuzzy hyperoperation.

The action of the complex fuzzy function $\diamondsuit = (\Delta, \nabla_{xy})$ on complex fuzzy elements of the complex fuzzy space (H, E^2) can be symbolized as follows:

$$(x, E^2) \diamondsuit (y, E^2) = (x \Delta y, \nabla_{xy}(E^2 \Box E^2)) = (\Delta(x, y), E^2).$$

Theorem 4.1. *To each complex fuzzy hyperstructure* $\left\langle (H, E^2); \diamondsuit \right\rangle$ *there is an associated fuzzy hyperstructure* $\langle (H, I), \diamondsuit \rangle$ *which is isomorphic to the* *complex fuzzy hyperstructure* $\left\langle (H, E^2); \diamondsuit \right\rangle$ *by the correspondence* $(x, E^2) \leftrightarrow (x, I)$.

Proof. Consider the complex fuzzy hyperstructure $\left\langle (H, E^2); \diamondsuit \right\rangle$ Now using the correspondence $(x, E^2) \leftrightarrow (x, I)$ we can redefine complex fuzzy hyperstructure $\diamondsuit = (\Delta, \nabla_{xy})$ with

$$\Delta : H \times H \to \mathcal{P}^*(H) \quad \nabla_{xy} : E^2 \times E^2 \to E^2$$

to be

$$\diamondsuit' : (H, I) \times (H, I) \to (H, I).$$

That is $\diamondsuit'$ defines a fuzzy heperoperation over (X, I). Thus, $\langle (H, I), \Delta \rangle$ is the associated fuzzy hyperstructure.

As a consequence of the above theorem and by using Theorem 4.3 of Davvaz et al (2013) we obtain the following corollary.

Corollary 4.1. *To each complex fuzzy hyperstructure* $\left\langle (H, E^2); \diamondsuit \right\rangle$, *there is an associated ordinary hyperstructure* $\langle H, \Delta \rangle$ *which is isomorphic to the complex fuzzy hyperstructure* $\left\langle (H, E^2); \diamondsuit \right\rangle$ *by the correspondence* $(x, E^2) \leftrightarrow x$.

Example 4.1. Let $Q = \{a\}$. Define the complex fuzzy hyperoperation $\diamondsuit = (\Delta, \nabla_{xy})$ over the complex fuzzy space (Q, E^2) such that

$$\Delta : Q \times Q \to \mathcal{P}^*(Q),$$

with $\Delta(a, a) = a\Delta a = \{a\}$, and $\nabla_{a\,a} : E^2 \times E^2 \to E^2$ with $\nabla_{a\,a}(r\, e^{i\theta_r}, s\, e^{i\theta_s}) = r \wedge s e^{i\wedge(\theta_r, \theta_s)}$. That is

$$(x,\ E^2) \diamondsuit\, (y,\ E^2) = \{(a, \nabla_{a\,a}(E^2 \Box E^2))\},$$

for $(a, E^2) \in (Q, E^2)$. Clearly $\left\langle (Q, E^2); \diamondsuit \right\rangle$ defines a uniform complex fuzzy hyperstructure.

Example 4.2. Let $R = \{a, b\}$. Define the complex fuzzy hyperoperation $\diamondsuit = (\Delta, \nabla_{xy})$ over the complex fuzzy space (R, E^2) such that $\Delta : R \times R \to \mathcal{P}^*(R)$ with

$$\begin{aligned} \Delta(a, a) =& a\Delta a \\ =& \{a\}, \end{aligned}$$

$$\begin{aligned}\Delta(a,b) =& \Delta(b,a) = \Delta(b,b)\\ =& \{a,b\},\end{aligned}$$

and $\nabla_{xy} : E^2 \times E^2 \to E^2$ such that $\nabla_{a\,a}(re^{i\theta_r}, se^{i\theta_s}) = r \wedge se^{i\wedge(\theta_r,\theta_s)}$,

$$\begin{aligned}\nabla_{b\,a}(r\,e^{i\theta_r}, s\,e^{i\theta_s}) =& \nabla_{b\,b}(r\,e^{i\theta_r}, s\,e^{i\theta_s})\\ =& r \vee se^{i\vee(\theta_r,\theta_s)}.\end{aligned}$$

That is

$$(x, E^2)\diamondsuit(y, E^2) = \{(x, \nabla_{xy}(E^2\square E^2), (y, \nabla_{yx}(E^2\square E^2))\}$$

for all $(x,\ E^2)$, $(y,\ E^2) \in (R, E^2)$. Thus $\left\langle (R, E^2); \diamondsuit \right\rangle$ defines a (nonuniform) complex fuzzy hyperstructure.

