International Journal of Pure and Applied Mathematics
Volume 108 No. 3 2016, 607-620
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v108i3.11

ijpam.eu

L-FUZZY (K,E)-SOFT INTERIOR OPERATORS AND L-FUZZY (K,E)-SOFT QUASI-UNIFORM SPACES

A.A. Ramadan[1], Ju-Mok Oh[2 §]

[1]Department of Mathematics
Faculty of Science
Beni-Suef University
Beni-Suef, EGYPT
[2]Department of Mathematics
Gangneung-Wonju University
Gangneung, Gangwondo, 210-702, KOREA

Abstract: This paper takes some investigations on L-fuzzy (K,E)-soft interior spaces and L-fuzzy (K,E)-soft quasi-uniform spaces which are a generalization of L-fuzzy topological structures and we gave some of its properties. Also, we give a systematic discussion on the relationship among these notions. Finally, we give their examples.

AMS Subject Classification: 03E72, 06A15, 06F07, 54A05
Key Words: complete residuated lattice, L-fuzzy (K,E)-soft interior space, L-fuzzy (K,E)-soft quasi-uniform space, L-fuzzy (K,E)-soft topological space

1. Introduction

Molodtsov [15,16] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1-3,7].The topological structures of soft sets have been developed by many researchers [4,5,17-20,23].

On the other hand, Hájek [8] introduced a complete residuated lattice which

Received: April 21, 2016
Published: July 21, 2016

§Correspondence author

is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structures [9-10,21,22].

It is well known that neighborhood systems and interior operators play an important role in topology and they are very good ways to characterize topology. Many authors [9-12,17-22] have studied neighborhood systems and interior operators in L-fuzzy topological spaces. Ramadan et al.[19,20] investigated the relationships between L-fuzzy (K, E)-soft quasi-uniform structures and L-fuzzy (K, E)-soft topological structures.

This paper takes some investigations on L-fuzzy (K, E)-soft interior spaces and L-fuzzy (K, E)-soft quasi-uniform spaces which are a generalization of L-fuzzy topological structures and we gave some of its properties. Also, we give a systematic discussion on the relationship among these notions. Finally, we give their examples.

2. Preliminaries

Definition 2.1. (see [8,9]) An algebra $(L, \wedge, \vee, \odot, \rightarrow, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $(L, \leq, \vee, \wedge, 0, 1)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \rightarrow z$ for $x, y, z \in L$.

In this paper, we assume that $L = (L, \vee, \wedge, \odot, \rightarrow, 0, 1)$ be a complete residuated lattice.

Lemma 2.2. (see [8,9]) *For each $x, y, z, w, x_i, y_i \in L$, the following properties hold:*

(1) *If $y \leq z$, then $x \odot y \leq x \odot z$.*

(2) *If $y \leq z$, then $x \rightarrow y \leq x \rightarrow z$ and $z \rightarrow x \leq y \rightarrow x$.*

(3) $x \rightarrow y = 1$ *iff* $x \leq y$.

(4) $x \rightarrow 1 = 1$ *and* $1 \rightarrow x = x$.

(5) $x \odot y \leq x \wedge y$.

(6) $x \odot (\bigvee_{i\in\Gamma} y_i) = \bigvee_{i\in\Gamma}(x \odot y_i)$ *and* $(\bigvee_{i\in\Gamma} x_i) \odot y = \bigvee_{i\in\Gamma}(x_i \odot y)$.

(7) $x \rightarrow (\bigwedge_{i\in\Gamma} y_i) = \bigwedge_{i\in\Gamma}(x \rightarrow y_i)$ *and* $(\bigvee_{i\in\Gamma} x_i) \rightarrow y = \bigwedge_{i\in\Gamma}(x_i \rightarrow y)$.

(8) $\bigvee_{i\in\Gamma} x_i \to \bigvee_{i\in\Gamma} y_i \geq \bigwedge_{i\in\Gamma}(x_i \to y_i)$ and $\bigwedge_{i\in\Gamma} x_i \to \bigwedge_{i\in\Gamma} y_i \geq \bigwedge_{i\in\Gamma}(x_i \to y_i)$.

(9) $(x \to y) \odot x \leq y$ and $(x \to y) \odot (y \to z) \leq (x \to z)$.

(10) $x \to y \leq (y \to z) \to (x \to z)$ and $x \to y \leq (z \to x) \to (z \to y)$.

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$.

(12) $y \to z \leq x \odot y \to x \odot z$ and $(x \to z) \odot (y \to w) \leq x \odot y \to z \odot w$.

A lattice L is called s-compact if $\bigvee_{j\in\Gamma} c_j \geq a$ for $c_j, a \in L$, there exists $j_\circ \in \Gamma$ such that $c_{j_\circ} \geq a$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.3. (see [4-6]) A map f is called an L-fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L-fuzzy set on X, for each $e \in E$. The family of all L-fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L-fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L-fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L-fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L-fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L-fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(6) 0_X (resp. 1_X) is an L-fuzzy soft set if $(0_X)_e(x) = 0$ (resp. $(1_X)_e(x) = 1$), for each $e \in E$, $x \in X$.

