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L-FUZZY (K, E)-SOFT CLOSURE OPERATORS INDUCED BY L-FUZZY (K, E)-SOFT QUASI-UNIFORM SPACES

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Abstract: In this paper, we investigate two L-fuzzy (K, E)-soft closure operators induced by an L-fuzzy (K, E)-soft quasi-uniform space in a complete residuated lattice L. We study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft closure operators and L-fuzzy (K, E)-soft quasi-uniformities. Finally, we give their examples.

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1. Introduction

Molodtsov [14,15] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,7].The topological structures of soft sets have been developed by many researchers [2-5,11,17,20].

On the other hand, Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structures [8,9,18]. Kim et al.[10,17] investigated the relationships between L-fuzzy (K, E)-soft quasi-uniform structures and L-fuzzy (K, E)-soft topological structures.

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In this paper, we investigate two L-fuzzy (K, E)-soft closure operators induced by an L-fuzzy (K, E)-soft quasi-uniform space in a complete residuated lattice L. We study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft closure operators and L-fuzzy (K, E)-soft quasi-uniformities. Finally, we give their examples.

2. Preliminaries

Definition 2.1. [8,9] An algebra $(L, \wedge, \vee, \odot, \to, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $(L, \leq, \vee, \wedge, 0, 1)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \to, \oplus, ^*)$ is a complete residuated lattice with an order reversing involution * which is defined by $x \oplus y = (x^* \odot y^*)^*$ unless otherwise specified and we denote $L_0 = L - \{0\}$.

Lemma 2.2. [8,9] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(6) $x \oplus (\bigwedge_i y_i) = \bigwedge_i (x \oplus y_i)$,

(7) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(8) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(9) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(10) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(12) $x \odot (x \to y) \leq y$ and $x \to y \leq (y \to z) \to (x \to z)$,

(13) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(14) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$,

(15) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(16) $x \odot y \odot (z \oplus w) \leq (x \odot z) \oplus (y \odot w)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.3. [3-5] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(6) 0_X (resp. 1_X) is an L-fuzzy soft set if $(0_X)_e(x) = 0$ (resp. $(1_X)_e(x) = 1$), for each $e \in E$, $x \in X$.

Definition 2.4. [3] Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X, E) \to (Y, K)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X, E) into (Y, K).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by

$$\varphi(f)_b(y) = \bigvee_{\varphi(x)=y} \Big(\bigvee_{\psi(e)=b} f_e(x)\Big).$$

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

Definition 2.5. [3-5, 10,17] A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(SO1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(SO2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \ \ \forall\ f, g \in (L^X)^E$,
(SO3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i\in I} \mathcal{T}_k(f_i)\ \forall\ f_i \in (L^X)^E,\ i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.

Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{T}^1)$ into $(Y, \mathcal{T}^2)$ is called L-fuzzy soft continuous if

$$\mathcal{T}^2_{\eta(k)}(f) \leq \mathcal{T}^1_k(\varphi^{-1}_{\psi,\eta}(f)) \ \ \forall\ f \in (L^Y)^{E_2}, k \in K_1.$$

Definition 2.6. [10] A map $\mathcal{C} : K \times (L^X)^E \times L_0 \to (L^X)^E$ is called an L-fuzzy (K, E)-soft closure operator if it satisfies the following conditions;

(SC1) $\mathcal{C}(k, 0_X, r) = 0_X$,
(SC2) $\mathcal{C}(k, f, r) \sqsupseteq f$,
(SC3) If $f_1 \sqsubseteq f_2$, then $\mathcal{C}(k, f_1, r) \sqsubseteq \mathcal{C}(k, f_2, r))$,
(SC4) If $r_1 \leq r_2$, then $\mathcal{C}(k, f, r_1) \sqsubseteq \mathcal{C}(k, f, r_2)$,
(SC5) $\mathcal{C}(k, f_1 \oplus f_2, r \odot s) \sqsubseteq \mathcal{C}(k, f_1, r) \oplus \mathcal{C}(k, f_2, s)$.

The pair $(X, \mathcal{C})$ is called an L-fuzzy (K, E)-soft closure space.

An L- fuzzy (K, E)-soft closure operator is called topological if

(T) $\mathcal{C}(k, \mathcal{C}(k, f, r), r) \sqsubseteq \mathcal{C}(k, f, r)$.

Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be L-fuzzy (K, E)-soft closure operators on X. Then $\mathcal{C}_1$ is finer than $\mathcal{C}_2$ if $\mathcal{C}_1(k, f, r) \sqsubseteq \mathcal{C}_2(k, f, r)$, for all $f \in (L^X)^E, r \in L_0$.

Let $(X, \mathcal{C}_X)$ be L-fuzzy (K_1, E_1)-soft closure spaces and $(Y, \mathcal{C}_Y)$ be L-fuzzy (K_2, E_2)-soft closure spaces. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be maps. Then $\varphi_{\psi,\eta}$ is called an L-fuzzy soft closed map if, for each $k \in K_1, f \in (L^X)^{E_1}, r \in L_0$,

$$\varphi_{\psi,\eta}(\mathcal{C}_X(k, f, r)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_{\psi,\eta}(f), r).$$

Definition 2.7 [17] An L- fuzzy (K, E)-soft quasi- uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1$.
(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.
(SU3) For every $u, v \in (L^{X\times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.
(SU4) If $\mathcal{U}_k(u) \neq 0$ then $1_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(1_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \le \bigvee\{\mathcal{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x, z) = \bigvee_{y \in X} v_e(x, y) \odot w_e(y, z),$$

The pair $(X, \mathcal{U})$ is called an L-fuzzy (K, E)-soft quasi-uniform space.

An L-fuzzy (K, E)-soft quasi-uniform space $(X, \mathcal{U})$ is said to be an L-fuzzy (K, E)-soft uniform space if

(U) $\mathcal{U}_k(u) \le \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x, y) = u_e(y, x)$ for each $k \in K$ and $u \in (L^{X \times X})^E$.

Let $(X, \mathcal{U}^1)$ be an L-fuzzy (K_1, E_1)-soft quasi-uniform space and $(Y, \mathcal{U}^2)$ be an L-fuzzy (K_2, E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{U}^1)$ into $(Y, \mathcal{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathcal{U}^2_{\eta(k)}(v) \le \mathcal{U}^1_k((\varphi \times \varphi)^{-1}_{\psi}(v)) \;\; \forall\, v \in (L^{Y \times Y})^{E_2}, k \in K_1.$$

3. L-fuzzy (K, E)-soft closure operators induced by L-fuzzy (K, E)-soft quasi-uniform spaces

Theorem 3.1. Let $(X, \mathcal{U})$ be an L-fuzzy (K, E)-soft quasi-uniform space. Define a map $\mathcal{C}^{\mathcal{U}}_l : K \times (L^X)^E \times L_0 \to (L^X)^E$, by:

$$\mathcal{C}^{\mathcal{U}}_l(k, f, r)_e(x) = \bigwedge_{\mathcal{U}_k(u) \ge r} \bigvee_{y \in X} (u_e(y, x) \odot f_e(y)).$$

Then the following properties hold.

(1) $(X, \mathcal{C}^{\mathcal{U}}_l)$ is an L- fuzzy (K, E)-soft closure space.

(2) For all $e \in E$, $f, g \in (L^X)^E$,

$$\bigwedge_{y \in X} (f_e(y) \to g_e(y)) \le \bigwedge_{x \in X} (\mathcal{C}^{\mathcal{U}}_l(k, f, r)_e(x) \to \mathcal{C}^{\mathcal{U}}_l(k, g, r)(x)).$$

(3) $\mathcal{C}^{\mathcal{U}}_l(\mathcal{C}^{\mathcal{U}}_l(k, f, r_1), r_1) \le \mathcal{C}^{\mathcal{U}}_l(k, f, r)$ for each $r_1 < r$.

Proof. (1) (SC1),(SC3) and (SC4) are easily proved.

(SC2) For $\mathcal{U}_k(u) \geq r$, by (SU4), since $1_\triangle \sqsubseteq u$, $\mathcal{C}_l^{\mathcal{U}}(k,f,r)_e(x) = \bigwedge_{\mathcal{U}_k(u)\geq r} \bigvee_{y\in X}(u_e(y,x)\odot f_e(y)) \geq u_e(x,x)\odot f_e(x) = f_e(x)$. So, $f \sqsubseteq \mathcal{C}_l^{\mathcal{U}}(k,f,r)$.