Definition 4.2. A *complex fuzzy hypergroup* is a complex fuzzy hyperstrucure $\left\langle (H, E^2), \diamondsuit \right\rangle$ satisfying the following axioms:

(i) $((x,\ E^2) \diamondsuit (y, E^2)) \diamondsuit (z, E^2) = (x,\ E^2) \diamondsuit ((y,\ E^2) \diamondsuit (z,\ E^2))$, for all $(x, E^2), (y, E^2), (z, E^2) \in (H, E^2)$,

(ii) $(x,\ E^2) \diamondsuit (H, E^2) = (H, E^2) \diamondsuit (x, E^2) = (H, E^2)$ for all $(x, E^2) \in (H, E^2)$.

Definition 4.3. A *complex fuzzy H_v -group* is a complex fuzzy hyperstructure $\left\langle (H, E^2); \diamondsuit \right\rangle$ satisfying the following conditions:

(i) $((x,\ E^2) \diamondsuit (y,\ E^2)) \diamondsuit (z,\ E^2) \cap (x,\ E^2) \diamondsuit ((y,\ E^2) \diamondsuit (z,\ E^2)) \neq \phi$, for all (x, E^2),

$$(y, E^2), (z, E^2) \in (H, E^2),$$

(ii) $(x, E^2)\diamondsuit(H,\ E^2) = (H, E^2)\diamondsuit (x, E^2) = (H, E^2)$, for all $(x, E^2) \in (H, E^2)$.

If $\left\langle (H, E^2); \diamondsuit \right\rangle$ satisfies only the first condition of Definition 4.2 (Definition 4.3) then it is called a *complex fuzzy semihypergroup* (*complex fuzzy H_v -semigroup*)

A *uniform complex fuzzy hypergroup (complex fuzzy H_v -group)* is a complex fuzzy hypergroup (complex fuzzy H_v-group) having uniform co-membership functions, i.e.

$$\diamondsuit = (\Delta, \nabla_x = \nabla) \text{ for } x \in H.$$

A complex fuzzy hypergroup (complex fuzzy H_v-group) $\langle (H, E^2); \diamondsuit \rangle$ is called a *commutative (or abelian) fuzzy hypergroup* if

$$((x,\ E^2) \diamondsuit\ (y,\ E^2)) = ((y,\ E^2) \diamondsuit\ (x,\ E^2))$$

for all $(x, E^2), (y, E^2) \in (H, E^2)$.

Definition 4.4. Let $\langle (H, E^2); \diamondsuit \rangle$ be a complex fuzzy hypergroup (complex fuzzy H_v-group) and let

$$U = \{(x, r\, e^{i\theta_x}) : \ \ x \in U_0\}$$

be a complex fuzzy subspace of (H, E^2) Then $(U; \diamondsuit)$ is called a *complex fuzzy sub-hypergroup* (*complex fuzzy H_v -subgroup*) of the complex fuzzy hypergroup $\langle (H, E^2); \diamondsuit \rangle$ if $\diamondsuit$ is closed on the complex fuzzy subspace U and $(U; \diamondsuit)$ satisfies the conditions of a complex fuzzy hypergroup (complex fuzzy H_v-group)

The next theorem gives a relation between complex fuzzy hypergroups and fuzzy hypergroups.

Theorem 4.2. *$\langle U; \diamondsuit \rangle$ is a complex fuzzy subhypergroup of the complex fuzzy hypergroup $\langle (H, E^2); \diamondsuit \rangle$ if and only if*

(i) $\langle U; \Delta \rangle$ is a fuzzy subhypergroup of the fuzzy hypergroup $\langle (H, E^2); \diamondsuit \rangle$,

(ii) $\nabla_{xy}(r_x e^{i\theta_x},\ r_y e^{i\theta_y}) = r_{x\Delta y} e^{i\theta_{x\,\Delta\,y}}$.

Proof. Assume that conditions (i) and (ii) are satisfied We want to show that $\langle U; \diamondsuit \rangle$ is a complex fuzzy sub-hypergroup of the complex fuzzy hypergroup $\langle (H, E^2); \diamondsuit \rangle$.