Definition 2.4. (see [4-6, 19-20]) A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$:

(SO1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(SO2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \ \ \forall\ f, g \in (L^X)^E$, (SO3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i\in I} \mathcal{T}_k(f_i) \; \forall \; f_i \in (L^X)^E, \; i \in I.$

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.
An L-fuzzy (K, E)-soft topology is called enriched if

(SR) $\mathcal{T}_k(\alpha \odot f) \geq \mathcal{T}_k(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

Lemma 2.5. (see [19-20]) *Define a binary mapping* $S : (L^X)^E \times (L^X)^E \to L$ *by*

$$S(f, g) = \bigwedge_{x\in X} \bigwedge_{e\in E} (f_e(x) \to g_e(x)) \quad \forall \; f, g \in (L^X)^E, \quad \forall \; e \in E.$$

Then $\forall f, g, h, m, n \in (L^X)^E$ *the following statements hold:*

(1) $f \sqsubseteq g$ *iff* $S(f, g) = 1$.

(2) If $f \sqsubseteq g$, *then* $S(h, f) \leq S(h, g)$ *and* $S(f, h) \geq S(g, h)$.

(3) $S(f, h) \odot S(h, g) \leq S(f, g)$. *Moreover,* $\bigvee_{h\in(L^X)^E}(S(f, h) \odot S(h, g)) = S(f, g)$

(4) $S(f, g) \odot S(m, n) \leq S(f \odot m, g \odot n)$.

Definition 2.6. (see [19-20]) An L-fuzzy (K, E)-soft quasi-uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions:

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.

(SU3) For every $u, v \in (L^{X\times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.

(SU4) If $\mathcal{U}_k(u) \neq 0$ then $1_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(1_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x, z) = \bigvee_{y\in X} v_e(x, y) \odot w_e(y, z),$$

The pair $(X, \mathcal{U})$ is called an L-fuzzy (K, E)-soft quasi-uniform space.

An L-fuzzy (K, E)-soft quasi-uniform space $(X, \mathcal{U})$ is said to be an L-fuzzy (K, E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x,y) = u_e(y,x)$ for each $k \in K$ and $u \in (L^{X\times X})^E$.

An L-fuzzy (K,E)-soft quasi-uniformity $\mathcal{U}$ on X is said to be stratified if

(SR) $\mathcal{U}_k(\alpha \odot u) \geq \alpha \odot \mathcal{U}_k(u), \quad \forall\, u \in (L^{X\times X})^E, \alpha \in L$.

3. L-Fuzzy (K,E)-Soft Interior Space and L-Fuzzy (K,E)-Soft Quasi-Uniform Space

Definition 3.1. A map $\mathcal{I} : K \times (L^X)^E \times L_0 \to (L^X)^E,\ L_0 = L - \{0\}$ is called an L-fuzzy (K,E)-soft interior operator on X if $\mathcal{I}$ satisfies the following conditions:

(SI1) $\mathcal{I}(k, 1_X, r) = 1_X$, and $\mathcal{I}(k, 0_X, r) = 0_X$,

(SI2) $\mathcal{I}(k, f, r) \sqsubseteq f$, for all $f \in (L^X)^E$,

(SI3) $S(f,g) \leq S(\mathcal{I}(k,f,r), \mathcal{I}(k,g,r))$ for all $f \in (L^X)^E$,

(SI4) If $r \leq s$, then $\mathcal{I}(k,f,s) \sqsubseteq \mathcal{I}(k,f,r)$,

(SI5) $\mathcal{I}(k, f \odot g, r \odot s) \sqsupseteq \mathcal{I}(k,f,r) \odot \mathcal{I}(k,g,s)$.

The pair $(X,\mathcal{I})$ is called an L-fuzzy (K,E)-soft interior space. An L-fuzzy (K,E)-soft interior space is called topological if

(SI6) $\mathcal{I}(k, \mathcal{I}(k,f,r), r) = \mathcal{I}(k,f,r), \quad \forall f \ \in (L^X)^E, r \in L_0$.

An L-fuzzy (K,E)-soft interior space $(X,\mathcal{I})$ is said to be stratified if

(R) $\mathcal{I}(k, \alpha \odot f, r) \geq \alpha \odot \mathcal{I}(k,f,r)$.

Theorem 3.2. *Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{I}_l^{\mathcal{U}} : K \times (L^X)^E \times L_0 \to (L^X)^E$, by:*

$$\mathcal{I}_l^{\mathcal{U}}(k,f,r)(x) = \bigvee_{\mathcal{U}_k(u)\geq r} S(u[x], f),$$

where $u_e[x](y) = u_e(y,x)$. Then the following properties hold:

(1) *$(X, \mathcal{I}_l^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft interior space.*

(2) *$\mathcal{I}_l^{\mathcal{U}}(\mathcal{I}_l^{\mathcal{U}}(k,f,r_1), r_1) \geq \mathcal{I}_l^{\mathcal{U}}(k,f,r)$ for each $r_1 < r$.*

(3) *If L is s-compact, then $\mathcal{I}_l^{\mathcal{U}}$ is a topological.*

(4) *If* $\mathcal{U}$ *is stratified, then* $\mathcal{I}_l^{\mathcal{U}}$ *is also stratified.*

Proof. (1) (SI1),(SI2) and (SI4) are easily proved.
(SI3)

$$\begin{aligned}
&S(\mathcal{I}_l^{\mathcal{U}}(k,f,r),\mathcal{I}_l^{\mathcal{U}}(k,g,r))\\
&= \bigwedge_{x\in X}\bigwedge_{e\in E}\big(\mathcal{I}_l^{\mathcal{U}}(k,f,r)(x)\to\mathcal{I}_l^{\mathcal{U}}(k,g,r)(x)\big)\\
&= \bigwedge_{x\in X}\bigwedge_{e\in E}\big(\bigvee_{\mathcal{U}(u)\geq r}S(u[x],f)\to\bigvee_{\mathcal{U}(v)\geq r}S(v[x],g)\big)\\
&\geq \bigwedge_{x\in X}\bigwedge_{e\in E}\bigwedge_{\mathcal{U}(u)\geq r}\big(S(u[x],f)\to S(u[x],g)\big)\ (\text{ by Lemma 2.2(8)})\\
&\geq \bigwedge_{\mathcal{U}(u)\geq r}\big(S(u[x],f)\to S(u[x],g)\big)\\
&\geq S(f,g).\ (\text{ by Lemma 2.5(5)})
\end{aligned}$$

(SI5) By Lemma 2.5 (4), we have

$$\begin{aligned}
\mathcal{I}_l^{\mathcal{U}}(k,f,r)(x)\odot\mathcal{I}_l^{\mathcal{U}}(k,g,s)(x) &= \big(\bigvee_{\mathcal{U}(u)\geq r}S(u[x],f)\big)\odot\big(\bigvee_{\mathcal{U}(v)\geq s}S(v[x],g)\big)\\
&\leq \bigvee_{\mathcal{U}(u)\odot\mathcal{U}(v)\geq r\odot s}\big(S(u[x],f)\odot S(v[x],g)\big)\\
&\leq \bigvee_{\mathcal{U}(u\odot v)\geq r\odot s}\big(S(u\odot v)[x],f\odot g\big)\\
&\leq \bigvee_{\mathcal{U}(w)\geq r\odot s}S(w[x],f\odot g)=\mathcal{I}_l^{\mathcal{U}}(k,f\odot g,r\odot s)(x).
\end{aligned}$$

(2) For $\mathcal{U}(u)\geq r$ and $r>r_1$, by (SU5), there exists $v\in(L^{X\times X})^E$ such that $\mathcal{U}(v)\geq r_1,\ v\circ v\leq u$.

$$\begin{aligned}
\mathcal{I}_l^{\mathcal{U}}(k,f,r)(x) &= \bigvee_{\mathcal{U}(u)\geq r}S(u[x],f)\\
&= \bigvee_{\mathcal{U}(u)\geq r}\bigwedge_{y\in X}\bigwedge_{e\in E}\big(u_e(y,x)\to f_e(y)\big)\\
&\leq \bigvee_{\mathcal{U}(v)\geq r_1}\bigwedge_{y\in X}\bigwedge_{e\in E}\big((v\circ v)_e(y,x)\to f_e(y)\big)
\end{aligned}$$

$$= \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{y\in X} \bigwedge_{e\in E} \big(\big(\bigvee_{z\in X} v_e(z,x) \odot v_e(y,z)\big) \to f_e(y)\big)$$
$$= \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{y\in X} \bigwedge_{z\in X} \bigwedge_{e\in E} \big(v_e(z,x) \odot v_e(y,z)\big) \to f_e(y)\big)$$
$$= \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{y\in X} \bigwedge_{z\in X} \bigwedge_{e\in E} \big(v_e(z,x) \to \big(v_e(y,z) \to f_e(y)\big)\big)$$
(by Lemma 2.2 (12))
$$= \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{z\in X} \bigwedge_{e\in E} \big(v_e(z,x) \to \bigwedge_{y\in X} \big(v_e(y,z) \to f_e(y)\big)\big).$$

Put $g_e(z) = \bigwedge_{y\in X}(v_e(y,z) \to f_e(y))$. Then $g_e(z) \leq \mathcal{I}_l^{\mathcal{U}}(k, f, r_1)(z)$ for all $z \in X$. Thus,

$$\begin{aligned}
\mathcal{I}_l^{\mathcal{U}}(k, f, r)(x) &= \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{z\in X} \bigwedge_{e\in E} (v_e(z,x) \to g_e(z)) \\
&\leq \bigvee_{\mathcal{U}(v)\geq r_1} \bigwedge_{z\in X} \bigwedge_{e\in E} (v_e(z,x) \to \mathcal{I}_l^{\mathcal{U}}(k, f, r_1)(z)) \\
&= \bigvee_{\mathcal{U}(v)\geq r_1} S(v[x], \mathcal{I}_l^{\mathcal{U}}(k, f, r_1)) \\
&= \mathcal{I}_l^{\mathcal{U}}(k, \mathcal{I}_l^{\mathcal{U}}(k, f, r_1), r_1)(x)
\end{aligned}$$

This implies that $(X, \mathcal{I}_l^{\mathcal{U}})$ is an L-fuzzy (K, E)-soft interior space.

(3) By (SU5), $r \leq \mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(u) \mid v \circ v \leq u\}$. Since L is s-compact, there exists $v_1 \in (L^{X\times X})^E$ with $v_1 \circ v_1 \sqsubseteq u$ such that $\mathcal{U}_k(v_1) \geq r$. By (2), the result holds.

(4) For any $f \in (L^X)^E$, $\alpha \in L$ and $\mathcal{U}_k(u) \geq r$, we have by Lemma 2.5 (4) and $(SI3)$:
$S(\mathcal{I}_l^{\mathcal{U}}(k, f, r), \mathcal{I}_l^{\mathcal{U}}(k, \alpha \odot f, r)) \geq S(f, \alpha \odot f) \geq \alpha$. That is

$$\alpha \odot \mathcal{I}_l^{\mathcal{U}}(k, f, r) \leq \mathcal{I}_l^{\mathcal{U}}(k, \alpha \odot f, r).$$

By a similar method of Theorem 3.2, we obtain the following corollary.

Corollary 3.3. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathcal{I}_r^{\mathcal{U}} : K \times (L^X)^E \times L_0 \to (L^X)^E$, by:

$$\mathcal{I}_r^{\mathcal{U}}(k,f,s)(x) = \bigvee_{\mathcal{U}_k(u)\geq s} S(u[[x]],f),\ \forall\ f\in (L^X)^E,\ x\in X,$$

where $u_e[[x]](y) = u_e(x,y)$. Then the following properties hold:

(1) $(X,\mathcal{I}_r^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft interior space.

(2) $\mathcal{I}_r^{\mathcal{U}}(\mathcal{I}_r^{\mathcal{U}}(k,f,s_1),s_1) \geq \mathcal{I}_r^{\mathcal{U}}(k,f,s)$ for each $s_1 < s$.

(3) If L is s-compact, then $\mathcal{I}_r^{\mathcal{U}}$ is a topological.

(4) If $\mathcal{U}$ is stratified, then $\mathcal{I}_r^{\mathcal{U}}$ is also stratified.

Theorem 3.4. *Let $(X,\mathcal{I})$ be an L-fuzzy (K,E)-soft interior space. Define a mapping $\mathcal{T}^{\mathcal{I}} : K \to L^{(L^X)^E}$ by:*

$$\mathcal{T}_k^{\mathcal{I}}(f) = \bigvee\{r\in L \mid S(f,\mathcal{I}(k,f,r)) = 1\}.$$

Then, $\mathcal{T}^{\mathcal{I}}$ is L-fuzzy (K,E)-soft topology on X. If $\mathcal{I}$ is stratified, then $\mathcal{T}^{\mathcal{I}}$ is enriched L-fuzzy (K,E)-topology on X.

Proof. (SO1) Since $1_X = \mathcal{I}(k,1_X,r)$ and $0_X = \mathcal{I}(k,0_X,r)$, we have $\mathcal{T}_k^{\mathcal{I}}(1_X) = 1, \mathcal{T}_k^{\mathcal{I}}(0_X) = 1$.

(SO2) By Theorem 3.2(1) and Lemma 2.5, we have

$$\begin{aligned}&S(\lambda_1,\mathcal{I}(k,f_1,r)) \odot S(f_2,\mathcal{I}(k,f_2,s))\\ &\leq S(f_1\odot f_2,\ \mathcal{I}(k,f_1,r)\odot\mathcal{I}(k,f_2,s))\\ &\leq S(f_1\odot f_2,\ \mathcal{I}(k,f_1\odot f_2,r\odot s))\end{aligned}$$

If $S(f_1,\mathcal{I}(k,f_1,r)) = 1$ and $S(f_2,\mathcal{I}(k,f_2,s)) = 1$, then $S(f_1\odot f_2,\ \mathcal{I}(k,f_1\odot f_2,r\odot s)) = 1$. Thus,

$$\mathcal{T}_k^{\mathcal{I}}(f_1\odot f_2) \geq \mathcal{T}_k^{\mathcal{I}}(f_1)\odot\mathcal{T}_k^{\mathcal{I}}(f_2).$$

(SO3) For a family of $\{f_i \mid i\in\Gamma\} \sqsubseteq (L^X)^E$, we have

$$S(\sqcup_{i\in\Gamma} f_i,\ \mathcal{I}(k,\sqcup_{i\in\Gamma} f_i,r)) \geq \bigwedge_{i\in\Gamma} S(f_i,\ \mathcal{I}(k,\sqcup_{i\in\Gamma} f_i,r))$$

$$\geq \bigwedge_{i\in\Gamma} S(f_i,\ \mathcal{I}(k,f_i,r)) = 1.$$

Hence, $\mathcal{T}^{\mathcal{I}}$ is an L-fuzzy (K,E)-topology on X.

(R) Let $\mathcal{I}$ be stratified. For $\alpha \in L_0$ and $f \in (L^X)^E$, we have

$$\begin{aligned} S(\alpha\odot f,\mathcal{I}(k,\alpha\odot f,r)) &\geq\ S(\alpha\odot f,\alpha\odot\mathcal{I}(k,f,r)) \\ &\geq S(f,\mathcal{I}(k,f,r)). \end{aligned}$$

Hence, $\mathcal{T}^{\mathcal{I}}$ is an enriched L-fuzzy (K,E)-topology on X.

From Theorems 3.2 and 3.4, we obtain the following corollary.

Corollary 3.5. *Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define the mappings $\mathcal{T}^{\mathcal{I}_l^{\mathcal{U}}}, \mathcal{T}^{\mathcal{I}_r^{\mathcal{U}}} : K \to L^{(L^X)^E}$ by:*

$$\mathcal{T}_k^{\mathcal{I}_l^{\mathcal{U}}}(f) = \bigvee\{r\in L \mid S(f,\mathcal{I}_l^{\mathcal{U}}(k,f,r)) = 1\},$$

$$\mathcal{T}_k^{\mathcal{I}_r^{\mathcal{U}}}(f) = \bigvee\{s\in L \mid S(f,\mathcal{I}_r^{\mathcal{U}}(k,f,s)) = 1\}.$$

Then, $\mathcal{T}^{\mathcal{I}_l^{\mathcal{U}}}$ and $\mathcal{T}^{\mathcal{I}_r^{\mathcal{U}}}$ are L-fuzzy (K,E)-soft topologies on X. If $\mathcal{U}$ is stratified, then $\mathcal{T}^{\mathcal{I}_l^{\mathcal{U}}}$ and $\mathcal{T}^{\mathcal{I}_r^{\mathcal{U}}}$ are enriched L-fuzzy (K,E)-topologies on X.

Example 3.6. Let $X = \{h_i \mid i = \{1,2,3\}\}$ with h_i=house and $E = \{e,b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0,1]$ by

$$x\odot y = \max\{0,x+y-1\},\ x\to y = \min\{1-x+y,1\}$$

Then $([0,1],\wedge,\to,0,1)$ is a complete residuated lattice.

(1) Put $v, v\odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v\odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix},\ (v\odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix}, \quad w_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : K = \{k_1, k_2\} \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v \odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

(2) $\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(x) = \bigvee_{\mathcal{U}(u)\geq r} S(u[x], f), \quad \forall\ f \in (L^X)^E,\ x \in X.$
If $r > 0.6$, then

$$\begin{aligned}
\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_1) &= S(1_{X\times X}[h_1], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\
\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_2) &= S(1_{X\times X}[h_2], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\
\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_3) &= S(1_{X\times X}[h_3], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x).
\end{aligned}$$

If $0.3 < r \leq 0.6$, then $\mathcal{U}_{k_1}(v) = 0.6$.

$$\begin{aligned}
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_1) = S(v[h_1], f) = f_e(h_1) \wedge (0.7 + f_e(h_2)) \wedge (0.6 + f_e(h_3)\\
&\wedge f_b(h_1) \wedge (0.3 + f_b(h_2)) \wedge (0.4 + f_b(h_3),\\
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_2) = S(v[h_2], f) = (0.4 + f_e(h_1)) \wedge f_e(h_2) \wedge (0.4 + f_e(h_3))\\
&\wedge(0.5 + f_b(h_1)) \wedge f_b(h_2) \wedge (0.4 + f_b(h_3),\\
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_3) = S(v[h_1], f) = (0.5 + f_e(h_1)) \wedge (0.5 + f_e(h_2)) \wedge f_e(h_3)\\
&\wedge(0.7 + f_b(h_1)) \wedge (0.5 + f_b(h_2)) \wedge f_b(h_3).
\end{aligned}$$

If $0 < r \leq 0.3$, then $\mathcal{U}_{k_1}(v \odot v) = 0.3$.

$$\begin{aligned}
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_1) = S(v \odot v[h_1], f)\\
&= f_e(h_1) \wedge f_b(h_1) \wedge (0.6 + f_b(h_2)) \wedge (0.8 + f_b(h_3),\\
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_2) = (0.8 + f_e(h_1)) \wedge f_e(h_2) \wedge (0.8 + f_e(h_3))\\
&\wedge f_b(h_2) \wedge (0.8 + f_b(h_3),\\
&\mathcal{I}_l^{\mathcal{U}}(k_1, f, r)(h_3) = f_e(h_3) \wedge f_b(h_3),
\end{aligned}$$

If $r > 0.5$, then

$$\begin{aligned}
\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_1) &= S(1_{X\times X}[h_1], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\
\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_2) &= S(1_{X\times X}[h_2], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\
\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_3) &= S(1_{X\times X}[h_3], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x).
\end{aligned}$$

If $0 < r \leq 0.5$, then $\mathcal{U}_{k_2}(w) = 0.5$.

$$\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_1) = S(w[h_1], f) = f_e(h_1) \wedge (0.6 + f_e(h_2)) \wedge (0.6 + f_e(h_3)$$
$$\wedge f_b(h_1) \wedge (0.7 + f_b(h_2)) \wedge (0.8 + f_b(h_3),$$
$$\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_2) = S(w[h_2], f) = (0.6 + f_e(h_1)) \wedge f_e(h_2) \wedge (0.4 + f_e(h_3))$$
$$\wedge(0.5 + f_b(h_1)) \wedge f_b(h_2) \wedge (0.7 + f_b(h_3),$$
$$\mathcal{I}_l^{\mathcal{U}}(k_2, f, r)(h_3) = S(w[h_3], f) = (0.5 + f_e(h_1)) \wedge (0.5 + f_e(h_2)) \wedge f_e(h_3)$$
$$\wedge(0.7 + f_b(h_1)) \wedge (0.5 + f_b(h_2)) \wedge f_b(h_3).$$

$$\mathcal{T}_{k_1}^{\mathcal{I}_l^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.6, & \text{if } f_e(h_1) = f_b(h_1) \leq f_e(h_2) + 0.7, f_e(h_1) \leq f(h_3) + 0.6, \\ & \quad f_b(h_1) \leq f_b(h_2) + 0.3, f_b(h_1) \leq f_b(h_3) + 0.4, \\ & \quad f_e(h_2) = f_b(h_2) \leq f_e(h_1) + 0.4, f_e(h_2) \leq f_e(h_3) + 0.4, \\ & \quad f_b(h_2) \leq f_b(h_1) + 0.5, f_b(h_2) \leq f_b(h_3) + 0.4, \\ & \quad f_e(h_3) = f_b(h_3) \leq f_e(h_1) + 0.5, f_e(h_3) \leq f_e(h_2) + 0.5, \\ & \quad f_b(h_3) \leq f_b(h_1) + 0.7, f_b(h_3) \leq f_b(h_2) + 0.5, \\ 0.3, & \text{if } f_e(h_1) = f_b(h_1) \leq f_b(h_2) + 0.6, f_b(h_1) \leq f_b(h_3) + 0.8, \\ & \quad f_e(h_2) = f_b(h_2) \leq f_e(h_1) + 0.8, \\ & \quad f_b(h_2) \leq f_b(h_1) + 0.8, f_b(h_2) \leq f_b(h_3) + 0.8, \\ & \quad f_e(h_3) = f_b(h_3), \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{T}_{k_2}^{\mathcal{I}_l^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.5, & \text{if } f_e(h_1) = f_b(h_1) \leq f_e(h_2) + 0.6, f_e(h_1) \leq f(h_3) + 0.6, \\ & \quad f_b(h_1) \leq f_b(h_2) + 0.7, f_b(h_1) \leq f_b(h_3) + 0.8, \\ & \quad f_e(h_2) = f_b(h_2) \leq f_e(h_1) + 0.6, f_e(h_2) \leq f_e(h_3) + 0.4, \\ & \quad f_b(h_2) \leq f_b(h_1) + 0.5, f_b(h_2) \leq f_b(h_3) + 0.7, \\ & \quad f_e(h_3) = f_b(h_3) \leq f_e(h_1) + 0.5, f_e(h_3) \leq f_e(h_2) + 0.5, \\ & \quad f_b(h_3) \leq f_b(h_1) + 0.7, f_b(h_3) \leq f_b(h_2) + 0.5, \\ 0, & \text{otherwise.} \end{cases}$$

(3) $\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(x) = \bigvee_{\mathcal{U}(u) \geq s} S(u[[x]], f), \ \ \forall\ f \in (L^X)^E,\ x \in X.$
If $r > 0.6$, then

$$\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_1) = S(1_{X \times X}[h_1], f) = \bigwedge_{e \in E} \bigwedge_{x \in X} f_e(x),$$
$$\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_2) = S(1_{X \times X}[h_2], f) = \bigwedge_{e \in E} \bigwedge_{x \in X} f_e(x),$$
$$\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_3) = S(1_{X \times X}[h_3], f) = \bigwedge_{e \in E} \bigwedge_{x \in X} f_e(x).$$

If $0.3 < r \leq 0.6$, then $\mathcal{U}_{k_1}(v) = 0.6$.

$$\begin{aligned}&\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_1) = f_e(h_1) \wedge (0.4 + f_e(h_2)) \wedge (0.5 + f_e(h_3)\\ &\wedge f_b(h_1) \wedge (0.5 + f_b(h_2)) \wedge (0.7 + f_b(h_3),\\ &\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_2) = (0.7 + f_e(h_1)) \wedge f_e(h_2) \wedge (0.5 + f_e(h_3))\\ &\wedge(0.3 + f_b(h_1)) \wedge f_b(h_2) \wedge (0.5 + f_b(h_3),\\ &\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_3) = (0.6 + f_e(h_1)) \wedge (0.4 + f_e(h_2)) \wedge f_e(h_3)\\ &\wedge(0.4 + f_b(h_1)) \wedge (0.4 + f_b(h_2)) \wedge f_b(h_3),\end{aligned}$$

If $0 < r \leq 0.3$, then $\mathcal{U}_{k_1}(v \odot v) = 0.3$.

$$\begin{aligned}&\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_1) = f_e(h_1) \wedge (0.8 + f_b(h_2)) \wedge f_b(h_1),\\ &\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_2) = f_e(h_2) \wedge (0.8 + f_b(h_1)) \wedge f_b(h_2),\\ &\mathcal{I}_r^{\mathcal{U}}(k_1, f, s)(h_3) = (0.8 + f_e(h_2)) \wedge f_e(h_3))\\ &\wedge(0.8 + f_b(h_1)) \wedge (0.8 + f_b(h_2)) \wedge f_b(h_3),\end{aligned}$$

If $r > 0.5$, then

$$\begin{aligned}&\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_1) = S(1_{X\times X}[h_1], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\ &\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_2) = S(1_{X\times X}[h_2], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x),\\ &\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_3) = S(1_{X\times X}[h_3], f) = \textstyle\bigwedge_{e\in E}\bigwedge_{x\in X} f_e(x).\end{aligned}$$

If $0 < r \leq 0.5$, then $\mathcal{U}_{k_2}(w) = 0.5$.

$$\begin{aligned}&\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_1) = f_e(h_1) \wedge (0.6 + f_e(h_2)) \wedge (0.5 + f_e(h_3)\\ &\wedge f_b(h_1) \wedge (0.5 + f_b(h_2)) \wedge (0.3 + f_b(h_3),\\ &\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_2) = (0.6 + f_e(h_1)) \wedge f_e(h_2) \wedge (0.5 + f_e(h_3))\\ &\wedge(0.7 + f_b(h_1)) \wedge f_b(h_2) \wedge (0.5 + f_b(h_3),\\ &\mathcal{I}_r^{\mathcal{U}}(k_2, f, s)(h_3) = 0.6 + f_e(h_1)) \wedge (0.4 + f_e(h_2)) \wedge f_e(h_3)\\ &\wedge(0.8 + f_b(h_1)) \wedge (0.7 + f_b(h_2)) \wedge f_b(h_3).\end{aligned}$$

$$\begin{aligned}&\mathcal{T}_{k_1}^{\mathcal{I}_r^{\mathcal{U}}}(f)\\ &= \begin{cases} 1, & \text{if } f = \alpha_X,\\ 0.6, & \text{if } f_e(h_1) = f_b(h_1) \leq f_e(h_2) + 0.4, f_e(h_1) \leq f(h_3) + 0.5,\\ & \quad f_b(h_1) \leq f_b(h_2) + 0.5, f_b(h_1) \leq f_b(h_3) + 0.7,\\ & \quad f_e(h_2) = f_b(h_2) \leq f_e(h_1) + 0.7, f_e(h_2) \leq f_e(h_3) + 0.5,\\ & \quad f_b(h_2) \leq f_b(h_1) + 0.3, f_b(h_2) \leq f_b(h_3) + 0.5,\\ & \quad f_e(h_3) = f_b(h_3) \leq f_e(h_1) + 0.6, f_e(h_3) \leq f_e(h_2) + 0.4,\\ & \quad f_b(h_3) \leq f_b(h_1) + 0.4, f_b(h_3) \leq f_b(h_2) + 0.4,\\ 0.3, & \text{if } f_e(h_1) = f_b(h_1) \leq f_b(h_2) + 0.8,\\ & \quad f_e(h_2) = f_b(h_2) \leq f_b(h_1) + 0.8,\\ & \quad f_e(h_3) = f_b(h_3) \leq f_b(h_2) + 0.8,\\ & \quad f_b(h_3) \leq f_b(h_1) + 0.8, f_b(h_3) \leq f_b(h_2) + 0.8,\\ 0, & \text{otherwise.}\end{cases}\end{aligned}$$

$$\mathcal{T}_{k_2}^{\mathcal{I}_r^{\mathcal{U}}}(f) = \begin{cases} 1, & \text{if } f = \alpha_X, \\ 0.5, & \text{if } f_e(h_1) = f_b(h_1) \leq f_e(h_2) + 0.6, f_e(h_1) \leq f(h_3) + 0.5, \\ & \quad f_b(h_1) \leq f_b(h_2) + 0.5, f_b(h_1) \leq f_b(h_3) + 0.3, \\ & \quad f_e(h_2) = f_b(h_2) \leq f_e(h_1) + 0.6, f_e(h_2) \leq f_e(h_3) + 0.5, \\ & \quad f_b(h_2) \leq f_b(h_1) + 0.7, f_b(h_2) \leq f_b(h_3) + 0.5, \\ & \quad f_e(h_3) = f_b(h_3) \leq f_e(h_1) + 0.6, f_e(h_3) \leq f_e(h_2) + 0.4, \\ & \quad f_b(h_3) \leq f_b(h_1) + 0.8, f_b(h_3) \leq f_b(h_2) + 0.7, \\ 0, & \text{otherwise.} \end{cases}$$

(4) For $f_e = f_b = (0.5, 0.1, 0.7)$,

$$\begin{aligned} \mathcal{I}_l^{\mathcal{U}}(k_1, f, r) &= (0.4, 0.1, 0.6), && \text{if } 0.3 \leq r < 0.6, \\ \mathcal{I}_l^{\mathcal{U}}(k_1, f, r) &= (0.5, 0.1, 0.7), && \text{if } 0 < r < 0.3, \\ \mathcal{I}_l^{\mathcal{U}}(k_2, f, r) &= (0.5, 0.1, 0.6), && \text{if } 0 < r < 0.5, \\ \mathcal{I}_r^{\mathcal{U}}(k_1, f, r) &= (0.5, 0.1, 0.4), && \text{if } 0.3 \leq r < 0.6, \\ \mathcal{I}_r^{\mathcal{U}}(k_1, f, r) &= (0.5, 0.1, 0.7), && \text{if } 0 < r < 0.3, \\ \mathcal{I}_r^{\mathcal{U}}(k_2, f, r) &= (0.5, 0.1, 0.7), && \text{if } 0 < r < 0.5. \end{aligned}$$

Thus, $\mathcal{T}_{k_1}^{\mathcal{I}_l^{\mathcal{U}}}(f) = 0.3, \mathcal{T}_{k_2}^{\mathcal{I}_l^{\mathcal{U}}}(f) = 0$ and $\mathcal{T}_{k_1}^{\mathcal{I}_r^{\mathcal{U}}}(f) = 0.3, \mathcal{T}_{k_2}^{\mathcal{I}_r^{\mathcal{U}}}(f) = 0.5$.

References

[1] H. Aktas, N. Cagman, Soft sets and soft groups, *Information Sciences*, **177** (2007) 2726-2735.

[2] M.I. Ali, F. Feng, X.Y. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, *Computers and Mathematics with Applications*, **57** (2009)1547-2553.

[3] A. Aygünoglu, H. Aygün, Introduction to fuzzy soft groups, *Computers and Mathematics with Appl.*, **58**(2009), 1279-1286.

[4] A. Aygünoglu, V. Cetkin, H. Aygün, An introduction to fuzzy soft topological spaces, *Hacettepe Journal of Math. and Stat.*, **43**(2)(2014), 193-204

[5] V. Cetkin, Alexander P. Šostak, H. Aygün, An Approach to the Concept of Soft fuzzy Proximity, *Abstract and Applied Analysis Stat.*, **2014**,**doi:** 10.1155/2014/782583.

[6] V. Cetkin, H. Aygün, On fuzzy Soft topogenous structure, *Journal of Intelligent and Fuzzy Systems*, **27(1)**(2014),247-255, **doi:** 10.3233/IFS-130993.

[7] F. Feng, Y.B. Jun, X. Liu, L. Li, An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics **234**(2010),10-20, **doi:** 10.1016/j.ins.2010.11.004.

[8] P. Hájek, Metamathematices of Fuzzy Logic, Kluwer Academic Publishers, Dordrecht, 1998, **doi:** 10.1007/978-94-011-5300-3.

[9] U. Höhle, E.P. Klement, *Non-classical logic and their applications to fuzzy subsets*, Kluwer Academic Publishers, Boston, 1995.

[10] U. Höhle, S.E. Rodabaugh, Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory, The Handbooks of Fuzzy Sets Series 3, Kluwer Academic Publishers, Boston, 1999,**doi:** 10.1007/978-1-4615-5079-2.

[11] Y.C. Kim, A.A.Ramadan, M.A. Usama, L-fuzzy Uniform spaces, *Journal of Fuzzy Mathematics*, **14** (2006),821-850.

[12] R. Lowen, Fuzzy uniform spaces, *J. Math. Anal. Appl.*, **82** (1981), 370-385.

[13] P.K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem, *Computers and Mathematics with Applications*, **44** (2002) 1077-1083.

[14] P.K. Maji, R. Biswas, R. Roy, Soft set theory, *Computers and Mathematics with Applications*, **45** (2003) 555-562.

[15] D. Molodtsov, Soft set theory first results, *Computers and Mathematics with Applications*, **37** (1999) 19-31.

[16] D. Molodtsov, V.Y. Leonov, D.V. Kovkov, Soft sets technique and its application, *Nechetkie Sistemy i Myagkie Vychisleniya*, **1** (1) (2006) 8-19.

[17] A.A. Ramadan, E.H.Elkordy, Y.C. Kim, Topological properties of L-fuzzy uniform spaces, *International Journal of Pure and Applied Mathematics* , **100** (2015), 347-360.

[18] A.A. Ramadan, E.H.Elkordy, Y.C. Kim, Relationships between L-fuzzy quasi-uniform stuctures and L-fuzzy topologies, *Journal of Intelligent and Fuzzy Systems*, **28** (2015), 2319-2327.

[19] A.A. Ramadan,Y.C. Kim, (K,E)-soft topologies and L-fuzzy (K,E)-soft neighborhood systems *International Journal of Pure and Applied Mathematics* (accepted)(2016).

[20] A.A. Ramadan,Y.C. Kim, L-fuzzy (K,E)-soft L-fuzzy (K,E)-soft topogenous orders and L-fuzzy (K,E)-soft topologies, *Journal of Intelligent and Fuzzy Systems*(submitted)(2016).

[21] S.E. Rodabaugh, A theory of fuzzy uniformities with applications to the fuzzy real lines, *J. Math. Anal. Appl.*, **129** (1988), 37-70.

[22] S.E. Rodabaugh, E.P. Klement, *Topological and Algebraic Structures in Fuzzy Sets*, The Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Kluwer Academic Publishers, Boston, Dordrecht, London, 2003.

[23] M. