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SOVEREIGN DEFAULT IN A CURRENCY AREA: A MONETARY GENERAL EQUILIBRIUM MODEL

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Abstract: Real exchange rate overvaluation can induce default. In a currency area formed by two countries, we prove that if prices differ even slightly then inflations will diverge persistently and monetary policy will be unable to suit all currency area members. In fact, it will affect area members inversely. We show that risk premia are time-varying and determined by real exchange rate overvaluation. Finally, we find a transmission mechanism from the overvaluation of the real exchange rate of a currency area member to his default.

AMS Subject Classification: 90A09, 91B28

Key Words: currency areas, sovereign default, cash-in-advance, monetary general equilibrium, stochastic pricing kernel, purchasing power parity, time-varying risk premia

1. Introduction

How can exchange rate overvaluation induce default? The existing literature cannot answer the question. On the one hand, optimum currency areas literature seems to ignore the question of default among area members almost entirely (for a survey see Broz [2005]). On the other, currency crises models deal with speculative attacks on government-controlled exchange rates. According to Jeanne (2000, p. 2), "the main message of the speculative-attack literature is that the reserve flight that occurs during a currency crisis is pro-

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voked by rational arbitrage." Again, this literature does not deal with default or with currency areas where devaluation of nominal local currencies is impossible. To the best of our knowledge, Arghyrou and Tsoukalas (2010) and Arghyrou and Kontonikas [2011] are the first to link exchange rate crises literature to an optimal currency area, the European Monetary Union, EMU. In their model the Greek government decides whether to commit or not to continued EMU participation. Using insights from the literature on currency crises (first generation currency crises model by Krugman [1979], second generation model by Obstfeld [1996] and third generation model by Krugman [1998]), these authors build a model of the eurozone debt crisis that relates real exchange rate deterioration to currency area participation.[1] The model however fails to explain the relation between deteriorating macroeconomic fundamentals and the bond yield spreads or default. Along the same ideas, Lane (2006, p. 47) finds that "there have been surprisingly persistent differences in national inflation rates within the euro area, such that the common monetary policy has not suited all member countries at all times". If Arghyrou and Kontonikas and Arghyrou and Tsoukalas' intuition is correct, then persistent diverging inflations with the resulting real exchange rate over- and undervaluations could lead to more defaults.[2]

We propose a monetary general equilibrium model of overvaluation and default. In this context, there are a number of questions that require an answer:

1) Why is inflation persistently different? We find that if inflations differ even slightly, then the effect of monetary policy on any two countries is inverse. If one country is overvalued then the other will be undervalued; and policy will necessarily exacerbate the position of one of the countries. As expected, a monetary contraction in the currency area induces a lower *purchasing power parity*, PPP, of the small country, country i, relative to the union's—lower inflation in i relative to inflation in the union—; while a monetary expansion induces a higher PPP. However, the effect would be the opposite on the large country, country j. A contraction induces higher PPP of j while an expansion produces a lower PPP. Hence, when designing monetary policy, if the Union's central bank designs the policy for country j when j is undervalued and i is overvalued, then the policy will have the opposite effect on the j. We exemplify with a two country currency area formed by Greece and Germany.

2) How can monetary policy accommodate both or what will be the effect of applying monetary policy to economies with diverging inflations? Mone-

[1] Exchange-rate risk hedging can be seen in González-Aréchiga *et al.* [2010].

[2] Further effects from financial deregulation, in the Latin America context, can be seen in Venegas-Martínez *et al.* [2009].

tary policy can not be applied to both countries equally if their inflations are different. It will affect one of them inversely to what the policy intends. Furthermore, a policy designed for a large country will have a large effect on the small country.

3) Can overvaluation induce default? And if so, what is the transmission mechanism from overvaluation to default? We find that it can. Assume that at each time t, country i's government evaluates whether to default or not by comparing the *continuation value* v^{nd}—the value of not defaulting—against the value of defaulting v^d. A higher overvaluation implies a lower utility of real euro consumption for households in i at each time t—$U\left(c^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1}\right)$—, thus inducing lower v^{nd}. Prolonged overvaluation will reduce v^{nd} until at some point in time $v^{nd} < v^d$. Default then becomes the optimal decision of a benevolent planner for whom, even after internalizing the adverse effects of default on economic activity, financial autarky has a higher payoff than debt repayment.

4) Is leaving the European Union the best option for Greece? Not necessarily, assuming a "mild" risk aversion and a high need to participate in the asset market in autarky—i.e. after default—, we contend that overvaluation increases the value of remaining in the union.

Finally, we find a stochastic pricing kernel that is capable of reproducing real risk premia. We show that the relative risk aversion coefficient between the Greek and the German long-term interest rates appears to be time-variant.

Our general equilibrium model is based on Mendoza and Yue (2012). They establish an explicit link between agents and the sovereign government in order to observe how government default affects economic activity. We also use insights from Alvarez, Atkeson and Kehoe (2008) in order to incorporate exchange rates in a monetary general equilibrium.

The remainder of the paper is organized as follows. Section 2 is a summary of Real exchange rate overvaluation. Section 3 presents an outline of our model. Section 4 introduces the model. In Section 5 we derive the Real euro pricing kernel and the time-varying risk premium. Section 6 concludes.

2. Real Exchange Rate Overvaluation

We have a pure exchange economy. We have two countries $i = 1, 2$. Each country has two agents: households and a government. These two countries form a *currency area* (or *monetary union*). That is, at some time $t = 0$, both countries renounced to their local currency and adopted the European union's currency, the euro, at a fixed exchange rate denoted

$$e^0_{i/€}.$$

For example, Spain substituted the peseta for the euro on January 1st, 2002, at a fixed rate of $e^0_{\text{Ptas}/€} = 166.386$ pesetas for 1 euro.

We are interested in modeling default under overvaluation of one of the two countries exchange rate, i.e. how *real exchange rate* imbalances may induce default of the local government whose exchange rate is overvalued. As real exchange rate we use the PPP. The PPP exchange rate between country i's currency and the euro at time t is

$$e^{t,PPP}_{i/€} = \frac{P^t_i}{P^t_€} \tag{1}$$

where $e^{t,PPP}_{i/€}$ is units of i's currency for 1 euro; P^t_i is the price level of a market basket of goods in country i at time t; and $P^t_€$ is the price level of the **same** market basket in terms of euros.

Overvaluation of i's currency with respect to the euro—equivalently, undervaluation of the euro w.r.t. i's currency—means that the PPP exchange rate at time t exceeds the time $t = 0$ exchange rate:

$$e^0_{i/€} < e^{t,PPP}_{i/€}.$$

Equivalently, by (1) the time $t = 0$ rate is lower than the ratio of the market basket price levels at time t

$$e^0_{i/€} < \frac{P^t_i}{P^t_€},$$

or

$$e^0_{i/€} P^t_€ < P^t_i.$$

Succinctly, overvaluation is a problem of diverging *inflations* – price level variations – between union members. For any of the previous equations to happen it must be true that the time t price level in country i grew faster than the time t price level in the union

$$\frac{P^0_i}{P^t_i} < \frac{P^0_€}{P^t_€}.$$

That is, the cost of the euro market basket converted to i's currency at time the $t = 0$ exchange rate is lower than the cost of i's basket evaluated in i's time

t currency. The effect of inflation is that it reduces real balances. Let y_t^i denote nominal balances at time t in country i's currency. Inflation—the change in price level from time t to time $t+1$—reduces the real spending capacity of nominal balances as follows

$$y_t^i \left(\frac{P_i^t}{P_i^{t+1}} \right).$$

The union's price level $P_{€}^t$ according to Eurostat and the European Central Bank is a *Harmonised Index of Consumer Prices*, HICP. "The HICP for the euro area as a whole is calculated as an average of the national HICP's for the euro area countries, weighted by the countries relative consumption expenditure in the euro area total. Weights are updated annually" (see European Central Bank [2014] and Eurostat [2014]). In our model we shall assume that weights are fixed

$$P_{€}^t = \alpha^i P_i^t + \alpha^j P_j^t, \;\; \alpha^i + \alpha^j = 1. \tag{2}$$

Price equilibrium at time t means that $P_i^t / P_{€}^t = 1$, equivalently, $P_i^t = P_j^t$. Furthermore, over-valuation,

$$P_i^t / P_{€}^t > 1 \quad \text{implies} \quad P_i^t > P_j^t.$$

To see this, if $P_i^t / P_{€}^t > 1$, then

$$\frac{P_i^t}{\alpha^i P_i^t + \alpha^j P_j^t} > 1, \tag{3}$$

which yields

$$P_i^t > \frac{\alpha^j P_j^t}{1 - \alpha^i}.$$

Hence, since $\alpha^i + \alpha^j = 1$, we obtain $P_i^t > P_j^t$. It follows that one country's overvaluation implies the other's undervaluation.

3. An Outline of the Model

We start by sketching out the basic structure of our model.Consider a two-country, cash-in-advance (CIA) economy with an infinite number of periods $t = 0, 1, 2, \ldots$. Both countries form a currency area, CA. The CA has a government and a central bank. Call one of the countries i and the other country j. Each country has a government and a continuum of households of measure one. Households in country i use euros to purchase a **single** home good; households in j use euros to purchase a **single** good in j. Euros in i were fixed at $e^0_{i/€}$ units of i's currency to 1 euro when country i joined the CA at time $t = 0$. The same process happened in country j.

Let M^i_t denote country i's stock of currency in period t, and let $\mu^i_t = M^i_t / M^i_{t-1}$ denote the growth rate of this stock—M^j_t denotes country j's stock of currency in period t and μ^j denotes the growth rate of this stock—. Similarly, let $\mu^{€}_t = M^{€}_t / M^{€}_{t-1}$ denote the growth rate of stock $M^{€}_t$ of euros. As usual, M^i_t and M^j_t are controlled by country i's and j's central banks respectively, and $M^{€}_t$ is controlled by the union's central bank.