(1) U is closed under $\diamondsuit$: Let $(x, r_x e^{i\theta_x}), (y, r_y e^{i\theta_x}) \in U$. Then

$$\begin{aligned}(x,\ r_x e^{i\theta_x}) \diamondsuit (y,\ r_y e^{i\theta_y}) =& (x\,\Delta\,y, \nabla_{xy}(r_x e^{i\theta_x}, r_y e^{i\theta_y}))\\ =& (x\,\Delta\,y,\ r_{x\Delta\,y} e^{i\theta_{x\Delta\,y}})\ \in U.\end{aligned}$$

(2) $\langle U; \diamondsuit \rangle$ satisfies the condition of a complex fuzzy hypergroup:
Let $(x, r_x e^{i\theta_x})\,(y,\ r_y e^{i\theta_y})$ and $(z, r_z e^{i\theta_z})$, be in U. Then:

$$\begin{aligned}((x,\ r_x e^{i\theta_x}) \diamondsuit (y,\ r_y e^{i\theta_y})) \diamondsuit (z\,, r_z e^{i\theta_z}) =& (x\,\Delta\,y\,, r_{x\,\Delta\,y} e^{i\theta_{x\,\Delta\,y}}) \diamondsuit (z,\ r_z e^{i\theta_z})\\ =& ((x\,\Delta y)\,\Delta\ z\,, r_{(x\,\Delta\,y)\Delta\,z} e^{i\theta_{(x\,\Delta\,y)\Delta\,z}})\end{aligned}$$

$$=(x\,\Delta\,(y\,\Delta\,z)\,,r_{x\,\Delta\,(y\Delta\,z)}r_x e^{i\theta_x}),$$

$$((x,\ r_x e^{i\theta_x})\diamondsuit(y,\ r_y e^{i\theta_y}))\diamondsuit(z\,,r_z e^{i\theta_z}) = (x,\ r_x e^{i\theta_x})\diamondsuit((y,\ r_y e^{i\theta_y})\diamondsuit(z\,,r_z e^{i\theta_z})).$$

Also for any $(x,r_x e^{i\theta_x})\in\ U$ we have

$$\begin{aligned}(x,r_x e^{i\theta_x})\diamondsuit U &= \bigcup_{y\in\ U}(x,r_x e^{i\theta_x})\diamondsuit(y,r_y e^{i\theta_y})\\ &= \bigcup_{y\in\ U}((x\Delta\,y),\ \nabla_{xy}(r_x e^{i\theta_x},r_y e^{i\theta_y}))\\ &= U \quad \text{(by assumptions).}\end{aligned}$$

Similarly

$$U\diamondsuit(x\,,r_x e^{i\theta_x})=U.$$

Therefore, by (1) and (2), $\langle U;\diamondsuit\rangle$ is a complex fuzzy sub-hypergroup.

Conversely, assume that $\langle U;\diamondsuit\rangle$ is a complex fuzzy sub-hypergroup then (i) follow directly from Theorem 4.1. For (ii) let $(x,r_x e^{i\theta_x}),(y,r_y e^{i\theta_y})\in U$, then

$$(x,\ r_x e^{i\theta_x})\diamondsuit(y,\ r_y e^{i\theta_y})=(x\ \Delta\,y,\nabla_{xy}(r_x e^{i\theta_x},r_y e^{i\theta_y}))\in\ U.$$

Thus $\nabla_{xy}(r_x e^{i\theta_x},r_y e^{i\theta_y})$ is the corresponding possible membership values for $x\Delta y$. That is

$$\nabla_{xy}(r_x e^{i\theta_x},r_y e^{i\theta_y})=r_{x\,\Delta\,y}e^{i\theta_{x\,\Delta\,y}}.$$

In order to prove that $\left\langle (H,E^2);\diamondsuit\right\rangle$ is a complex fuzzy H_v-subgroup the same method can be applied.

By using the above theorem and Theorem 4.10 of Davvaz et al (2013) we get the following corollary.

Corollary 4.2. $\langle U;\diamondsuit\rangle$ *is a complex fuzzy subhypergroup of the complex fuzzy hypergroup* $\left\langle (H,E^2);\diamondsuit\right\rangle$ *if and only if*

(i) $\langle U;\Delta\rangle$ *is an ordinary subhypergroup of the fuzzy hypergroup*

(ii) $\nabla_{xy}(r_x e^{i\theta_x},\ r_y e^{i\theta_y})=r_{x\Delta y}e^{i\theta_{x\,\Delta\,y}}.$

5. Conclusion

In this paper, we continue the study of fuzzy hypergroup to the context of complex fuzzy hypergroup. We have defined the notion of a complex fuzzy hypergroup and its sub-hypergroups using the notion of a complex fuzzy space.

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