(SC5)

$$\begin{aligned}
&\mathcal{C}_l^{\mathcal{U}}(k,f,r)(x) \oplus \mathcal{C}_l^{\mathcal{U}}(k,g,s)(x)\\
&= \Big(\bigwedge_{\mathcal{U}(u)\geq r} \bigvee_{y\in X} (u_e(y,x)\odot f_e(y))\Big) \oplus \Big(\bigwedge_{\mathcal{U}(v)\geq s} \bigvee_{y\in X} (v_e(y,x)\odot g_e(y))\Big)\\
&\geq \bigwedge_{\mathcal{U}(u)\geq r, \mathcal{U}(v)\geq s} \bigvee_{y\in X} \Big((u_e(y,x)\odot f_e(y)) \oplus (v_e(y,x)\odot g_e(y))\Big)\\
&\text{(by Lemma 2.2 (16))}\\
&\geq \bigvee_{\mathcal{U}(u)\odot\mathcal{U}(v)\geq r\odot s} \bigvee_{y\in X} \Big((u_e\odot u_e)(y,x)\odot (f\oplus g)_e(y)\Big)\\
&\geq \mathcal{C}_l^{\mathcal{U}}(k, f\oplus g, r\odot s)(x).
\end{aligned}$$

(2)

$$\begin{aligned}
&\bigwedge_{x\in X} (\mathcal{C}_l^{\mathcal{U}}(k,f,r)_e(x) \to \mathcal{C}_l^{\mathcal{U}}(k,g,r))\\
&= \bigwedge_{x\in X} \Big(\bigwedge_{\mathcal{U}_k(u)\geq r} \bigvee_{y\in X} (u_e(y,x)\odot f_e(y))\\
&\to \bigwedge_{\mathcal{U}_k(u)\geq r} \bigvee_{y\in X} (u_e(y,x)\odot g_e(y))(x)\Big)\\
&= \bigwedge_{x\in X} \bigwedge_{\mathcal{U}_k(u)\geq r} \bigwedge_{y\in X} \Big((u_e(y,x)\odot f_e(y)) \to (u_e(y,x)\odot g_e(y))(x)\Big)\\
&\geq \bigwedge_{y\in X} (f_e(y) \to g_e(y))
\end{aligned}$$

(3) For $\mathcal{U}(u) \geq r$ and $r > r_1$, by (SU5), there exists $v \in (L^{X\times X})^E$ such that $\mathcal{U}(v) \geq r_1,\ v\circ v \sqsubseteq u$.

$$\begin{aligned}
\mathcal{C}_l^{\mathcal{U}}(k,f,r)_e(x) &= \bigwedge_{\mathcal{U}(u)\geq r} \bigvee_{y\in X} (u_e(y,x)\odot f_e(y))\\
&\geq \bigwedge_{\mathcal{U}(v)\geq r_1} \bigvee_{y\in X} (v_e\circ v_e(y,x)\odot f_e(y))
\end{aligned}$$

$$= \bigwedge_{\mathcal{U}(v)\geq r_1} \bigvee_{y\in X} \bigvee_{z\in X} (v_e(z,x) \odot v_e(y,z) \odot f_e(y))$$

$$\geq \bigwedge_{\mathcal{U}(v)\geq r_1} \bigvee_{z\in X} (v_e(z,x) \odot \bigwedge_{\mathcal{U}(v)\geq r_1} \bigvee_{y\in X} (v_e(y,z) \odot f_e(y)))$$

$$\geq \bigwedge_{\mathcal{U}(v)\geq r_1} \bigvee_{z\in X} (v_e(z,x) \odot \mathcal{C}_l^{\mathcal{U}}(k,f,r)_e(z))$$

$$= \mathcal{C}_l^{\mathcal{U}}(k, \mathcal{C}_l^{\mathcal{U}}(k,f,r),r)_e(x).$$

This implies that $(X, \mathcal{C}_l^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft closure space.

By a similar method of Theorem 3.1, we obtain the following corollary.

Corollary 3.2. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{C}_r^{\mathcal{U}} : K \times (L^X)^E \times L_0 \to (L^X)^E$, by:

$$\mathcal{C}_r^{\mathcal{U}}(k,f,s)_e(x) = \bigwedge_{\mathcal{U}_k(u)\geq r} \bigvee_{y\in X} (u_e(x,y) \odot f_e(y)).$$

Then the following properties hold.

(1) $(X, \mathcal{C}_r^{\mathcal{U}})$ is an L- fuzzy (K,E)-soft closure space.

(2) For all $e \in E, f,g \in (L^X)^E$,

$$\bigwedge_{y\in X} (f_e(y) \to g_e(y))$$
$$\leq \bigwedge_{x\in X} (\mathcal{C}_r^{\mathcal{U}}(k,f,s)_e(x) \to \mathcal{C}_r^{\mathcal{U}}(k,g,s)(x)).$$

(2) $\mathcal{C}_r^{\mathcal{U}}(\mathcal{C}_r^{\mathcal{U}}(k,f,s_1),s_1) \leq \mathcal{C}_r^{\mathcal{U}}(k,f,s)$ for each $s_1 < s$.

Theorem 3.3. Let $(X,\mathcal{C})$ be an L-fuzzy (K,E)-soft closure space. Define a mapping $\mathcal{T}^{\mathcal{C}} : K \to L^{(L^X)^E}$ by:

$$\mathcal{T}_k^{\mathcal{C}}(f) = \bigvee \{r \in L \mid f^* = \mathcal{C}(k,f^*,r))\}.$$

Then, $\mathcal{T}^{\mathcal{C}}$ is L-fuzzy (K,E)-soft topology on X.

Proof. (SO1) Since $1_X = \mathcal{C}(k,1_X,r)$ and $0_X = \mathcal{C}(k,0_X,r)$, we have $\mathcal{T}_k^{\mathcal{C}}(1_X) = 1, \mathcal{T}_k^{\mathcal{C}}(0_X) = 1$.

(SO2) Let $f_1^* = \mathcal{C}(k,f_1^*,r)$ and $f_2^* = \mathcal{C}(k,f_2^*,s)$, then $f_1^* \oplus f_2^* = \mathcal{C}(k,f_1^*,r) \oplus \mathcal{C}(k,f_2^*,s) = \mathcal{C}(k,f_1^* \oplus f_2^*, r \odot s)$. Thus,

$$\mathcal{T}_k^{\mathcal{C}}(f_1 \odot f_2) \geq \mathcal{T}_k^{\mathcal{C}}(f_1) \odot \mathcal{T}_k^{\mathcal{C}}(f_2).$$

(SO3) For a family of $\{f_i \mid i \in \Gamma, f_i^* = \mathcal{C}(k, f_i^*, r)\}$, we have

$$\mathcal{C}(k, \sqcap_{i\in\Gamma} f_i^*, r) \leq \sqcap_{i\in\Gamma}\mathcal{C}(k, f_i^*, r) = \sqcap_{i\in\Gamma} f_i^*$$

Thus, $\mathcal{T}_k^{\mathcal{C}}(\sqcup_{i\in\Gamma} f_i) \geq \bigwedge_{i\in\Gamma} \mathcal{T}_k^{\mathcal{C}}(f_i)$. Hence $\mathcal{T}^{\mathcal{C}}$ is an L-fuzzy (K,E)-topology on X.

From Theorems 3.1 and 3.3, we obtain the following corollary.

Corollary 3.4. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi- uniform space. Define the mappings $\mathcal{T}^{\mathcal{C}_l^{\mathcal{U}}}, \mathcal{T}^{\mathcal{C}_r^{\mathcal{U}}} : K \to L^{(L^X)^E}$ by:

$$\mathcal{T}_k^{\mathcal{C}_l^{\mathcal{U}}}(f) = \bigvee\{r \in L \mid f^* = \mathcal{C}_l^{\mathcal{U}}(k, f^*, r)\},$$

$$\mathcal{T}_k^{\mathcal{C}_r^{\mathcal{U}}}(f) = \bigvee\{s \in L \mid f^* = \mathcal{C}_r^{\mathcal{U}}(k, f^*, s)\}.$$

Then, $\mathcal{T}^{\mathcal{C}_l^{\mathcal{U}}}$ and $\mathcal{T}^{\mathcal{C}_r^{\mathcal{U}}}$ are L-fuzzy (K,E)-soft topologies on X.

Theorem 3.5. Let $(X,\mathcal{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta} : (X,\mathcal{C}_l^{\mathcal{U}}) \to (Y,\mathcal{C}_l^{\mathcal{V}})$ and $\varphi_{\psi,\eta} : (X,\mathcal{C}_r^{\mathcal{U}}) \to (Y,\mathcal{C}_r^{\mathcal{V}})$ are L-fuzzy soft closed maps.

Proof.

$$\begin{aligned} &v_{\psi(e)}(\varphi_{\psi,\eta}(z), \varphi_{\psi,\eta}(x)) = (\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v_{\psi(e)})(z,x) \\ &= (\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)_e(z,x). \end{aligned}$$

$$\mathcal{C}_l^{\mathcal{U}}(\eta(k), \varphi_{\psi,\eta}(f), r)_{\psi(e)}(y) = \bigvee_{x\in\varphi_{\psi,\eta}^{-1}(\{y\})} \mathcal{C}_l^{\mathcal{U}}(\eta(k), \varphi_{\psi,\eta}(f), r)_{\psi(e)}(\varphi_{\psi,\eta}(x))$$

$$= \bigvee_{x\in\varphi_{\psi,\eta}^{-1}(\{y\})} \bigwedge_{\mathcal{V}_{\eta(k)}(u)\geq r} \bigvee_{w\in Y} (v_{\psi(e)}(w, \varphi_{\psi,\eta}(x)) \odot \varphi_{\psi,\eta}(f)_{\psi(e)}(w))$$

$$\geq \bigvee_{x\in\varphi_{\psi,\eta}^{-1}(\{y\})} \bigwedge_{\mathcal{V}_{\eta(k)}(u)\geq r} \bigvee_{w\in Y} (v_{\psi(e)}(\varphi_{\psi,\eta}(z), \varphi_{\psi,\eta}(x)) \odot \varphi_{\psi,\eta}(f)_{\psi(e)}(\varphi_{\psi,\eta}(z)))$$

$$\geq \bigvee_{x\in\varphi^{-1}_{\psi,\eta}(\{y\})} \bigwedge_{\mathcal{V}_{\eta(k)}(u)\geq r} \bigvee_{w\in Y} (v_{\psi(e)}(\varphi_{\psi,\eta}(z),\varphi_{\psi,\eta}(x)) \odot \varphi_{\psi,\eta}(f)_{\psi(e)}(\varphi_{\psi,\eta}(z)))$$

$$\geq \bigvee_{x\in\varphi^{-1}_{\psi,\eta}(\{y\})} \bigwedge_{\mathcal{U}_k((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v))\geq r} \bigvee_{z\in X} ((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)(z,x) \odot \varphi^{-1}_{\psi,\eta}(f)_e(z))$$

$$\geq \bigvee_{x\in\varphi^{-1}_{\psi,\eta}(\{y\})} \mathcal{C}^{\mathcal{U}}_l(k,\varphi^{-1}_{\psi,\eta}(f),r)_e(x)$$

$$= \varphi_{\psi,\eta}(\mathcal{C}^{\mathcal{U}}_l(k,\varphi^{-1}_{\psi,\eta}(f),r))_{\psi(e)}(y).$$

Theorem 3.6. Let $(X,\mathcal{C}_X)$ be an L-fuzzy (K_1,E_1)-soft closure space and $(Y,\mathcal{C}_Y)$ be an L-fuzzy (K_2,E_2)-soft closure space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathcal{C}_X) \to (Y,\mathcal{C}_Y)$ is L-fuzzy soft closure map, then a map $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{C}_X}) \to (Y,\mathcal{T}^{\mathcal{C}_Y})$ is L-fuzzy soft continuous.

Proof. Since $\varphi^{-1}_{\psi,\eta}(\mathcal{C}_Y(\eta(k),g,r)_{\psi(e)})(x) \geq \mathcal{C}_X(k,\varphi^{-1}_{\psi,\eta}(g),r)_e(x)$, for $\mathcal{C}_Y(\eta(k),g,r)_{\psi(e)} = g_{\psi(e)}$, we have

$$\varphi^{-1}_{\psi,\eta}(g_{\psi(e)})(x) = \varphi^{-1}_{\psi,\eta}(g)_e(x) \geq \mathcal{C}_X(k,\varphi^{-1}_{\psi,\eta}(g),r)_e(x).$$

Hence $\mathcal{T}^{\mathcal{C}_Y}(g) \leq \mathcal{T}^{\mathcal{C}_X}(\varphi^{-1}_{\psi,\eta}(g))$.

From Theorems 3.5 and 3.6, we obtain the following corollary.

Corollary 3.7. Let $(X,\mathcal{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then two maps $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{C}^{\mathcal{V}}_l}) \to (Y,\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_l})$ and $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{C}^{\mathcal{V}}_r}) \to (Y,\mathcal{T}^{\mathcal{C}^{\mathcal{U}}_r})$ are L-fuzzy soft continuous.

Example 3.8. Let $X = \{h_i \mid i = \{1,2,3\}\}$ with h_i=house and $E = \{e,b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0,1]$ by

$$x \odot y = \max\{0, x+y-1\},\ x \to y = \min\{1-x+y, 1\}$$

$$x \oplus y = \min\{1-x+y, 1\},\ x^* = 1-x.$$

Then $([0,1],\wedge,\to,0,1)$ is a complete residuated lattice.

(1) Put $v, v \odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix}, v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v\odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix}, (v\odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix}, w_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : K = \{k_1, k_2\} \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v\odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

(2) $\mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(x) = \bigwedge_{\mathcal{U}(u)\geq r} \bigvee_{y\in X} (u(y,x) \odot f_e(x)), \ \forall f \in (L^X)^E, \ x \in X.$
If $r > 0.6$, then

$$\begin{aligned} \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_1) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_2) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_3) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E. \end{aligned}$$

If $0.3 < r \leq 0.6$, then $\mathcal{U}_{k_1}(v) = 0.6$.

$$\begin{aligned} \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_1) &= f_e(h_1) \vee (f_e(h_2) - 0.7) \vee (f_e(h_3) - 0.6), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_1) &= f_b(h_1) \vee (f_b(h_2) - 0.3) \vee (f_b(h_3) - 0.4), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_2) &= (f_e(h_1) - 0.4) \vee f_e(h_2) \vee (f_e(h_3) - 0.4), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_2) &= (f_b(h_1) - 0.5) \vee f_b(h_2) \vee (f_b(h_3) - 0.6), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_3) &= (f_e(h_1) - 0.5) \vee (f_e(h_2) - 0.5) \vee f_e(h_3), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_3) &= (f_b(h_1) - 0.7) \vee (f_b(h_2) - 0.5) \vee f_b(h_3). \end{aligned}$$

If $0 < r \leq 0.3$, then $\mathcal{U}_{k_1}(v \odot v) = 0.3$.

$$\begin{array}{l} \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_1) = f_e(h_1), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_1) = f_b(h_1) \vee (f_b(h_2) - 0.6) \vee (f_b(h_3 - 0.8), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_2) = (f_e(h_1) - 0.8) \vee f_e(h_2) \vee (f_e(h_3) - 0.8), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_2) = f_b(h_2) \vee (f_b(h_3 - 0.8), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e(h_3) = f_e(h_3), \\ \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b(h_3) = f_b(h_3). \end{array}$$

If $r > 0.5$, then

$$\begin{array}{l} \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_1) = \bigvee_{x \in X} f_e(x), \forall e \in E \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_2) = \bigvee_{x \in X} f_e(x), \forall e \in E \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_3) = \bigvee_{x \in X} f_e(x), \forall e \in E. \end{array}$$

If $0 < r \leq 0.5$, then $\mathcal{U}_{k_2}(w) = 0.5$.

$$\begin{array}{l} \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_1) = f_e(h_1) \vee (f_e(h_2) - 0.6) \vee (f_e(h_3) - 0.6) \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_b(h_1) = f_b(h_1) \vee (f_b(h_2) - 0.7) \vee (f_b(h_3) - 0.8), \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_2) = (f_e(h_1) - 0.6) \vee f_e(h_2) \vee (f_e(h_3) - 0.4) \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_b(h_2) = (f_b(h_1) - 0.5) \vee f_b(h_2) \vee (f_b(h_3) - 0.7), \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_3) = (f_e(h_1) - 0.5) \vee (f_e(h_2) - 0.5) \vee f_e(h_3) \\ \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e(h_3) = (f_b(h_1) - 0.7) \vee (f_b(h_2) - 0.5) \vee f_b(h_3). \end{array}$$

$$\mathcal{T}_{k_1}^{\mathcal{C}_l^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.6, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_e(h_2) - 0.7, f_e^*(h_1) \geq f_e^*(h_3) - 0.6, \\ & \quad f_b^*(h_1) \geq f_b^*(h_2) - 0.3, f_b^*(h_1) \geq f_b^*(h_3) - 0.4, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_e^*(h_1) - 0.4, f_e^*(h_2) \geq f_e^*(h_3) - 0.4, \\ & \quad f_b^*(h_2) \geq f_b^*(h_1) - 0.5, f_b^*(h_2) \geq f_b^*(h_3) - 0.4, \\ & \quad f_e^*(h_3) = f_b^*(h_3) \geq f_e^*(h_1) - 0.5, f_e^*(h_3) \geq f_e^*(h_2) - 0.5, \\ & \quad f_b^*(h_3) \geq f_b^*(h_1) - 0.7, f_b^*(h_3) \geq f_b^*(h_2) - 0.5, \\ 0.3, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_b^*(h_2) - 0.6, f_b^*(h_1) \geq f_b^*(h_3) - 0.8, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_e^*(h_1) - 0.8, \\ & \quad f_b^*(h_2) \geq f_b^*(h_1) - 0.8, f_b^*(h_2) \geq f_b^*(h_3) - 0.8, \\ & \quad f_e^*(h_3) = f_b^*(h_3), \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{T}_{k_2}^{\mathcal{C}_l^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.5, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_e^*(h_2) - 0.6, f_e^*(h_1) \geq f^*(h_3) - 0.6, \\ & \quad f_b^*(h_1) \geq f_b^*(h_2) - 0.7, f_b^*(h_1) \geq f_b^*(h_3) - 0.8, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_e^*(h_1) - 0.6, f_e^*(h_2) \geq f_e^*(h_3) - 0.4, \\ & \quad f_b^*(h_2) \geq f_b^*(h_1) - 0.5, f_b^*(h_2) \geq f_b^*(h_3) - 0.7, \\ & \quad f_e^*(h_3) = f_b^*(h_3) \geq f_e^*(h_1) - 0.5, f_e^*(h_3) \geq f_e^*(h_2) - 0.5, \\ & \quad f_b^*(h_3) \geq f_b^*(h_1) - 0.7, f_b^*(h_3) \geq f_b^*(h_2) - 0.5, \\ 0, & \text{otherwise.} \end{cases}$$

(3) $\mathcal{C}_r^{\mathcal{U}}(k_1, f, r)_e(x) = \bigwedge_{\mathcal{U}(u)\geq r} \bigvee_{y\in X}(u(x,y) \odot f_e(x)), \ \ \forall\, f \in (L^X)^E, \ x \in X.$
If $r > 0.6$, then

$$\begin{aligned} \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_1) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E, \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_2) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E, \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_3) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E. \end{aligned}$$

If $0.3 < r \leq 0.6$, then $\mathcal{U}_{k_1}(v) = 0.6$.

$$\begin{aligned} \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_1) &= f_e(h_1) \vee (f_e(h_2) - 0.4) \vee (f_e(h_3) - 0.5) \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_1) &= f_b(h_1) \vee (f_b(h_2) - 0.5) \vee (f_b(h_3) - 0.7), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_2) &= (f_e(h_1) - 0.7) \vee f_e(h_2) \vee (f_e(h_3) - 0.5) \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_2) &= (f_b(h_1) - 0.3) \vee f_b(h_2) \vee (f_b(h_3) - 0.5), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_3) &= (f_e(h_1) - 0.6) \vee (f_e(h_2) - 0.4) \vee f_e(h_3) \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_3) &= (f_b(h_1) - 0.4) \vee (f_b(h_2) - 0.4) \vee f_b(h_3), \end{aligned}$$

If $0 < r \leq 0.3$, then $\mathcal{U}_{k_1}(v \odot v) = 0.3$.

$$\begin{aligned} \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_1) &= f_e(h_1), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_1) &= (f_b(h_2) - 0.8) \vee f_b(h_1), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_2) &= f_e(h_2), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_2) &= (f_b(h_1) - 0.8) \vee f_b(h_2), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_e(h_3) &= (f_e(h_2) - 0.8) \vee f_e(h_3)), \\ \mathcal{C}_r^{\mathcal{U}}(k_1, f, s)_b(h_3) &= (f_b(h_1) - 0.8) \vee (f_b(h_2) - 0.8) \vee f_b(h_3). \end{aligned}$$

If $r > 0.5$, then

$$\begin{aligned} \mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_1) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E, \\ \mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_2) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E, \\ \mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_3) &= \textstyle\bigvee_{x\in X} f_e(x), \forall e \in E. \end{aligned}$$

If $0 < r \leq 0.5$, then $\mathcal{U}_{k_2}(w) = 0.5$.

$$\begin{array}{l}
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_1) = f_e(h_1) \vee (f_e(h_2) - 0.6) \vee (f_e(h_3) - 0.6) \\
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_b(h_1) = f_b(h_1) \vee (f_b(h_2) - 0.5) \vee (f_b(h_3) - 0.3), \\
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_2) = (f_e(h_1) - 0.6) \vee f_e(h_2) \vee (f_e(h_3) - 0.5) \\
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_b(h_2) = (f_b(h_1) - 0.7) \vee f_b(h_2) \vee (f_b(h_3) - 0.5), \\
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_e(h_3) = (f_e(h_1) - 0.6) \vee (f_e(h_2) - 0.4) \vee f_e(h_3) \\
\mathcal{C}_r^{\mathcal{U}}(k_2, f, s)_b(h_3) = (f_b(h_1) - 0.8) \vee (f_b(h_2) - 0.7) \vee f_b(h_3).
\end{array}$$

$$\mathcal{T}_{k_1}^{\mathcal{C}_r^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.6, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_e^*(h_2) - 0.4, f_e^*(h_1) \geq f^*(h_3) - 0.5, \\ & \quad f_b^*(h_1) \geq f_b^*(h_2) - 0.5, f_b^*(h_1) \geq f_b^*(h_3) - 0.7, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_e^*(h_1) - 0.7, f_e^*(h_2) \geq f_e^*(h_3) - 0.5, \\ & \quad f_b^*(h_2) \geq f_b^*(h_1) - 0.3, f_b^*(h_2) \geq f_b^*(h_3) - 0.5, \\ & \quad f_e^*(h_3) = f_b^*(h_3) \geq f_e^*(h_1) - 0.6, f_e^*(h_3) \geq f_e^*(h_2) - 0.4, \\ & \quad f_b^*(h_3) \geq f_b^*(h_1) - 0.4, f_b^*(h_3) \geq f_b^*(h_2) - 0.4, \\ 0.3, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_b^*(h_2) - 0.8, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_b^*(h_1) - 0.8, \\ & \quad f_e^*(h_3) = f_b^*(h_3) \geq f_b^*(h_2) - 0.8, \\ & \quad f_b^*(h_3) \geq f_b^*(h_1) - 0.8, f_b^*(h_3) \geq f_b^*(h_2) - 0.8, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{T}_{k_2}^{\mathcal{C}_r^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.5, & \text{if } f_e^*(h_1) = f_b^*(h_1) \geq f_e^*(h_2) - 0.6, f_e^*(h_1) \geq f^*(h_3) - 0.5, \\ & \quad f_b^*(h_1) \geq f_b^*(h_2) - 0.5, f_b^*(h_1) \geq f_b^*(h_3) - 0.3, \\ & \quad f_e^*(h_2) = f_b^*(h_2) \geq f_e^*(h_1) - 0.6, f_e^*(h_2) \geq f_e^*(h_3) - 0.5, \\ & \quad f_b^*(h_2) \geq f_b^*(h_1) - 0.7, f_b^*(h_2) \geq f_b^*(h_3) - 0.5, \\ & \quad f_e^*(h_3) = f_b^*(h_3) \geq f_e^*(h_1) - 0.6, f_e^*(h_3) \geq f_e^*(h_2) - 0.4, \\ & \quad f_b^*(h_3) \geq f_b^*(h_1) - 0.8, f_b^*(h_3) \geq f_b^*(h_2) - 0.7, \\ 0, & \text{otherwise.} \end{cases}$$

(4) For $f_e = (0.5, 0.1, 0.6), f_b = (0.7, 0.6, 0.3)$,
For $0.3 \leq r < 0.6$,

$$\begin{array}{l}
\mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e = (0.5, 0.2, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b = (0.7, 0.6, 0.3), \\
\mathcal{C}_r^{\mathcal{U}}(k_1, f, r)_e = (0.5, 0.2, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b = (0.7, 0.6, 0.3).
\end{array}$$

For $0 < r \leq 0.3$,

$$\mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_e = (0.5, 0.1, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b = (0.7, 0.6, 0.3),$$
$$\mathcal{C}_r^{\mathcal{U}}(k_1, f, r)_e = (0.5, 0.1, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_1, f, r)_b = (0.7, 0.6, 0.3).$$

For $0 < r \leq 0.5$,

$$\mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_e = (0.5, 0.2, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_b = (0.7, 0.6, 0.3)$$
$$\mathcal{C}_r^{\mathcal{U}}(k_2, f, r)_e = (0.5, 0.1, 0.6), \mathcal{C}_l^{\mathcal{U}}(k_2, f, r)_b = (0.7, 0.6, 0.3)$$

Thus, $\mathcal{T}_{k_1}^{\mathcal{C}_l^{\mathcal{U}}}(f^*) = 0.3, \mathcal{T}_{k_2}^{\mathcal{C}_l^{\mathcal{U}}}(f^*) = 0$ and $\mathcal{T}_{k_1}^{\mathcal{C}_r^{\mathcal{U}}}(f^*) = 0.3, \mathcal{T}_{k_2}^{\mathcal{C}_r^{\mathcal{U}}}(f^*) = 0.5$.

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