Money supply in each central bank is linked to inflation by the equation of exchange:

$$M^i \cdot V^i = P_i \cdot Q_i, \tag{4}$$

where M^i is money supply, V^i is velocity of money, P_i is price level and Q_i is quantity of assets, goods and services sold during the year. In equilibrium real money demand is simply Q_i/V^i.

Let $s_t = (\mu^i_t, \mu^j_t, \mu^{€}_t)$ be the aggregate event in period t, the *state*, while $s^t = (s_1, \ldots, s_t)$ denotes the history of aggregate events through period t. Let $g(s_t)$ denote the density of the probability distribution over the histories of aggregate events.

The real value of a euro at i at time t is given by the PPP rate

$$e^{t,PPP}_{i/€} = \frac{P_i(s_t)}{P_{€}(s_t)}.$$

The PPP rate is a function of the state, but, we shall omit the additional notation—$e^{t,PPP}_{i/€}(s_t)$—in order to simplify. Furthermore, we know that if country i is overvalued then country j will be undervalued, i.e.

$$e^{t,PPP}_{j/€} < e^{t,PPP}_{i/€},$$

or simply,

$$P_j(s_t) < P_i(s_t),$$

that is, the cost of the market basket at i is higher than the price of the same basket at j given s_t.

Trade in this economy in periods $t \geq 1$ occurs in three separate locations: 1) an asset market available to both countries and the Union government; 2) a goods market in country i; and 3) a goods market in country j. In the asset market the two countries' governments sell and buy euro bonds via open market operations. The euro bonds promise delivery of an amount of euros in the asset market in the next period.

In each goods market, households use euros to buy *the* local good—there is only one good in each country—subject to a CIA constraint. At time t, euros buy $\left(e_{i/€}^{t,PPP}\right)^{-1}$ of the local good. Households also sell their endowment of the local good for euros at the time $t = 0$ exchange rate. If country i is overvalued, then at time t $\left(e_{i/€}^{t,PPP}\right)^{-1}$ buys less than $\left(e_{i/€}^{0}\right)^{-1}$.

In period 0 there is an initial round of trade in bonds in the asset market with no trade in goods markets.

Each household in i must pay a real fixed cost γ^i in euros

$$\gamma^i \left(e_{i/€}^{t,PPP}\right)^{-1}$$

for each transfer of cash between the asset market and the goods market. This fixed cost is constant over time for any specific household, but it varies across households in both countries according to a probability distribution $F(\gamma^i)$ with density $f(\gamma^i)$; and according to the PPP exchange rate at time t. Households are indexed by their fixed cost γ^i. The fixed costs for households in each country are in units of the local good. We assume $F(0) > 0$, so that a positive mass of households has a zero fixed cost.

The only source of uncertainty in this economy is shocks to money growth in the two countries and in the CA central bank. We emphasize the physical separation of the markets. Households in country i enter the period with the cash $P_i^{t-1} y_{t-1}^i \left(e_{i/€}^{0}\right)^{-1}$ they obtained from selling their home good endowments in $t-1$, where P_i^{t-1} is the price level and y_{t-1}^i is their endowment of country i's good. Each government conducts an open market operation in the asset market, which determines the realizations of money growth rates μ_t^1 and μ_t^2 in the two countries and the current price levels in the two countries, P_i^t and P_j^t.

The household then splits into a worker and a shopper. Each period the worker sells the household endowment y_t^i for cash $P_i^t y_t^i \left(e_{i/€}^{0}\right)^{-1}$ and rejoins the shopper at the end of the period. The shopper takes the household's cash $P_i^{t-1} y_{t-1}^i \left(e_{i/€}^{0}\right)^{-1}$ with real value

$$n_t^i \left(e_{i/€}^{t,PPP}\right)^{-1} := \frac{P_i^{t-1} y_t^i \left(e_{i/€}^{t-1,PPP}\right)^{-1}}{P_i^t \left(e_{i/€}^{t,PPP}\right)^{-1}}$$

and shops for goods. The shopper can choose to pay the fixed cost $\gamma^i \left(e_{i/€}^{t,PPP}\right)^{-1}$

to transfer an amount of cash $P_i^t x_t^i \left(e_{i/€}^0\right)^{-1}$ with real euro value $x_t^i \left(e_{i/€}^{t,PPP}\right)^{-1}$ to or from the asset market. This fixed cost is paid in cash obtained in the asset market. If the shopper pays the fixed cost, then the CIA constraint in country i in real euros is that consumption equals

$$c_t^i = n_t^i + x_t^i;$$

and if the shopper does not pay the fixed cost, this constraint is

$$c_t^i = n_t^i.$$

Note that overvaluation of i's currency with respect to euro reduces real euro consumption as

$$c_t^i \left(e_{i/€}^0\right)^{-1} > c_t^i \left(e_{i/€}^{t,PPP}\right)^{-1}.$$

The Asset Market

In order for a household to make transfers **from** the asset market at t, it must have bought government bonds at some prior time. Bonds are claims to cash in the asset market with payoffs contingent on the rates of money growth μ_t^1 and μ_t^2 in the current period. The household enters the period with bonds; this cash can be either reinvested in the asset market or, if the fixed cost is paid, transferred to the goods market. With B_t^i denoting the current payoff of the state-contingent bonds purchased in the past, q_t^i the current price of the bonds, and $\int q_t^i \theta_t^i ds_{t+1}$ the household's purchases of new bonds, the asset market constraint is

$$B_t^i \left(e_{i/€}^0\right)^{-1} = \int q_t^i \left(e_{i/€}^0\right)^{-1} \theta_t^i ds_{t+1} + P_i^t \left(x_t^i + \gamma^i\right) \left(e_{i/€}^0\right)^{-1}$$

where θ_t^i is the quantity of government i's bond bought at price q_t^i and payoff at some future time if the fixed cost is paid; and

$$B_t^i \left(e_{i/€}^0\right)^{-1} = \int q_t^i \left(e_{i/€}^0\right)^{-1} \theta_t^i ds_{t+1} \tag{5}$$

otherwise, i.e. current payoff is entirely reinvested. At the beginning of period $t+1$, the household starts with cash $P_i^t y_t^i$ in the goods market and a portfolio of contingent bonds θ_t^i in the asset market. Note that (5) is "nominal": the real euro buying capacity of this cash at time t is

$$B_t^i \left(e_{i/€}^{t,PPP}\right)^{-1},$$

and overvaluation implies

$$B_t^i \left(e_{i/€}^0\right)^{-1} > B_t^i \left(e_{i/€}^{t,PPP}\right)^{-1}.$$

In equilibrium, households with a sufficiently low fixed cost pay it and transfer cash between the goods and asset markets while others do not. We refer to households that pay the fixed cost as *active* and those that do not as *inactive*. Inactive households simply consume their current real balances.

Throughout, we assume that the shoppers are not allowed to store cash from one period to the next. This assumption implies that the CIA constraint holds with equality and greatly simplifies the analysis. For some models in which agents are allowed to store cash and end up doing so in equilibrium, see the work of Alvarez, Atkeson, and Edmond (2003) and Khan and Thomas (2007).

We also assume throughout that in the asset market, households hold their assets in interest-bearing bonds rather than cash. Note that as long as nominal interest rates are positive, bonds dominate cash held in the asset market.

4. The Model

In period 0 there is an initial round of trade in bonds in the asset market with no trade in goods markets. In the asset market in period 0, country i's households of type γ^i have:

1. M_0^i units of country i's money,
2. $\overline{B}_i(\gamma^i)$ units of country i's government debt (bonds)—i.e. claims to $\overline{B}_i(\gamma^i)$ units of country i's currency—, and
3. $\overline{B}_i^{\square,j}$ units of country j's government debt—i.e. claims to $\overline{B}_i^{\square,j}$ units of country j's currency—,

bonds are claims on $\overline{B}_i(\gamma^i)$ units of i's currency—the overbar distinguishes claims from payoffs as in (5)—and $\overline{B}_i^{\square,j}$ units of j's currency both converted to euros in the asset market in that period. As in Alvarez, Atkeson and Kehoe (2008), we will call households of type γ^i the *active* households; and those that do not participate of the asset market as *inactive* households.

Likewise, in the asset market in period 0 country j's households start with

1. M_0^j units of country j money,

2. $\overline{B}_j$ units of country i's government debt—i.e. claims of country i's debt in hands of country j's households—, and
3. $\overline{B}_j^{\square}(\gamma^j)$ units of country j's government debt in the asset market.

We require that

$$\int \overline{B}_i(\gamma^i) f(\gamma^i) d\gamma^i + \overline{B}_j = \overline{B}$$

and

$$\overline{B}_i^{\square,j} + \int \overline{B}_j^{\square}(\gamma^j) f(\gamma^j) d\gamma^j = \overline{B}^{\square,j}.$$

Where $\overline{B}$ denotes the stock of outstanding i currency bonds at $t = 0$ and $\overline{B}^{\square,j}$ denotes the stock of outstanding j currency bonds at $t = 0$.

Agents must convert local currencies to euros using the time $t = 0$ exchange rate since all trades in the asset market are made in time $t = 0$ euros

$$\int \overline{B}_i(\gamma^i) f(\gamma^i) d\gamma^i \left(e^0_{i/€}\right)^{-1} + \overline{B}_j \left(e^0_{i/€}\right)^{-1} = \overline{B} \left(e^0_{i/€}\right)^{-1}$$

and

$$\overline{B}_i^{\square,j} \left(e^0_{j/€}\right)^{-1} + \int \overline{B}_j^{\square}(\gamma^j) f(\gamma^j) d\gamma^j \left(e^0_{j/€}\right)^{-1} = \overline{B}^{\square,j} \left(e^0_{j/€}\right)^{-1}.$$

If country i becomes overvalued with respect to country j at time t, the spending capacity of the outstanding stock of country i's debt is reduced, i.e. it buys less $\overline{B}\left(e^0_{i/€}\right)^{-1} > \overline{B}\left(e^{t,PPP}_{i/€}\right)^{-1}$, while the spending capacity of j's increases at time t $\overline{B}^{\square,j}\left(e^0_{j/€}\right)^{-1} < \overline{B}^{\square,j}\left(e^{t,PPP}_{j/€}\right)^{-1}$—recall that overvaluation of i w.r.t. to j implies j's undervaluation w.r.t. i.

