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L-FUZZY (K, E)-SOFT QUASI-UNIFORMITIES AND L-FUZZY (K, E)-SOFT PRECLOSURE OPERATORS

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Abstract: In this paper, we introduce the notion of L-fuzzy (K, E)-soft preclosure spaces. We investigate two L-fuzzy (K, E)-soft preclosure spaces induced by an L-fuzzy (K, E)-soft quasi-uniform space. Also, we study the relationship among L-fuzzy (K, E)-soft preclosure spaces, L-fuzzy (K, E)-soft topological spaces and L-fuzzy (K, E)-soft quasi-uniform space. Finally, we give their examples.

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1. Introduction

Molodtsov [14,15] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1, 11-16,19].The topological structures of soft sets have been developed by many researchers [2-5,10,17,20,21].

On the other hand, Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structures [8-10,17,18]. Ramadan et al.[10,17] investigated the relationships between L-fuzzy (K, E)-soft quasi-uniform structures and L-fuzzy (K, E)-soft topological structures in a complete residuated lattice.

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2. Preliminaries

Definition 2.1. [8,9] An algebra $(L, \wedge, \vee, \odot, \to, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $(L, \leq, \vee, \wedge, 0, 1)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \to, \oplus, ^*)$ is a complete residuated lattice with an order reversing involution * which is defined by $x \oplus y = (x^* \odot y^*)^*$ unless otherwise specified and we denote $L_0 = L - \{0\}$.

Lemma 2.2. [8,9] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(6) $x \oplus (\bigwedge_i y_i) = \bigwedge_i (x \oplus y_i)$,

(7) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(8) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(9) $(\bigvee_i x_i) \to (\bigvee_i y_i) \geq \bigwedge_i (x_i \to y_i)$,

(10) $(\bigwedge_i x_i) \to (\bigwedge_i y_i) \geq \bigwedge_i (x_i \to y_i)$,

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(12) $x \odot (x \to y) \leq y$ and $x \to y \leq (y \to z) \to (x \to z)$,

(13) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(14) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$,

(15) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(16) $x \odot y \odot (z \oplus w) \leq (x \odot z) \oplus (y \odot w)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.3. [3-5] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(6) 0_X (resp. 1_X) is an L-fuzzy soft set if $(0_X)_e(x) = 0$ (resp. $(1_X)_e(x) = 1$), for each $e \in E$, $x \in X$.

Definition 2.4. [3] Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X, E) \to (Y, K)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X, E) into (Y, K).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by

$$\varphi(f)_b(y) = \bigvee_{\varphi(x)=y} \Big(\bigvee_{\psi(e)=b} f_e(x) \Big).$$

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

Definition 2.5. [3-5, 10,17] A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(SO1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(SO2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \;\; \forall\, f, g \in (L^X)^E$,
(SO3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i \in I} \mathcal{T}_k(f_i) \; \forall\; f_i \in (L^X)^E,\; i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.

Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{T}^1)$ into $(Y, \mathcal{T}^2)$ is called L-fuzzy soft continuous if

$$\mathcal{T}^2_{\eta(k)}(f) \leq \mathcal{T}^1_k(\varphi^{-1}_{\psi,\eta}(f)) \;\; \forall\; f \in (L^Y)^{E_2}, k \in K_1.$$

Definition 2.6 [17] An L- fuzzy (K, E)-soft quasi- uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.

(SU3) For every $u, v \in (L^{X\times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.

(SU4) If $\mathcal{U}_k(u) \neq 0$ then $1_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(1_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x, z) = \bigvee_{y \in X} v_e(x, y) \odot w_e(y, z),$$

The pair $(X,\mathcal{U})$ is called an L-fuzzy (K, E)-soft quasi-uniform space.

An L-fuzzy (K, E)-soft quasi-uniform space $(X,\mathcal{U})$ is said to be an L-fuzzy (K, E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x, y) = u_e(y, x)$ for each $k \in K$ and $u \in (L^{X\times X})^E$.