4.1. The Household's Problem

Let us describe the problem of the household of type γ^i in country i. Let $P_i(s_t)$ denote the price level in country i's currency in the home goods market in period t. Observe that prices are a function of the state up to time t, not only of time. We have two markets: goods and assets.

The goods market
In each period $t \geq 1$, in the goods market, households start the period with currency i's real balances $n^i(s_t, \gamma^i)$. They then choose transfers of real balances

between the goods market and the asset market $x^i(s_t, \gamma^i)$, an indicator variable $z^i(s_t, \gamma^i)$ equal to zero if these transfers are zero and one if they are positive, and consumption of country i's good $c^i(s_t, \gamma^i)$ subject to the CIA constraint and the transition law

$$c^i(s_t, \gamma^i) = n^i(s_t, \gamma^i) + x^i(s_t, \gamma^i) z^i(s_t, \gamma^i), \tag{6}$$

$$n^i(s_{t+1}, \gamma^i) = \left(\frac{P_i(s_t)}{P_i(s_{t+1})} \right) y_t^i, \tag{7}$$

where in (6) at $t = 1$, the term $n^i(s_1, \gamma)$ is given by $M_0^i / P_i(s_1)$

Real balances (7) can be expressed in real euros as

$$\begin{aligned} n^i(s_{t+1}, \gamma^i) \left(e_{i/€}^{t+1,PPP} \right)^{-1} &= \left(\frac{P_i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1}}{P_i(s_{t+1}) \left(e_{i/€}^{t+1,PPP} \right)^{-1}} \right) y_t^i \\ &= \left(\frac{\alpha^i P_i(s_t) + \alpha^j P_j(s_t)}{\alpha^i P_i(s_{t+1}) + \alpha^j P_j(s_{t+1})} \right) y_t^i \end{aligned} \tag{8}$$

Analogously, transfers can be expressed in real euros as

$$\begin{aligned} x^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} &= x^i(s_t, \gamma^i) \left(\frac{P_€(s_t)}{P_i(s_t)} \right) \\ &= x^i(s_t, \gamma^i) \left(\frac{\alpha^i P_i(s_t) + \alpha^j P_j(s_t)}{P_i(s_t)} \right). \end{aligned} \tag{9}$$

Finally, combining (6) with (8) and (9) we have that the CIA constraint in real euros is

$$\begin{aligned} c^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} &= n^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} \\ &\quad + x^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} z^i(s_t, \gamma^i), \\ \text{i.e.} \qquad & \\ c^i(s_t, \gamma^i) \left(\frac{P_€(s_t)}{P_i(s_t)} \right) &= n^i(s_t, \gamma^i) \left(\frac{P_€(s_t)}{P_i(s_t)} \right) \\ &\quad + x^i(s_t, \gamma^i) \left(\frac{P_€(s_t)}{P_i(s_t)} \right) z^i(s_t, \gamma^i). \end{aligned} \tag{10}$$

Observe that overvaluation of i's currency at time t reduces household's real euro consumption

$$c^i(s_t,\gamma^i)e^0_{i/€} > c^i(s_t,\gamma^i)\left(\frac{P_€(s_t)}{P_i(s_t)}\right).$$

This implies that the other union member's real euro consumption increases.

The asset market

In the asset market at time $t \geq 1$, country i's households begin by receiving cash payments $B^i(s_t,\gamma)$ on their bonds. They purchase new bonds and make cash transfers to the goods market subject to the sequence of budget constraints

$$B^i(s_t,\gamma^i) = \int q^i(s_t,s_{t+1})B^i(s_t,s_{t+1},\gamma^i)ds_{t+1} + P_i(s_t)\left[x^i(s_t,\gamma^i)+\gamma^i\right]z^i(s_t,\gamma^i). \tag{11}$$

$B^i(s_t,s_{t+1})$ are claims to i's currency in the next asset market issued by i's government at time t at prices $q^i(s_t,s_{t+1})$, given s_t. Equation (11) states that cash payments received at the beginning of each stage—the left side of the equality—can either be reinvested—the first term on the right side of the equality—or transferred to the goods market if γ^i is paid—the last term on the right—. The cost γ^i is paid in the local good.

Using the same procedure as in the goods market, bond holdings can be expressed in real euros as follows

$$\begin{aligned} B^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} &= \int q^i(s_t,s_{t+1})\left(e^{t,PPP}_{i/€}\right)^{-1}B^i(s_t,s_{t+1},\gamma^i)ds_{t+1} \\ &\quad + P_i(s_t)\left[x^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} + \gamma^i\left(e^{t,PPP}_{i/€}\right)^{-1}\right]z^i(s_t,\gamma^i) \end{aligned} \tag{12}$$

or, equivalently,

$$\begin{aligned} B^i(s_t,\gamma^i)\left(\frac{P_€(s_t)}{P_i(s_t)}\right) &= \int q^i(s_t,s_{t+1})\left(\frac{P_€(s_t)}{P_i(s_t)}\right)B^i(s_t,s_{t+1},\gamma^i)ds_{t+1} \\ &\quad + P_i(s_t)\left[x^i(s_t,\gamma^i)\left(\frac{P_€(s_t)}{P_i(s_t)}\right) + \gamma^i\left(\frac{P_€(s_t)}{P_i(s_t)}\right)\right]z^i(s_t,\gamma^i). \end{aligned}$$

Assume that both consumption $c^i(s_t,\gamma^i)$ and real bond holdings $B^i(s_t,\gamma)\left(e^{t,PPP}_{i/€}\right)^{-1}$ are uniformly bounded by some (possibly large) constants.

The problem of country i's households of type γ^i is to maximize utility

$$\sum_{t=1}^{\infty} \beta^t \int U\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right) g(s_t)ds_t \tag{13}$$

subject to (8),(10), and (12). Observe that overvaluation of i's currency with respect to the euro at time t, i.e. $P_i(s_t) > P_€(s_t)$, affects the spending capacity of agents in country i: it reduces real money holdings, utility of consumption is an inverse function of prices, and transfers from the asset market are also adversely affected by increasing $P_i(s_t)$.

Households in country $j \neq i$, solve the analogous problem, with $P_j(s_t)$ denoting the price level in country j's goods market. However, recall that i's overvaluation implies j's undervaluation: country j's households benefit from country i's higher inflation. Since each transfer of cash between the asset market and country i's goods market consumes γ^i units of the i's good, the total goods cost in real euros of carrying out all transfers between i's households and the asset market in t is

$$\gamma^i \left(e_{i/€}^{t,PPP}\right)^{-1} \int z(s_t,\gamma^i) f(\gamma^i)d\gamma^i,$$

and likewise for the foreign households. The resource constraint in the home country is given by

$$\int \left[c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1} + \gamma^i \left(e_{i/€}^{t,PPP}\right)^{-1} z(s_t,\gamma^i)\right] f(\gamma^i)d\gamma^i = y_t^i \left(e_{i/€}^{t,PPP}\right)^{-1}, \tag{14}$$

for all t, s_t, with the analogous constraint in country j. The fixed costs are paid for with cash obtained in the asset market. Thus, country i's money market-clearing condition in $t \geq 1$ is given by

$$\int \left(n^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1} + \left[x^i(s_t,\gamma^i)+\gamma^i\right]\left(e_{i/€}^{t,PPP}\right)^{-1} z^i(s_t,\gamma^i)\right) f(\gamma^i)d\gamma^i = \frac{M^i(s_t)}{P_i(s_t)\left(e_{i/€}^{t,PPP}\right)^{-1}}, \tag{15}$$

for all s_t. The money market-clearing condition for the foreign country is analogous. Equation (15) is very important. Indeed, according to the Maastricht criteria, inflation would not exceed more than 1.5 percentage points higher than the average of the three best performing (lowest inflation) member states of the EU.

First order condition of the household problem with respect to $c^i(s_t, \gamma^i)$ is

$$\beta U' \left(c^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} \right) g(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} - \lambda_2 \left(e_{i/€}^{t,PPP} \right)^{-1} = 0 \qquad (16)$$

where λ_2 is the Lagrange multiplier of the CIA constraint, equation (10) with $e_{i/€}^{t,PPP}$. The first order condition with respect to transfers is

$$\lambda_2 \left(e_{i/€}^{t,PPP} \right)^{-1} z^i(s_t, \gamma^i) + \lambda_3 P_i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} z^i(s_t, \gamma^i) = 0 \qquad (17)$$

where λ_3 is the Lagrange multiplier of the real bond holdings, equation (12).

We assume that the utility function takes the standard constant-relative risk-aversion, CRRA, form

$$U \left(c \left(e_{i/€}^{PPP} \right)^{-1} \right) = \frac{\left(c \left(e_{i/€}^{PPP} \right)^{-1} \right)^{1-\eta} - 1}{1 - \eta}, \quad \text{with } \eta > 0, \eta \neq 1. \qquad (18)$$

The parameter η measures the degree of relative risk aversion that is implicit in the utility function, a greater η implies a greater risk aversion. The first two derivatives of the CRRA utility function with respect to c are

$$U' \left(c \left(e_{i/€}^{PPP} \right)^{-1} \right) = \left(c \left(e_{i/€}^{PPP} \right)^{-1} \right)^{-\eta} \left(e_{i/€}^{PPP} \right)^{-1} \qquad (19)$$

and

$$U'' \left(c \left(e_{i/€}^{PPP} \right)^{-1} \right) = - \left(c \left(e_{i/€}^{PPP} \right)^{-1} \right)^{-1-\eta} \eta \left(e_{i/€}^{PPP} \right)^{-2}. \qquad (20)$$

4.2. The Sovereign Government

Country i's government issues one-period bonds denominated in i's currency contingent on the aggregate state s_t.

In period t, given state s_t, country i's government pays off outstanding bonds $B^i(s_t) \left(e_{i/€}^{0} \right)^{-1}$ in euros and issues claims to euros in the next asset market of the form $B^i(s_t, s_{t+1}) \left(e_{i/€}^{0} \right)^{-1}$ at prices $q^i(s_t, s_{t+1}) \left(e_{i/€}^{0} \right)^{-1}$.

Let $\overline{B}^i \left(e_{i/€}^{0} \right)^{-1}$ denote the stock of outstanding country i's euro bonds at the beginning of period $t = 0$. The set of bond face values is $B^i = [b^i_{\text{min}}, b^i_{\text{max}}] \subset$

$\mathbb{R}$, where $b^i_{\min} \leq 0 \leq b^i_{\max}$. Mendoza and Yue (2012) set the lower bond limit at

$$b^i_{\min} > -\frac{\overline{y}}{r},$$

where $\overline{y}$ is the country's production and r is the interest rate that the government pays for its debt. This boundary is the largest debt that the country could repay with full commitment. Our model does not have production so we will just assume that $b^i_{\min}$ exists.

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz (1981) model, when the country defaults it does not repay at date t and the punishment is exclusion from *world* (*currency area* in our case) asset markets in the same period. The country reenters credit markets with an exogenous probability ϕ^i, and when it does it starts with a fresh record and zero debt. We add to the Eaton-Gersovitz setup an endogenous link between the sovereign default and private economic activity. This link follows from the assumption that both households and the government are excluded from world asset markets when default occurs.

Recall that with higher inflation comes diminishing spending capacity, thus more and more households need to be active in the asset market as overvaluation deepens. If they are excluded from asset markets then consumption is greatly reduced:

$$c = n.$$

This is agrees with the empirical evidence of severe adverse effects from sovereign default on private credit (asset) markets and foreign trade documented by Kaletsky (1985), Rose (2005), Kohlscheen and O'Connell (2008), Reinhart and Rogoff (2010), and Reinhart (2010), and with evidence on inefficient reallocation across foreign and domestic inputs in the aftermath of sovereign default found by Gopinath and Neiman (2010).

The sovereign government chooses a debt policy (amounts and default or repayment) along with private consumption so as to solve a recursive social planner's problem. The state variables are the bond position $B(s_t)$ and s_t. If a government purchases its own bonds, it increases the money supply $\mu > 0$, in effect creating money. When it sells bonds, it is decreasing money supply $\mu < 0$.

The planner takes as given the bond pricing function $q(s_t, s_{t+1})$. The planner's payoff is given by:

$$V(B^i(s_t), s_t) = \max\left\{v^{nd}(B^i(s_t), s_t), v^d(s_t)\right\}, \tag{21}$$

where $v^{nd}(B^i(s_t), s_t)$ is the value of continuing in the credit relationship with foreign lenders (i.e. no default), and v^d is the value of default. If $B^i(s_t) \geq 0$—i.e. the government is a creditor—, the value function is simply $v^{nd}(B^i(s_t), s_t)$ because in this case the economy uses the credit market to save, receiving a return equal to the world's risk-free rate $r_t^{\star}$.

The continuation value is given by the choice of $B^i(s_t, s_{t+1})$ and $c^i(s_t)$ that solves this constrained maximization problem:

$$v^{nd}(B^i(s_t), s_t) = \max_{c^i(s_t), B^i(s_t, s_{t+1})} \left\{ U\left(c^i(s_t, \gamma^i) \left(e_{i/€}^{t,PPP} \right)^{-1} \right) + \beta E\left[V(B^i(s_{t+1}), s_{t+1}) \right] \right\}, \quad (22)$$

subject to:

$$B^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} = M^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} - M^i(s_{t-1}) \left(e_{i/€}^{t-1,PPP} \right)^{-1} + \int q^i(s_t, s_{t+1}) B^i(s_t, s_{t+1}) \left(e_{i/€}^{t,PPP} \right)^{-1} ds_{t+1} \quad (23)$$

$$\int \left[c^i(s_t, \gamma^i) + \gamma^i z^i(s_t, \gamma^i) \right] \left(e_{i/€}^{t,PPP} \right)^{-1} f(\gamma^i) d\gamma^i = y_t^i \left(e_{i/€}^{t,PPP} \right)^{-1} \quad (24)$$

$$\int \left[n^i(s_t, \gamma^i) + \left(x^i(s_t, \gamma^i) + \gamma^i \right) z^i(s_t, \gamma^i) \right] \left(e_{i/€}^{t,PPP} \right)^{-1} f(\gamma^i) d\gamma^i = \frac{M^i(s_t)}{P^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1}} \quad (25)$$

where (23) is the government budget constraint at s_t with $t \geq 1$; (24) is the resource constraint in country i's economy; and (25) is country i's money market clearing condition.

Let ϕ^i denote the probability that country i reenters world capital markets, and $1 - \phi^i$ the probability that it remains in financial autarky. Then, the value of default is

$$v^d(0, s_t) = \max_{c_t, B^i(s_t, s_{t+1})} \left\{ U\left(c^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} \right) + \beta(1 - \phi^i) E\left[v^d(0, s_{t+1}) \right] + \beta \phi^i E\left[V(0, s_{t+1}) \right] \right\}, \quad (26)$$

subject to

$$c^i(s_t)\left(e^{t,PPP}_{i/€}\right)^{-1} = n^i(s_t)\left(e^{t,PPP}_{i/€}\right)^{-1} \tag{27}$$

$$\frac{M^{i,\square}(s_t)\left(e^{t,PPP}_{i/€}\right)^{-1} - M^i(s_{t-1})\left(e^{t-1,PPP}_{i/€}\right)^{-1}}{M^i(s_{t-1})\left(e^{t-1,PPP}_{i/€}\right)^{-1}} = 2\% \tag{28}$$

Note that $v^d(0,s_t)$ takes into account the fact that in case of default at date t, the country has no access to financial markets that period, and hence the country consumes the total income given by the resource constraint (27) in the default scenario. The value of default at t also takes into account that at $t+1$ the economy may reenter world capital markets with probability ϕ^i and associated value $V(0,s_{t+1})$, or remain in financial autarky with probability $1-\phi^i$ and associated value $v^d(s_{t+1})$. Finally, the government constraint under default (28) assumes that if country i defaults, then the union government takes over the money supply in country i, $M^{i,\square}$, and limits its growth in real euros to 2%. The definition of the default set and the probability of default are standard from Eaton-Gersovitz (see Arellano [2008]).

First order conditions for the value of continuation, v^{nd}, are given by the derivative of the Lagrangian with respect to consumption and bond position

$$\frac{\partial \mathcal{L}_{v^{nd}}}{\partial c^i(s_t)} = U^{\square}\left(c^i(s_t)\left(e^{t,PPP}_{i/€}\right)^{-1}\right) + \lambda_2 f^i(\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} = 0 \tag{29}$$

where λ_2 is the lagrange multiplier of the resource constraint (24) in the v^{nd} problem.

The derivative with respect to the bond position $B^i(s_t,s_{t+1})$ is

$$\frac{\partial \mathcal{L}_{v^{nd}}}{\partial B^i(s_t,s_{t+1})} = -\lambda_1 q^i(s_t,s_{t+1})\left(e^{t,PPP}_{i/€}\right)^{-1} = 0, \tag{30}$$

where λ_1 is the lagrange multiplier of the government budget constraint (23) in the v^{nd} problem.

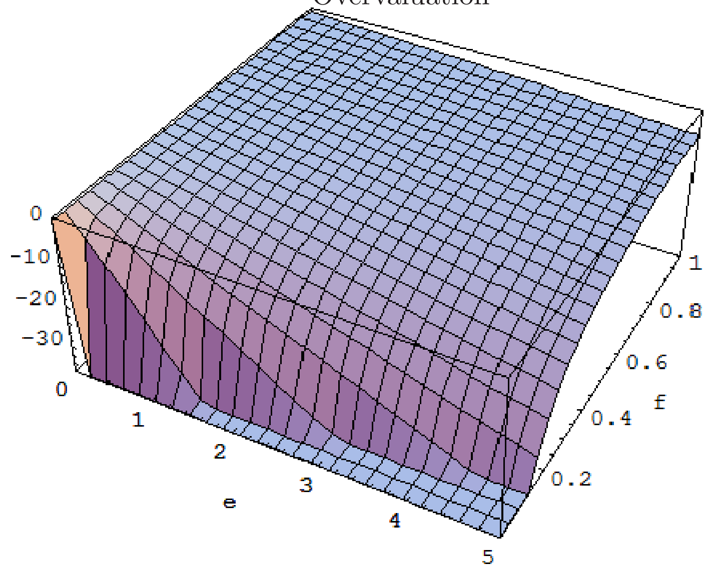
Combining (29) and (19) we have

$$\left(c^i(s_t)\left(e^{t,PPP}_{i/€}\right)^{-1}\right)^{-\eta} = -\lambda_2 f^i(\gamma^i)$$

Solving for $c^i(s_t)$ we have

$$c^i(s_t) = e^{t,PPP}_{i/€}\left(-\lambda_2 f^i(\gamma^i)\right)^{-1/\eta}.$$

Figure I
Overvaluation



Fixed parameters $\lambda_2 = 1$ and $\eta = 3$.