Let $(X,\mathcal{U}^1)$ be an L-fuzzy (K_1, E_1)-soft quasi-uniform space and $(Y,\mathcal{U}^2)$ be an L-fuzzy (K_2, E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X,\mathcal{U}^1)$ into $(Y,\mathcal{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathcal{U}^2_{\eta(k)}(v) \leq \mathcal{U}^1_k((\varphi \times \varphi)^{-1}_{\psi,\eta}(v)) \;\; \forall\; v \in (L^{Y\times Y})^{E_2}, k \in K_1.$$

3. L-Fuzzy (K,E)-Soft Quasi-Uniformities and L-Fuzzy (K,E)-Soft Preclosure Operators

Definition 3.1. A map $\mathcal{C}: K \times (L^X)^E \to (L^X)^E$ is called an L-fuzzy (K,E)-soft preclosure operator if it satisfies the following conditions:

(PC1) $\mathcal{C}(k, 0_X) = 0_X$,

(PC2) $\mathcal{C}(k, f \oplus g) \sqsubseteq \mathcal{C}(k,f) \oplus \mathcal{C}(k,g)$ for each $f, g \in (L^X)^E$,

(PC3) If $f \sqsubseteq g$, then $\mathcal{C}(k,f) \sqsubseteq \mathcal{C}(k,g)$,

(PC4) $\mathcal{C}(k,f) \sqsupseteq f$ for all $f \in (L^X)^E$.

The pair $(X, \mathcal{C})$ is called an L-fuzzy (K,E)-soft preclosure space.

An L-fuzzy (K,E)-soft preclosure space is called stratified if

(R) $\mathcal{C}(k, \alpha \to f) \sqsubseteq \alpha \to \mathcal{C}(k,f)$ for all $f \in (L^X)^E, k \in K$ and $\alpha \in L$.

Let $(X, \mathcal{C}_X)$ be (K_1, E_1)-soft preclosure space and $(X, \mathcal{C}_Y)$ be be (K_2, E_2)-soft preclosure space. Let $\varphi: X \to Y$, $\psi: E_1 \to E_2$ and $\eta: K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}: (X, \mathcal{C}_X) \to (X, \mathcal{C}_Y)$ is called an L-fuzzy soft closure map if

$$\varphi_{\psi,\eta}(\mathcal{C}_X(k,f)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_{\psi,\eta}(f)).$$

Theorem 3.2. Let $(X, \mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{C}_l^{\mathcal{U}}: K \times (L^X)^E \to (L^X)^E$ by, $\forall\, f \in (L^X)^E, e \in E,\ x \in X$,

$$\mathcal{C}_l^{\mathcal{U}}(k,f)_e(x) = \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y \in X} (u_e(y,x) \odot f_e(y))).$$

Then $(X, \mathcal{C}_l^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft preclosure space. If $\mathcal{U}$ is stratified, then $\mathcal{C}_l^{\mathcal{U}}$ is also stratified.

Proof. (PC1) For $\mathcal{U}_k(u) \neq 0$, $1_\triangle \sqsubseteq u$. For each $e \in E$,

$$\begin{aligned}\mathcal{C}_l^{\mathcal{U}}(k,0_X)_e(x) &= \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y \in X} (u_e(y,x) \odot 0_X(y))) \\ &\geq \bigwedge_u (\mathcal{U}_k(u) \to 0) = \bigvee_u \mathcal{U}_k(u) \to 0 = 1 \to 0 = 0.\end{aligned}$$

Hence, $\mathcal{C}_l^{\mathcal{U}}(k, 0_X) = 0_X$.

(PC2) Since

$$\begin{aligned}&(a \odot b) \odot (c^* \odot d^*) = (a \odot c^*) \odot (b \odot d^*) \\ &\text{iff } (a \odot b) \to (c \oplus d) = (a \odot b) \to (c^* \odot d^*)^*\end{aligned}$$

$$= ((a \to c)^* \odot (b \to d)^*)^* = (a \to c) \oplus (b \to d),$$

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k,f)_e(x) \oplus \mathcal{C}_l^{\mathcal{U}}(k,g)_e(x) \\
&= \Big(\bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)))\Big) \\
&\oplus \Big(\bigwedge_v (\mathcal{U}_k(v) \to \bigvee_{y\in X} (v_e(y,x) \odot g_e(y)))\Big) \\
&= \bigwedge_{u,v} \Big(\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)))\Big) \\
&\oplus \Big((\mathcal{U}_k(v) \to \bigvee_{y\in X} (v_e(y,x) \odot g_e(y)))\Big) \\
&= \bigwedge_{u,v} \Big(\mathcal{U}_k(u) \odot \mathcal{U}_k(v) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)) \\
&\oplus \bigvee_{y\in X} (v_e(y,x) \odot g_e(y))\Big)
\end{aligned}$$

$$\begin{aligned}
&\geq \bigwedge_{u,v} \Big(\mathcal{U}_k(u) \odot \mathcal{U}_k(v) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)) \oplus (v_e(y,x) \odot g_e(y))\Big) \\
&\text{(by Lemma 2.2 (16))} \\
&\geq \bigwedge_{u,v} \Big(\mathcal{U}_k(u \odot v) \to \bigvee_{y\in X} ((u_e \odot v_e)(y,x) \odot (f_e \oplus g_e)(y))\Big) \\
&\geq \mathcal{C}_l^{\mathcal{U}}(k,(f \odot g)_e)(x).
\end{aligned}$$

(PC3) It is simply proved from the definition of $\mathcal{C}_l^{\mathcal{U}}$.
(PC4) For $\mathcal{U}_k(u) \neq 0$, $1_\triangle \sqsubseteq u$.

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k,f)_e(x) = \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y))) \\
&\geq \bigwedge_u (\mathcal{U}_k(u) \to (1_\triangle \odot f_e(x))) = \bigvee_u \mathcal{U}_k(u) \to (1_\triangle \odot f_e(x)) \\
&= 1 \to f_e(x) = f_e(x).
\end{aligned}$$

This implies that $(X, \mathcal{C}_l^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft preclosure space.
(R) Let $\mathcal{U}$ be stratified. Then

$$\begin{aligned}
&\alpha \to \mathcal{C}_l^{\mathcal{U}}(k,f)_e(x) = \alpha \to \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y))) \\
&= \bigwedge_u \Big(\alpha \to (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)))\Big) \\
&= \bigwedge_u \Big((\alpha \odot \mathcal{U}_k(u)) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)))\Big) \\
&\geq \bigwedge_u \Big(\mathcal{U}_k(\alpha \odot u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y)))\Big) \\
&\geq \bigwedge_u \Big(\mathcal{U}_k(\alpha \odot u) \to \bigvee_{y\in X} ((\alpha \odot u)_e(y,x) \odot (\alpha \to f_e(y)))\Big) \\
&\geq \mathcal{C}_l^{\mathcal{U}}(k, \alpha \to f)_e(x).
\end{aligned}$$

Corollary 3.3. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{C}_r^{\mathcal{U}} : K \times (L^X)^E \to (L^X)^E$ by, $\forall\ f \in (L^X)^E, e \in E,\ x \in X$,

$$\mathcal{C}_r^{\mathcal{U}}(k,f)_e(x) = \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(x,y) \odot f_e(y))).$$

Then $(X, \mathcal{C}_r^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft preclosure space. If $\mathcal{U}$ is stratified, then $\mathcal{C}_r^{\mathcal{U}}$ is also stratified.

Theorem 3.4. Let $(X,\mathcal{C})$ be an L-fuzzy (K,E)-soft pre-closure space. Define a map $\mathcal{T}_k^{\mathcal{C}} : K \to L^{(L^X)^E}$ by:

$$\mathcal{T}_k^{\mathcal{C}}(f) = \bigwedge_{e\in E} \bigwedge_{x\in X} (\mathcal{C}(k,f^*)_e(x) \to f_e^*(x)).$$

Then, $\mathcal{T}_k^{\mathcal{C}}$ is an L-fuzzy (K,E)-soft topology on X. If $\mathcal{C}$ is stratified, then $\mathcal{T}_k^{\mathcal{C}}$ is an enriched L-fuzzy (K,E)-soft topology.

Proof. (SO1)

$$\mathcal{T}_k^{\mathcal{C}}(0_X) = \bigwedge_{e\in E} \bigwedge_{x\in X} (\mathcal{C}(k,1_X)_e(x) \to (1_X)_e(x)) = 1 \to 1 = 1,$$

$$\mathcal{T}_k^{\mathcal{C}}(1_X) = \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,0_X)_e(x) \to (0_X)_e(x)) = 0 \to 0 = 1.$$

(SO2) By Lemma 2.2(14), we havre

$$\begin{aligned}
&\mathcal{T}_k^{\mathcal{C}}(f\odot g)\\
&= \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,(f\odot g)^*)_e(x) \to (f\odot g)^*_e(x))\\
&= \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f^*\oplus g^*)_e(x) \to (f^*\oplus g^*)_e(x))\\
&\geq \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f^*)_e(x)\oplus \mathcal{C}(k,g^*)_e(x) \to (f^*\oplus g^*)_e(x))\\
&\geq \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f^*)_e(x) \to f^*_e(x))\odot(\mathcal{C}(k,g^*)_e(x) \to g^*_e(x))\\
&\geq \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f^*)_e(x) \to f^*_e(x))\\
&\odot \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,g^*)_e(x) \to g^*_e(x))\\
&= \mathcal{T}_k^{\mathcal{C}}(f)\odot\mathcal{T}_k^{\mathcal{C}}(g).
\end{aligned}$$

(SO3) By Lemma 2.2(10),we have

$$\begin{aligned}
\mathcal{T}_k^{\mathcal{C}}(\sqcup_i f_i) &= \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,\sqcap_i f_i^*)_e(x) \to (\sqcap_i f_i^*)_e(x))\\
&\geq \bigwedge_{e\in E}\bigwedge_{x\in X}(\bigwedge_i \mathcal{C}(k,f_i^*)_e(x) \to \bigwedge_i (f_i)^*_e(x))\\
&\geq \bigwedge_{e\in E}\bigwedge_{x\in X}\bigwedge_i(\mathcal{C}(k,f_i^*)_e(x) \to (f_i)^*_e(x))\\
&= \bigwedge_i\bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f_i^*)_e(x) \to (f_i)^*_e(x))\\
&= \bigwedge_i \mathcal{T}_k^{\mathcal{C}}(f_i).
\end{aligned}$$

(R) By Lemma 2.2 (12)and Theorem 3.2(2), we have

$$\mathcal{T}_k^{\mathcal{C}}(\alpha\odot f) = \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,(\alpha\odot f)^*)_e(x) \to (\alpha\odot f_e)^*(x))$$

$$= \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,\alpha\to f)_e(x)\to(\alpha\to f^*_e(x))$$
$$\geq \bigwedge_{e\in E}\bigwedge_{x\in X}((\alpha\to\mathcal{C}(k,f)_e(x))\to(\alpha\to f^*_e(x))$$
$$\geq \bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}(k,f)_e(x)\to f^*_e(x))$$
$$= \mathcal{T}^{\mathcal{C}}_k(f).$$

From Theorems 3.2 and 3.4, we obtain the following corollary.

Corollary 3.5. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l},\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r}: K\to L^{(L^X)^E}$ by:

$$\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l}(f)=\bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}^{\mathcal{U}}_l(k,f^*)_e(x)\to f^*_e(x))),$$

$$\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r}(f)=\bigwedge_{e\in E}\bigwedge_{x\in X}(\mathcal{C}^{\mathcal{U}}_r(k,f^*)_e(x)\to f^*_e(x))).$$

Then, $\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l}$ and $\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r}$ are L-fuzzy (K,E)-soft topologies on X. If $\mathcal{U}$ is stratified, then $\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l}$ and $\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r}$ are enriched L-fuzzy (K,E)-soft topologies.

Theorem 3.6. Let $(X,\mathcal{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi: X\to Y$, $\psi: E_1\to E_2$ and $\eta: K_1\to K_2$ be mappings. If $\varphi_{\psi,\eta}:(X,\mathcal{U})\to(Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta}:(X,\mathcal{C}^{\mathcal{U}}_l)\to(Y,\mathcal{C}^{\mathcal{V}}_l)$ and $\varphi_{\psi,\eta}:(X,\mathcal{C}^{\mathcal{U}}_r)\to(Y,\mathcal{C}^{\mathcal{V}}_r)$ are L-fuzzy soft continuous.

Proof. Since

$$v_{\psi(e)}(\varphi_{\psi,\eta}(z),\varphi_{\psi,\eta}(x))=(\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v_{\psi(e)})(z,x),$$

$$\mathcal{C}^{\mathcal{V}}_l(\eta(k),\varphi_{\psi,\eta}(f))_{\psi(e)}(\varphi_{\psi,\eta}(x))$$
$$=\bigwedge_{v}(\mathcal{V}_{\eta(k)}(v)\to\bigvee_{y\in Y}(v_{\psi(e)}(y,\varphi_{\psi,\eta}(x))\odot(\varphi_{\psi,\eta}(f))_{\psi(e)}(y))$$
$$\geq\bigwedge_{v}(\mathcal{V}_{\eta(k)}(v)\to\bigvee_{z\in X}(v_{\psi(e)}(\varphi_{\psi,\eta}(z),\varphi_{\psi,\eta}(x))\odot(\varphi_{\psi,\eta}(f))_{\psi(e)}(\varphi_{\psi,\eta}(z)))$$

$$\geq \bigwedge_{v}(\mathcal{U}_k((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)) \to \bigvee_{z\in X}((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)(z,x)\odot f_e(z))$$
$$\geq \mathcal{C}_l^{\mathcal{U}}(k,f)_e(x).$$

$$\begin{aligned}\varphi_{\psi,\eta}(\mathcal{C}_l^{\mathcal{U}}(k,f))_{\psi(e)}(y) &= \bigvee_{x\in\varphi_{\psi,\eta}^{-1}(\{y\})}\mathcal{C}_l^{\mathcal{U}}(k,f)_e(x)\\ &\leq \bigvee_{x\in\varphi_{\psi,\eta}^{-1}(\{y\})}\mathcal{C}_l^{\mathcal{V}}(\eta(k),\varphi_{\psi,\eta}(f))_{\psi(e)}(\varphi_{\psi,\eta}(x))\\ &= \mathcal{C}_l^{\mathcal{V}}(\eta(k),\varphi_{\psi,\eta}(f))_{\psi(e)}(y).\end{aligned}$$

Theorem 3.7. Let $(X,\mathcal{C}_X)$ be an L-fuzzy (K_1,E_1)-soft preclosure space and $(Y,\mathcal{C}_Y)$ be an L-fuzzy (K_2,E_2)-soft preclosure space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathcal{C}_X) \to (Y,\mathcal{C}_Y)$ is L-fuzzy soft closed map, then a map $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{C}_X}) \to (Y,\mathcal{T}^{\mathcal{C}_Y})$ is L-fuzzy soft continuous.

Proof. By Lemma 2.2, since $\varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k),f)_{\psi(e)})(x) \leq \mathcal{C}_X(\varphi_{\psi,\eta}^{-1}(f))_e(x)$ we have

$$\begin{aligned}&\mathcal{T}^{\mathcal{C}_Y}(f) \to \mathcal{T}^{\mathcal{C}_X}(\varphi_{\psi,\eta}^{-1}(f))\\ &= \bigwedge_{e\in E}\bigwedge_{y\in Y}\Big(\mathcal{C}_Y(\eta(k),f^*)_{\psi(e)}(y) \to f^*_{\psi(e)}(y)\Big)\\ &\to \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\mathcal{C}_X(k,\varphi_{\psi,\eta}^{-1}(f^*))_e(x) \to \varphi_{\psi,\eta}^{-1}(f^*)_e(x)\Big)\\ &\geq \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k),f^*)_{\psi(e)})(x) \to \varphi_{\psi,\eta}^{-1}(f^*)_e(x)\Big)\\ &\to \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\mathcal{C}_X(k,\varphi_{\psi,\eta}^{-1}(f^*))_e(x) \to \varphi_{\psi,\eta}^{-1}(f^*)_e(x)\Big)\\ &\geq \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\mathcal{C}_X(k,\varphi_{\psi,\eta}^{-1}(f^*))_e(x) \to \varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k),f^*)_{\psi(e)})(x)\Big).\end{aligned}$$

Since $\varphi_{\psi,\eta}$ is an L-fuzzy soft closed map,

$$\varphi_{\psi,\eta}(\mathcal{C}_X(k,\varphi_{\psi,\eta}^{-1}(f^*)) \sqsubseteq \mathcal{C}_Y(\eta(k),\varphi_{\psi,\eta}(\varphi_{\psi,\eta}^{-1}(f^*))) \sqsubseteq \mathcal{C}_Y(\eta(k),f^*).$$

So, $\mathcal{C}_X(k, \varphi^{-1}_{\psi,\eta}(f^*) \sqsubseteq \varphi^{-1}_{\psi,\eta}(\mathcal{C}_Y(\eta(k), f^*))$. Hence
$\mathcal{T}^{\mathcal{C}_Y}(f) \leq \mathcal{T}^{\mathcal{C}_X}(\varphi^{-1}_{\psi,\eta}(f))$.

From Theorems 3.4 and 3.6, we obtain the following corollary.

Corollary 3.8. Let $(X,\mathcal{U})$ be an L-fuzzy (K_1, E_1)-soft uniform space and $(Y, \mathcal{V})$ be an L-fuzzy (K_2, E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y, \mathcal{V})$ is L-fuzzy soft uniformly continuous, then two maps $\varphi_{\psi,\eta} : (X, \mathcal{T}^{\mathcal{C}^{\mathcal{V}}_l}) \to (Y, \mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l})$ and $\varphi_{\psi,\eta} : (X, \mathcal{T}^{\mathcal{C}^{\mathcal{V}}_r}) \to (Y, \mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r})$ are L-fuzzy soft continuous.

Example 3.9. Let $X = \{h_i \mid i = \{1, 2, 3\}\}$ with h_i=house and $E = \{e, b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\},\ x \to y = \min\{1 - x + y, 1\}$$

Then $([0, 1], \wedge, \to, 0, 1)$ is a complete residuated lattice.

(1) Put $v, v \odot v, w \in ([0, 1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v \odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix},\ (v \odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ w_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : K = \{k_1, k_2\} \to [0, 1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v \odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

(2) From Theorem 3.2, since

$$\mathcal{C}_l^{\mathcal{U}}(k,f)_e(x) = \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X} (u_e(y,x) \odot f_e(y))),$$

$$\begin{aligned}
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_e(h_1) &= (\textstyle\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to (f_e(h_1) \vee (f_e(h_2) - 0.7) \\
&\quad \vee (f_e(h_3) - 0.6)\Big) \wedge (0.3 \to f_e(h_1)) \\
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_b(h_1) &= (\textstyle\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to (f_b(h_1) \vee (f_b(h_2) - 0.3) \\
&\quad \vee (f_b(h_3) - 0.4)\Big) \wedge \Big((0.3 \to (f_b(h_1) \vee (f_b(h_2) - 0.6) \\
&\quad \vee (f_b(h_3) - 0.8))\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_e(h_2) &= (\textstyle\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to ((f_e(h_1) - 0.4) \vee f_e(h_2) \\
&\quad \vee (f_e(h_3) - 0.4))\Big) \wedge \Big(0.3 \to (f_e(h_1) - 0.8) \\
&\quad \vee f_e(h_2) \vee (f_e(h_3) - 0.8)\Big) \\
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_b(h_2) &= (\textstyle\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to ((f_b(h_1) - 0.5) \vee f_b(h_2) \\
&\quad \vee (f_b(h_3) - 0.4)\Big) \wedge \Big(0.3 \to (f_b(h_2) \vee (f_b(h_3) - 0.8)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_e(h_3) &= (\textstyle\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to ((f_e(h_1) - 0.5) \vee (f_e(h_2) - 0.5) \\
&\quad \vee f_e(h_3))\Big) \wedge \Big(0.3 \to f_e(h_3))\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_l^{\mathcal{U}}(k_1,f)_b(h_3) &= (\textstyle\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to ((f_b(h_1) - 0.7) \vee (f_b(h_2) - 0.5) \\
&\quad \vee f_b(h_3))\Big) \wedge \Big(0.3 \to f_b(h_3)\Big)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k_2,f)_e(h_1) = (\textstyle\bigvee_{x\in X} f_e(x)) \\
&\wedge\Big(0.5 \to (f_e(h_1) \vee (f_e(h_2) - 0.6) \vee (f_e(h_3) - 0.6))\Big)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k_2,f)_b(h_1) = (\textstyle\bigvee_{x\in X} f_b(x)) \\
&\wedge\Big(0.5 \to (f_b(h_1) \vee (f_b(h_2) - 0.7) \vee (f_b(h_3) - 0.8))\Big)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k_2,f)_e(h_2) = (\textstyle\bigvee_{x\in X} f_e(x)) \\
&\wedge\Big(0.5 \to ((f_e(h_1) - 0.6) \vee f_e(h_2) \vee (f_e(h_3) - 0.4))\Big)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k_2,f)_b(h_2) = (\textstyle\bigvee_{x\in X} f_b(x)) \\
&\wedge\Big(0.5 \to ((f_b(h_1) - 0.5) \vee f_b(h_2) \vee (f_b(h_3) - 0.7))\Big)
\end{aligned}$$

$$\mathcal{C}_l^{\mathcal{U}}(k_2,f)_e(h_3) = (\bigvee_{x\in X} f_e(x)) \\ \wedge\Big(0.5 \to ((f_e(h_1)-0.5)\vee(f_e(h_2)-0.5)\vee f_e(h_3))\Big)$$

$$\mathcal{C}_l^{\mathcal{U}}(k_2,f)_b(h_3) = (\bigvee_{x\in X} f_b(x)) \\ \wedge\Big(0.5 \to ((f_b(h_1)-0.7)\vee(f_b(h_2)-0.5)\vee f_b(h_3))\Big)$$

For $f_e = (0.4, 0.9, 0.3)$ and $f_b = (0.7, 0.1, 0.6)$,

$$\mathcal{C}_l^{\mathcal{U}}(k_1,f)_e = (0.8,0.9,0.8),\ \ \mathcal{C}_l^{\mathcal{U}}(k_1,f)_b = (0.7,0.6,0.6)$$

$$\mathcal{C}_l^{\mathcal{U}}(k_2,f)_e = (0.9,0.9,0.9),\ \ \mathcal{C}_l^{\mathcal{U}}(k_2,f)_e = (0.7,0.7,0.7)$$

$$\mathcal{T}_{k_1}^{\mathcal{C}_l^{\mathcal{U}}}(f^*) = 0.5,\ \mathcal{T}_{k_2}^{\mathcal{C}_l^{\mathcal{U}}}(f^*) = 0.4.$$

(3) From Corollary 3.3, since

$$\mathcal{C}_r^{\mathcal{U}}(k,f)_e(x) = \bigwedge_u (\mathcal{U}_k(u) \to \bigvee_{y\in X}(u_e(x,y)\odot f_e(y))),$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1,f)_e(h_1) \ &= (\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to (f_e(h_1)\vee(f_e(h_2)-0.4) \\ &\vee(f_e(h_3)-0.5)\Big)\vee(0.3\to f_e(h_1))\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1,f)_b(h_1) \ &= (\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to (f_b(h_1)\vee(f_b(h_2)-0.5) \\ &\vee(f_b(h_3)-0.7)\Big)\wedge\Big(0.3\to((f_b(h_2)-0.8)\vee f_b(h_1))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1,f)_e(h_2) \ &= (\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to ((f_e(h_1)-0.7)\vee f_e(h_2) \\ &\vee(f_e(h_3)-0.5))\Big)\wedge\Big(0.3\to f_e(h_2)\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1,f)_b(h_2) \ &= (\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to ((f_b(h_1)-0.3)\vee f_b(h_2) \\ &\vee(f_b(h_3)-0.5)\Big)\wedge\Big(0.3\to((f_b(h_1)-0.8)\vee f_b(h_2))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1,f)_e(h_3) \ &= (\bigvee_{x\in X} f_e(x)) \wedge \Big(0.6 \to ((f_e(h_1)-0.6)\vee(f_e(h_2)-0.4) \\ &\vee f_e(h_3))\Big)\wedge\Big(0.3\to((f_e(h_2)-0.8)\vee f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_1, f)_b(h_3) \;&= (\bigvee_{x\in X} f_b(x)) \wedge \Big(0.6 \to ((f_b(h_1) - 0.4)\vee \\ &(f_b(h_2) - 0.4) \vee f_b(h_3))\Big) \wedge \Big(0.3 \to ((f_b(h_1) - 0.8) \\ &\vee(f_b(h_2) - 0.8) \vee f_b(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_e(h_1) \;&= (\bigvee_{x\in X} f_e(x)) \\ &\wedge\Big(0.5 \to (f_e(h_1) \vee (f_e(h_2) - 0.6) \vee (f_e(h_3) - 0.5)\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_b(h_1) \;&= (\bigvee_{x\in X} f_b(x)) \\ &\wedge\Big(0.5 \to (f_b(h_1) \vee (f_b(h_2) - 0.5) \vee (f_b(h_3) - 0.3)\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_e(h_2) \;&= (\bigvee_{x\in X} f_e(x)) \\ &\wedge\Big(0.5 \to ((f_e(h_1) - 0.6) \vee f_e(h_2) \vee (f_e(h_3) - 0.5))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_b(h_2) \;&= (\bigvee_{x\in X} f_b(x)) \\ &\wedge\Big(0.5 \to ((f_b(h_1) - 0.7) \vee f_b(h_2) \vee (f_b(h_3) - 0.5)\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_e(h_3) \;&= (\bigvee_{x\in X} f_e(x)) \\ &\wedge\Big(0.5 \to ((f_e(h_1) - 0.6) \vee (f_e(h_2) - 0.4) \vee f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{C}_r^{\mathcal{U}}(k_2, f)_b(h_3) \;&= (\bigvee_{x\in X} f_b(x)) \\ &\wedge\Big(0.5 \to ((f_b(h_1) - 0.8) \vee (f_b(h_2) - 0.7) \vee f_b(h_3))\Big)\end{aligned}$$

For $f_e = (0.4, 0.9, 0.3)$ and $f_b = (0.7, 0.1, 0.6)$,

$$\mathcal{C}_r^{\mathcal{U}}(k_1, f)_e = (0.9, 0.9, 0.9),\;\; \mathcal{C}_r^{\mathcal{U}}(k_1, f)_b = (0.7, 0.7, 0.7)$$

$$\mathcal{C}_r^{\mathcal{U}}(k_2, f)_e = (0.8, 0.9, 0.9),\;\; \mathcal{C}_r^{\mathcal{U}}(k_2, f)_e = (0.7, 0.6, 0.7)$$

$$\mathcal{T}_{k_1}^{\mathcal{C}_r^{\mathcal{U}}}(f^*) = 0.4,\;\; \mathcal{T}_{k_2}^{\mathcal{C}_r^{\mathcal{U}}}(f^*) = 0.4.$$

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