Figure I shows consumption in country i—vertical axis—at different levels of overvalu-ation—e denotes $e_{i/€}^{t,PPP}$—and for any probability of being active in the asset market, $f^i(\gamma^i) \in (0, 1]$. Relative risk aversion is fixed at $\eta = 3$ (a "mild" relative risk aversion). Observe that the relation is inverse, i.e. higher levels of overvaluation induce lower levels of consumption in country i; and this is true for any probability of being active in the asset market although seems sharper at lower levels of $f^i(\gamma^i)$. This situation penalizes higher inflation in country i.

We now repeat the procedure for the v^d problem. First order condition for the value of default, v^d, is

$$\frac{\partial \mathcal{L}_{v^d}}{\partial c^i(s_t)} = U' \left(c^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} \right) - \lambda_1 = 0, \tag{31}$$

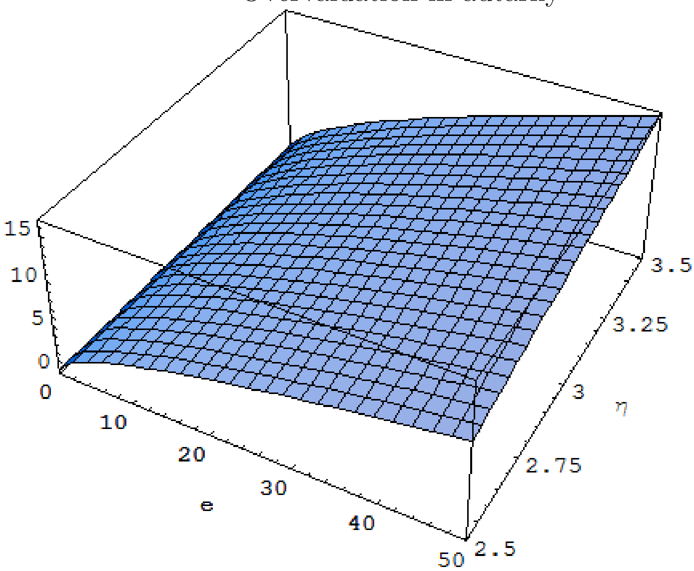
where λ_1 is the lagrange multiplier of the resource constraint (27) in the v^d problem. Combining (31) and (19), we have

$$\left(c^i(s_t) \left(e_{i/€}^{t,PPP} \right)^{-1} \right)^{-\eta} \left(e_{i/€}^{t,PPP} \right)^{-1} = \lambda_1.$$

Solving for $c^i(s_t)$ we have

$$c^i(s_t) = e^{t,PPP}_{i/€} \left(e^{t,PPP}_{i/€} \lambda_1 \right)^{-1/\eta}.$$

Figure II
Overvaluation in autarky



Fixed parameters $\lambda_1 = 1$.

At the same level of relative risk aversion as the v^{nd} case, the relation between overvaluation and consumption is positive. That is, in autarky in a currency area, a higher overvaluation of country i relative to the euro induces higher consumption in country i. This could make unappealing for country i to reenter asset markets once it has defaulted. It also entices higher local inflation.

4.3. Equilibrium

Let c denote the sequence of functions $c(s_t, \gamma)$ and use similar notation for the other variables. An *equilibrium* in this economy is a collection of bond and goods prices $(q, q^{\star})$ and $(P, P^{\star})$, together with bond holdings $(B, B^{\star})$ and allocations for home and foreign households (c, x, z, n) and $(c^{\star}, x^{\star}, z^{\star}, n^{\star})$, such that for each transfer cost γ, the bond holdings and the allocations solve the households' utility maximization problems, the governments' budget constraints hold, and the resource constraints and the money market-clearing conditions are satisfied.

We will next solve for the equilibrium consumption and real balances of both *active households* (those that pay the fixed cost and transfer cash between asset and goods markets) and *inactive households* (those that do not). We then characterize the link between the consumption of active households and asset prices. We focus on households in the home country, the analysis of households in the foreign country is similar.

Notice that constraints (10), (14), (15) and the definition of $P_{€}(s_t)$ in (2) imply that the price level in country i is

$$\begin{aligned} P_i(s_t) &= y_t^i \frac{\left(P_{€}(s_t)\right)^2}{M^i(s_t)} \\ &= y_t^i \frac{\left(\alpha^i P_i(s_t) + \alpha^j P_j(s_t)\right)^2}{M^i(s_t)}. \end{aligned} \tag{32}$$

Using the fact that price equilibrium in the currency area requires $P_i(s_t) = P_j(s_t)$, and that by definition $\alpha^i + \alpha^j = 1$—recall equation (2) above—we have that the equilibrium price in country i given s_t is

$$P_i(s_t) = \frac{M^i(s_t)}{y_t^i} \tag{33}$$

This is the same result found by Alvarez, Atkeson and Kehoe (2008, equation (16), p.13), which is an evidence that our result is correct. However, note that equation (32) implies that if prices deviate from equilibrium—i.e. $P_i(s_t) \neq P_j(s_t)$, precisely over or under-valuation—even slightly as the Maastricht criteria indicate, then the price level in country i given s_t has the following quadratic form

$$P_i(s_t) = \frac{M^i(s_t) - 2\alpha^i \alpha^j P_j(s_t) y_t^i \pm \sqrt{-M^i(s_t)}\sqrt{-M^i(s_t) + 4\alpha^i\alpha^j P_j(s_t) y_t^i}}{2y_t^i(\alpha^i)^2}.$$

By substituting this equation in (1) with (2) we can observe the behavior of $e_{i/€}^{t,PPP}$ as a function of M^i and M^j. The result is that the effect of a monetary expansion or contraction on the real exchange rate , PPP, of a small country depends strongly on the policy implemented in the large country. In order to test the effect, we use data from Germany (country j) and Greece (country i),

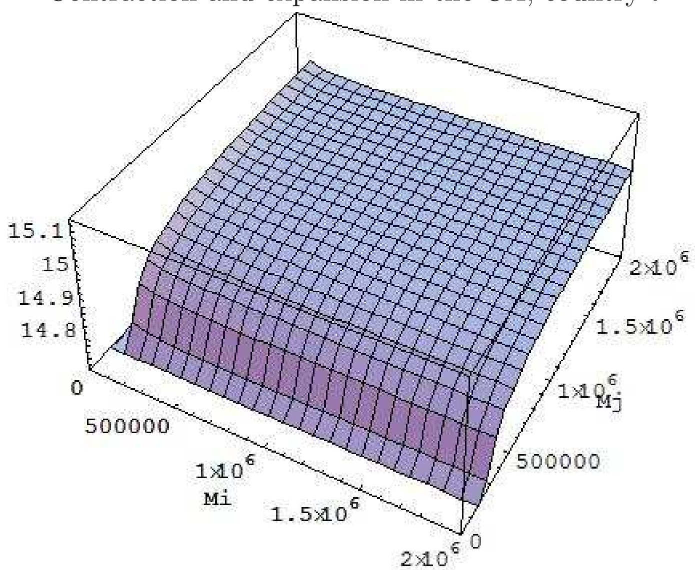
Table I
Selected data from Greece and Germany

	Greece, i	Germany, j
$\alpha^{i,j}$ (%)	6.13	93.87
$y^{i,j}$ (billion USDs)	249.45	3,820.00
$P_{i,j}$ (HICP)	170.6	128.1

Source: HICP data for April 2012 obtained from Eurostat; and GDP data, y, from the IMF; α's are estimated using $\alpha^i = y^i/(y^i + y^j)$.

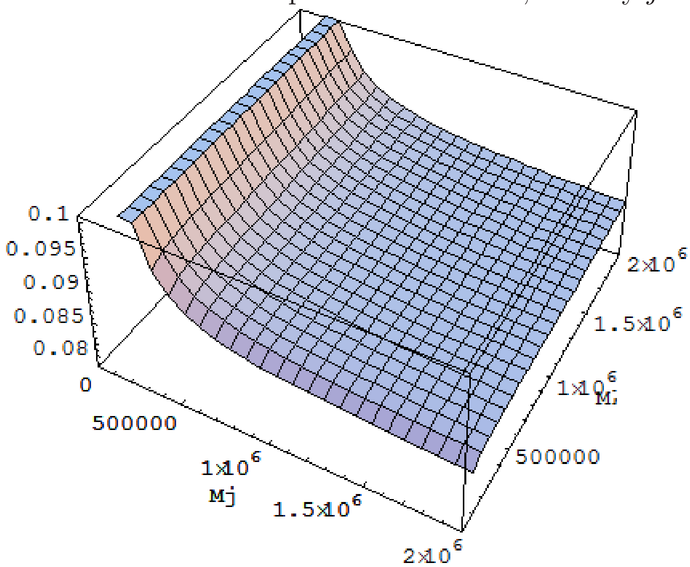
Figure III shows the behavior of the PPP of Greece relative to a currency area formed by Greece and Germany with a monetary contraction with M^i and M^j heading towards 0; and expansion with M^i and M^j moving away from 0

Figure III
Contraction and expansion in the CA, country i



The real exchange rate is observed in the vertical axis. A monetary contraction in the Union implies a lower $e^{t,PPP}_{i/€}$, and a monetary expansion generates a higher $e^{t,PPP}_{i/€}$. It is also important to observe that the largest effect on $e^{t,PPP}_{i/€}$ is due to M^j in both cases. That is, the PPP of a small country is highly sensitive to the monetary policy of the Union. Finally, as expected, the effect on the large country's PPP is the inverse. In Figure IV, we observe $e^{t,PPP}_{j/€}$ on the vertical axis.

Figure IV
Contraction and expansion in the CA, country j



Observe that a monetary contraction generates a higher $e^{t,PPP}_{j/€}$, while a monetary expansion induces a lower German PPP.

Now, recall the inflation rate—$P_i(s_t)/P_i(s_{t+1})$—in country i at time t is π^i_t and also equal to $\pi^i_t = \mu^i_t$ and the real euro money holdings in i are

$$n^i_t \left(e^{t,PPP}_{i/€}\right)^{-1} = \frac{y^i_t \left(e^{t-1,PPP}_{i/€}\right)^{-1}}{\mu^i_t \left(e^{t,PPP}_{i/€}\right)^{-1}}$$

Again, using the fact that price equilibrium requires $P_i(s_t) = P_j(s_t)$, and by definition $\alpha^i + \alpha^j = 1$, the previous equation reduces to

$$n^i_t = \frac{y^i_t}{\mu^i_t}.$$

Hence, the **equilibrium real euro consumption** of inactive households in country i given s_t is

$$c^i(s_t) = \frac{y^i_t}{\mu^i_t}.$$

In this economy, inflation is distorting because it reduces the consumption of any household that chooses to be inactive. This effect induces some households to use real resources to pay the fixed cost, thereby reducing the total amount of resources available for consumption.

Assuming that a complete set of nominal claims are traded in the asset market, the competitive equilibrium allocations and asset prices can be found from the solution to the following planning problem for country i, together with that to the analogous problem for country j. Choose $z^i(s_t, \gamma^i) \in [0, 1]$, $c^i(s_t, \gamma^i) \geq 0$, and $c^i(s_t) \geq 0$ to solve

$$\max \sum_{t=1}^{\infty} \beta^t \int \int U\left(c^i(s_t, \gamma^i) \left(e^{t,PPP}_{i/€} \right)^{-1} \right) f(\gamma^i) g(s_t) d\gamma^i ds_t \tag{34}$$

subject to the resource constraint (14) and the following additional constraint

$$\begin{aligned} c^i(s_t, \gamma^i) \left(e^{t,PPP}_{i/€} \right)^{-1} &= z^i(s_t, \gamma^i) c^i_A(s_t, \gamma^i) \left(e^{t,PPP}_{i/€} \right)^{-1} \\ &+ \left[1 - z^i(s_t, \gamma^i) \right] \left(\frac{y^i_t \left(e^{t-1,PPP}_{i/€} \right)^{-1}}{\mu^i_t \left(e^{t,PPP}_{i/€} \right)^{-1}} \right). \end{aligned} \tag{35}$$

Where $c^i_A(\cdot\,, \cdot)$ denotes the consumption of active households in country i. By substituting (35) in (14) and (34) and maximizing $U(\cdot)$ with respect to the modified resource constraint we have that the first order condition is

$$\beta^t U'(\cdot) g(s_t) = \lambda(s_t) \tag{36}$$

where $\lambda(s_t)$ stands for the multiplier on the resource constraint. The first-order condition clearly implies that all households that pay the fixed cost choose the same consumption level, which means that $c^i_A(s_t, \gamma^i)$ is independent of γ^i. Since this problem is static, this consumption level depends on the current money growth shock μ^i_t only. Hence, we denote this consumption as $c^i_A(\mu^i_t)$.

It is clear that z has a cutoff rule: for each shock μ^i, there is some fixed cost level $\bar{\gamma}^i(\mu)$ at which the households with $\gamma^i \leq \bar{\gamma}^i(\mu^i)$ pay this fixed cost and consume $c^i_A(\mu^i)$, and all other households do not pay and consume instead y^i/μ^i. For each μ^i, the planning problem reduces to choosing $c^i_A(\mu^i)$ and $\bar{\gamma}^i(\mu^i)$, to solve

$$\max U\left(c^i_A(\mu^i) \left(e^{t,PPP}_{i/€} \right)^{-1} \right) F(\bar{\gamma}^i(\mu^i)) + U\left(\frac{y^i_t \left(e^{t-1,PPP}_{i/€} \right)^{-1}}{\mu^i_t \left(e^{t,PPP}_{i/€} \right)^{-1}} \right) \left[1 - F(\bar{\gamma}^i(\mu^i)) \right]$$

subject to

$$c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}F(\bar{\gamma}^i(\mu^i))+\int_0^{\bar{\gamma}^i(\mu^i)}\gamma^i\left(e_{i/€}^{t,PPP}\right)^{-1}f(\gamma^i)d\gamma^i$$
$$+\left(\frac{y_t^i\left(e_{i/€}^{t-1,PPP}\right)^{-1}}{\mu_t^i\left(e_{i/€}^{t,PPP}\right)^{-1}}\right)\left[1-F(\bar{\gamma}^i(\mu^i))\right]=y^i\left(e_{i/€}^{t,PPP}\right)^{-1} \tag{37}$$

The first-order conditions with respect to $c_A^i(\mu^i)$ is

$$U'\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)=\lambda \tag{38}$$

where λ is the Lagrange multiplier. Combining (19) and (38), we have that $U'(\cdot)$ is equal to λ

$$\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)^{-\eta}\left(e_{i/€}^{t,PPP}\right)^{-1}=\lambda.$$

Moreover, the first order condition with respect to $\bar{\gamma}^i$ is

$$-U\left(\frac{y_t^i\left(e_{i/€}^{t-1,PPP}\right)^{-1}}{\mu_t^i\left(e_{i/€}^{t,PPP}\right)^{-1}}\right)F'(\bar{\gamma}^i(\mu^i))+U\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)F'(\bar{\gamma}^i(\mu^i))$$
$$-\lambda\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}F'(\bar{\gamma}^i(\mu^i))+\bar{\gamma}^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}F'(\bar{\gamma}^i(\mu^i))\right. \tag{39}$$
$$\left.+F(\bar{\gamma}^i(\mu^i))\left(e_{i/€}^{t,PPP}\right)^{-1}-\frac{y_t^i\left(e_{i/€}^{t-1,PPP}\right)^{-1}}{\mu_t^i\left(e_{i/€}^{t,PPP}\right)^{-1}}F'(\bar{\gamma}^i(\mu^i))\right)=0.$$

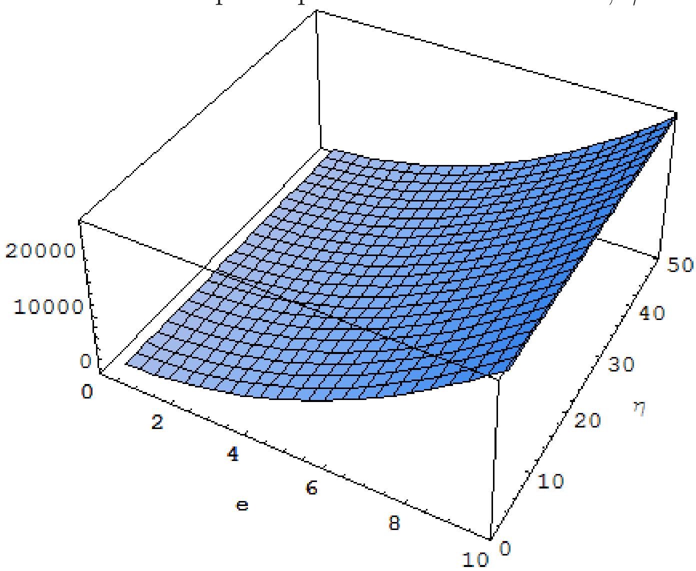
Equations (38) and (39) can be summarized as

$$U\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)-U\left(\frac{y_t^i\left(e_{i/€}^{t-1,PPP}\right)^{-1}}{\mu_t^i\left(e_{i/€}^{t,PPP}\right)^{-1}}\right)$$
$$-U'\left(c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)\left\{c_A^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}+\bar{\gamma}^i(\mu^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right.$$

$$\left. - \frac{y_t^i \left(e_{i/€}^{t-1,PPP}\right)^{-1}}{\mu_t^i \left(e_{i/€}^{t,PPP}\right)^{-1}} \right\} = 0. \quad (40)$$

Solving (40) with respect to $\bar{\gamma}^i(\mu^i)$ and plotting it against η and μ^i we have Figure V

Figure V
The need to participate in the asset market, $\bar{\gamma}^i$



In Figure V, we use the same data from Table I: $y^i = 249.45$ billion US dollars; furthermore, we assume $\lambda = 1$ and $\mu^i = 1$, i.e. the monetary policy is neutral. Observe that $\bar{\gamma}^i$—in the vertical axis—is a direct function of overvaluation, i.e a higher $e_{i/€}^{t,PPP}$ implies higher $\bar{\gamma}^i$. This means that a higher overvaluation implies lower real consumption for households in i, thus a higher need to participate in the asset market to make up for the loss in purchasing power, but a higher cost of participating, $\bar{\gamma}^i$. If $\bar{\gamma}^i$ is high then fewer households will be able to participate in the asset market.

We now have our main conclusion in this section.

Proposition 1. . *The equilibrium* **real euro** *consumption of active households is*

$$c^i(s_t, \gamma^i) = \begin{cases} c_A^i(\mu_t^i) & \text{if } \gamma^i \leq \bar{\gamma}^i(\mu_t^i) \\ y_t^i/\mu_t^i & \text{otherwise,} \end{cases}$$

where the functions $c_A^i(\mu^i)$ and $\bar{\gamma}^i(\mu^i)$ are the solutions to (38) and (39).

5. Real Euro Pricing Kernel and Time-Varying Risk Premium

We now characterize the link between the consumption of active households and asset prices. In order to find the *pricing kernel* (also known in the literature as a *stochastic discount factor*), recall that the household's objective is to maximize the utility of real euro consumption. That is, households in country i face the problem of maximizing expected utility over consumption, given a budget constraint and a market clearing condition,

$$\max \sum_t \beta^t \int U\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right) g(s_t)ds_t$$

equivalently

$$\max\ E_t\left\{\sum_t \beta^t U\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)\right\}. \tag{41}$$

If we simplify the problem to a single period from t to $t+1$, the problem is

$$\max_{c_t,\theta_t}\left[U\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)+\beta E_t\left\{U\left(c^i(s_{t+1},\gamma^i)\left(e_{i/€}^{t+1,PPP}\right)^{-1}\right)\right\}\right] \tag{42}$$

where θ_t is the holding of government i's bond claim by households at time t.

The real euro budget constraint of the economy is

$$c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}=n^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}+\left[x^i(s_t)+\gamma^i\right]\left(e_{i/€}^{t,PPP}\right)^{-1} \tag{43}$$

and the nominal value of the asset market constraint is

$$\begin{aligned}\theta_{t-1}B^i(s_{t-1},\gamma^i)\left(e_{i/€}^{0}\right)^{-1} &= \int_{s_{t+1}} q^i(s_t,s_{t+1})\theta_t\left(e_{i/€}^{0}\right)^{-1} g(s_t)ds_{t+1}\\ &\quad + P_i(s_t)\left[x^i(s_t)+\gamma^i\right]\left(e_{i/€}^{0}\right)^{-1}.\end{aligned} \tag{44}$$

We are studying the case of active households, thus in equations (43) and (44), households in i are assumed to make transfers from the asset market to the goods market: $z^i(s_t,\gamma^i)=1$.

The left side of (44) is the return that households begin period t with—from investments in the previous $t-1$ stage—. The right side is split in two parts: on the one hand, households reinvest part of their earnings in government i's bonds at price $q^i(\cdot,\cdot)$; on the other, they transfer the rest to the goods markets paying cost γ^i.

We combine (43) and (44) to obtain the time t constraint

$$\begin{aligned} c^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} &= n^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} + \theta_{t-1}B^i(s_{t-1},\gamma^i)(P_i(s_t))^{-1} \\ &\quad - E_t\left\{q^i(s_t,s_{t+1})\theta_t\right\}(P_i(s_t))^{-1} \end{aligned} \tag{45}$$

In order to simplify the constraint at time $t+1$ we make the additional assumption that households in i transfer all previous earnings to the goods market, i.e. reinvestment is zero. Thus, (44) becomes at $t+1$

$$\theta_t B^i(s_t,\gamma^i)\left(e^{0}_{i/€}\right)^{-1} = P_i(s_{t+1})\left[x^i(s_{t+1})+\gamma^i\right]\left(e^{0}_{i/€}\right)^{-1} \tag{46}$$

and the constraint of the household at time $t+1$ is

$$c^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1} = n^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1} + \theta_t B^i(s_t,\gamma^i)(P_i(s_{t+1}))^{-1} \tag{47}$$

Forming a Lagrangian with equations (42), (45) and (47)

$$\begin{aligned} \mathcal{L} = {} & U\left(c^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1}\right) + \beta E_t\left\{U\left(c^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1}\right)\right\} \\ & + \lambda_t\left(n^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1} + \theta_{t-1}B^i(s_{t-1},\gamma^i)(P_i(s_t))^{-1}\right. \\ & -E_t\left\{q^i(s_t,s_{t+1})\theta_t\right\}(P_i(s_t))^{-1} \\ & \left.-c^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1}\right) \\ & + \lambda_{t+1}\left(n^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1} + \theta_t B^i(s_t,\gamma^i)(P_i(s_{t+1}))^{-1}\right. \\ & \left.-c^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1}\right). \end{aligned} \tag{48}$$

Taking first order conditions, we have

$$\begin{aligned} \lambda_t &= U'\left(c^i(s_t,\gamma^i)\left(e^{t,PPP}_{i/€}\right)^{-1}\right) \\ \lambda_{t+1} &= \beta U'\left(c^i(s_{t+1},\gamma^i)\left(e^{t+1,PPP}_{i/€}\right)^{-1}\right) g(s_{t+1}) \\ \lambda_{t+1}B^i(s_t,\gamma^i)(P_i(s_{t+1}))^{-1} &= \lambda_t q^i(s_t,s_{t+1})g(s_t)(P_i(s_t))^{-1}. \end{aligned}$$

Simplify to get

$$\left(\beta\frac{U'\left(c^i(s_{t+1},\gamma^i)\left(e_{i/€}^{t+1,PPP}\right)^{-1}\right)g(s_{t+1})P_i(s_t)}{U'\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)g(s_t)P_i(s_{t+1})}\right)B^i(s_t,\gamma^i)=q^i(s_t,s_{t+1}). \tag{49}$$

Hence, the pricing kernel for country i's currency assets in real euros can also be written as

$$m_{t+1}^i=\beta\frac{U'\left(c^i(s_{t+1},\gamma^i)\left(e_{i/€}^{t+1,PPP}\right)^{-1}\right)g(s_{t+1})P_i(s_t)}{U'\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)g(s_t)P_i(s_{t+1})} \tag{50}$$

while the pricing kernel for country j's currency assets in real euros is

$$m_{t+1}^j=\beta\frac{U'\left(c^j(s_{t+1},\gamma^j)\left(e_{j/€}^{t+1,PPP}\right)^{-1}\right)g(s_{t+1})P_j(s_t)}{U'\left(c^j(s_t,\gamma^j)\left(e_{j/€}^{t,PPP}\right)^{-1}\right)g(s_t)P_j(s_{t+1})} \tag{51}$$

These kernels are the state-contingent real-euro prices for countries i and j currency normalized by the probabilities of the state. Note that both kernels are inversely related to prices $P_i(\cdot)$ and $P_j(\cdot)$, so higher prices at $t+1$ imply lower pricing kernels in $t+1$.

These pricing kernels can price any asset in i's or j's currency converted to real euros. In particular, the pricing kernels imply that any asset purchased in period t with an i's currency return of $R_{t+1}^i = B^i(s_t,\gamma^i)/q^i(s_t,s_{t+1})$—the payment on government i's bond at $t+1$ divided by the price of the bond at t—between periods t and $t+1$ satisfies the Euler equation

$$1 = E_t m_{t+1}^i R_{t+1}^i \tag{52}$$

where, for simplicity we drop the s_t notation. The same can be done for every possible j currency asset with return R_{t+1}^j. Note that $\exp(r_t^i)$ is the i-currency return on an i-currency-denominated bond with interest rate r_t^i, and $\exp(r_t^j)$ is the expected return on a j-currency-denominated bond with interest rate r_t^j; the Euler equations imply that

$$r_t^i = -\log E_t m_{t+1}^i \quad \text{and} \quad r_t^j = -\log E_t m_{t+1}^j. \tag{53}$$

The pricing kernels for i and j currencies have a natural relation in real terms

$$m^j_{t+1} = m^i_{t+1}\left(\frac{e^{t+1,PPP}_{i/j}}{e^{t,PPP}_{i/j}}\right).$$

Where $e^{t,PPP}_{i/j}$ is the real exchange rate between currencies i and j. Recall that the national currencies do not exist, both countries adopted the euro at time $t = 0$, however the real relation between currencies can be characterized as

$$e^{t,PPP}_{i/j} = e^{t,PPP}_{i/€}\left(e^{t,PPP}_{j/€}\right)^{-1}.$$

The i-currency return on a j-government bond is

$$\exp(r^j_t)\left(\frac{e^{t+1,PPP}_{i/j}}{e^{t,PPP}_{i/j}}\right)$$

obtained by converting one unit of i's currency in period t to $\left(e^{t,PPP}_{i/j}\right)^{-1}$ of j's units; buying a bond paying interest rate $\exp(r^j_t)$; and then converting the resulting j's units back into i's in $t+1$ at the exchange rate $e^{t+1,PPP}_{i/j}$.

Equilibrium requires that

$$1 = E_t m^i_{t+1} R^i_{t+1} = E_t\left\{\left[m^i_{t+1}\left(\frac{e^{t+1,PPP}_{i/j}}{e^{t,PPP}_{i/j}}\right)\right] R^j_{t+1}\right\} \tag{54}$$

Since (54) holds for every j-currency return, $m^i_{t+1}\left(\frac{e^{t+1,PPP}_{i/j}}{e^{t,PPP}_{i/j}}\right)$ is an equilibrium pricing kernel for j-currency assets. Complete asset markets have only one j-currency pricing kernel, so

$$\log e^{t+1,PPP}_{i/j} - \log e^{t,PPP}_{i/j} = \log m^j_{t+1} - \log m^i_{t+1}. \tag{55}$$

Alvarez, Atkeson and Kehoe (2008, p. 7, equation (1)) define country j's nominal *risk premium* p^j_t. We use their definition to define country j's *real risk premium* as

$$p^j_t = r^j_t + \left|E_t\left(\log e^{t+1,PPP}_{i/j}\right)\right| - \left|\log e^{t,PPP}_{i/j}\right| - r^i_t. \tag{56}$$

That is, p^j_t is the difference between the expected log i-currency return on a j government bond, $r^j_t + \left|E_t\left(\log e^{t+1,PPP}_{i/j}\right)\right| - \left|\log e^{t,PPP}_{i/j}\right|$, and the log return on a i-government bond, r_t^i. Clearly, the i-currency return on a j-government bond is risky because the future exchange rate $e_{i/j}^{t+1,PPP}$ is unknown at t. Also, recall that an equilibrium condition in the monetary union is that prices must be equal in both countries, $P_t^i = P_t^j$. Overvaluation or undervaluation of any country w.r.t. PPP will increase j's risk premium, p_t^j. In fact, higher prices in j will lead to a faster increase of p^j than higher prices in i.

Substituting (53) and (55) into the risk premium (56) gives that

$$\begin{aligned} p_t^j = & \left| E_t\left(\log m_{t+1}^j\right) - E_t\left(\log m_{t+1}^i\right) + \log e_{i/j}^{t,PPP}\right| \\ & - \left(\log E_t m_{t+1}^j - \log E_t m_{t+1}^i + \left|\log e_{i/j}^{t,PPP}\right|\right), \end{aligned} \tag{57}$$

which, in equilibrium reduces to Alvarez, Atekson and Kehoe's (2008, equation (28), pg. 17)

$$p_t^j = \left| E_t\left(\log m_{t+1}^j\right) - E_t\left(\log m_{t+1}^i\right)\right| - \left(\log E_t m_{t+1}^j - \log E_t m_{t+1}^i\right).$$

Hence, a currency's risk premium depends on the difference between the expected value of the log and the log of the expectation of the pricing kernel. Jensen's inequality—$\varphi(E(X)) \leq E(\varphi(X))$—implies that fluctuations in the risk premium are driven by fluctuations in the conditional variability of the pricing kernel.

Recall that overvaluation of country i's currency implies that prices in country i are higher than prices in j. Also, remember that the pricing kernels are inversely related to prices, hence overvaluation in country i will lead it's pricing kernel to be lower than country j's. These facts together with Jensen's inequality imply that p_t^j is a growing function of overvaluation in country i. That is, risk in country j grows with overvaluation in country i. Conversely, undervaluation of country j's currency with respect to country i's reduces country i's risk premium. In this sense, country i and j share risk.

In order to apply the pricing kernel, substitute (19) in (49) to obtain

$$m_{t+1}^i = \left(\beta \frac{\left(c^i(s_t,\gamma^i)\left(e_{i/€}^{t,PPP}\right)^{-1}\right)^{\eta} e_{i/€}^{t,PPP} g(s_{t+1}) P_i(s_t)}{\left(c^i(s_{t+1},\gamma^i)\left(e_{i/€}^{t+1,PPP}\right)^{-1}\right)^{\eta} e_{i/€}^{t+1,PPP} g(s_t) P_i(s_{t+1})} \right).$$

or

$$m^i_{t+1} = \beta \left(\frac{\left(c^i(s_t, \gamma^i) \left(e^{t,PPP}_{i/€} \right)^{-1} \right)^{\eta}}{\left(c^i(s_{t+1}, \gamma^i) \left(e^{t+1,PPP}_{i/€} \right)^{-1} \right)^{\eta}} \right) \left(\frac{e^{t,PPP}_{i/€}}{e^{t+1,PPP}_{i/€}} \right) \left(\frac{g(s_{t+1})}{g(s_t)} \right) \left(\frac{P_i(s_t)}{P_i(s_{t+1})} \right)$$

$$= \beta \left(\frac{\left(c^i(s_t, \gamma^i) \left(e^{t,PPP}_{i/€} \right)^{-1} \right)^{\eta}}{\left(c^i(s_{t+1}, \gamma^i) \left(e^{t+1,PPP}_{i/€} \right)^{-1} \right)^{\eta}} \right) \nabla e^{t,PPP}_{i/€} \cdot (\nabla g(s_t))^{-1} \cdot \pi^i_t$$

In Table II we estimate the stochastic pricing kernels of Greece and Germany. Greece is country i and Germany is country j. Table II presents three cases: a first case with $\eta = 0.5$ which represents a relative risk loving coefficient; case two is estimated with $\eta = 3$, a "mild" risk aversion; and in case three we use $\eta = 50$, an "extreme" risk aversion.

Table II

Case 1: Stochastic pricing kernels with $\eta = 0.5$

Dates	m^i_{t+1}	m^j_{t+1}	∇m^i_{t+1} (%)	∇m^j_{t+1} (%)
Q4-2013	1.018	1.030		
Q1-2014	1.051	1.024	3.2	−0.6
Q2-2014	1.013	1.027	−3.6	0.4
Q3-2014	1.043	1.019	3.0	−0.8

Case 2: Stochastic pricing kernels with $\eta = 3$

Dates	m^i_{t+1}	m^j_{t+1}	∇m^i_{t+1} (%)	∇m^j_{t+1} (%)
Q4-2013	0.990	1.030		
Q1-2014	1.058	1.006	6.8	−2.3
Q2-2014	1.020	1.017	−3.6	1.1
Q3-2014	0.980	0.994	−3.9	−2.3

Case 3: Stochastic pricing kernels with $\eta = 50$

Dates	m^i_{t+1}	m^j_{t+1}	∇m^i_{t+1} (%)	∇m^j_{t+1} (%)
Q4-2013	0.587	1.033		
Q1-2014	1.199	0.727	104.4	−29.6
Q2-2014	1.167	0.842	−2.6	15.9
Q3-2014	0.304	0.622	−73.9	−26.1

Source: Own estimation with selected data from http://stats.oecd.org

Where ∇m_{t+1}^i and ∇m_{t+1}^j are the stochastic pricing kernels' relative variations from one period to the next. Observe that the kernels are extremely sensitive to the relative risk aversion coefficient.

Finally, Table III shows Greece's real risk premium, p_t^i, relative to the German long-term interest rate compared with the difference between both countries' long term interest rates

Table III
Greece's p_t^i relative to Germany.

	$r_t^i - r_t^j(\%)$ nominal	$p_t^i(\%)$ eq. (56)	$p_t^i(\%)$ $\eta = 2.5$	$p_t^i(\%)$ $\eta = 0.5$	$p_t^i(\%)$ $\eta = 0.8$	$p_t^i(\%)$ $\eta = 0.92$
Q4-2013	6.9	6.9	6.8	6.0	6.0	6.0
Q1-2014	6.0	5.4	3.9	5.3	5.1	5.0
Q2-2014	4.8	5.1	3.9	5.4	5.1	5.0
Q3-2014	5.0	4.1	2.9	5.0	4.7	4.5

Source: Own estimation with selected data from http://stats.oecd.org

Where $r_t^i - r_t^j$ is the nominal difference between the long term interest rates of Greece and Germany, the nominal risk premium. We assume $g(s_t) = g(s_{t+1})$ for all computations. The second column is the real market risk premium estimated using (56). The last four columns are calculated using (57). Table III suggests that our model is capable of replicating market behavior and that the relative risk aversion coefficient is time variant. Regard that the observed p_t^i estimated using (56) coincide with risk premiums estimated with different η's. Table III also indicates that market risk premiums vary between risk-loving and "mild" risk aversion.

6. Conclusions

As mentioned in the introduction our objective was to model real exchange rate overvaluation in a currency area. Using insights from Alvarez, Atkeson and Kehoe (2008) and Mendoza and Yue (2012) we build a monetary general equilibrium model that allows us to shed light on several important issues regarding prices within a currency area. For example, we find that persistent diverging inflations between European Union members is due to monetary policy, which exacerbates divergence. We find that monetary policy can not be applied equally to economies with diverging inflations. We find that real exchange rate overvaluation can induce default, that is, Arghyrou and Tsoukalas

(2010) and Arghyrou and Kontonikas (2011) intuition is correct. Thus, allowing overvaluation to grow could result in more defaults among EMU members.

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List of notation

$e^0_{i/€}$, the exchange rate at which country i entered the currency area fixed time $t = 0$.

P_i^t, Harmonised Index of Consumer Prices, HICP, in country i at time t, analogous for country j.

π_t^i, is inflation in country i at time t.

$P_€^t$, HICP of the currency area at time t.

$e^{t,PPP}_{i/€}$, country i's purchasing power parity at time t w.r.t. the currency area; analogous for country j.

y_t^i, nominal balances at time t in country i's currency; analogous for country j.

α^i, weight of country i in the CA's HICP; analogous for country j.

M_t^i, is money stock in country i at time t; analogous for country j.

μ_t^i, is money stock growth in country i at time t; analogous for country j.

$s_t = (\mu_t^i, \mu_t^j, \mu_t^{\square})$, aggregate event in period t or the *state*.

$s^t = (s_1, \ldots, s_t)$, history of aggregate events through period t.

$g(s_t)$, the density of the probability distribution over the histories of aggregate events.

γ^i, real fixed cost paid by households in country i in order to make cash transfers from the asset market to the goods market.

$F(\gamma^i)$, probability distribution of γ^i; analogous for country j.

$f(\gamma^i)$, probability density function of γ^i; analogous for country j.

n_t^i, real balances in country i at time t.

x_t^i, amount of cash transfered from the asset market to the goods market by households in country i at time t.

c_t^i, counsumption of households in country i at time t.

q_t^i, price of country i's government bond at time t.

B_t^i, payoff of country i's government bond at time t.

θ_t^i, quantity of government i's bond held by households.

$B^i(s_t, s_{t+1})$, are claims to i's currency in the next asset market issued by i's government at time t at prices

$q^i(s_t, s_{t+1})$, given s_t

$\overline{B}_i(\gamma^i)$, units of country i's government debt (bonds)

$\overline{B}_i^{\square,j}$, units of country j's government debt—i.e. claims to $\overline{B}_i^{\square,j}$ units of country j's currency—.

$\overline{B}$, denotes the stock of outstanding i currency bonds at $t = 0$

$\overline{B}^{\square,j}$, denotes the stock of outstanding j currency bonds at $t = 0$

$z^i(s_t, \gamma^i)$, indicator function equal to 1 for households that are active in the asset market and equal to 0 for inactive households.

β, discount factor.

η, parameter that measures the degree of relative risk aversion. $\eta > 1$ indicates risk aversion; $\eta = 1$ is risk neutrality; and $\eta < 1$ is risk affection.

v^{nd}, value of not defaulting.

v^d, value of defaulting.

ϕ^i, probability that country i reenters world capital markets after defaulting.

λ's are Lagrange multipliers.

c_A^i, consumption of active households in country i.

$\bar{\gamma}^i$, threshold level of γ^i below which households will be active in the asset market.

m_t^i, pricing kernel for country i's currency assets in real euros.

$R_{t+1}^i = \frac{B^i(s_t,\gamma^i)}{q^i(s_t,s_{t+1})}$, i's currency return at time $t+1$.

r_t^i, interest rate on an i-currency-denominated bond.

Note: There is an equivalent notation of country j for every symbol above.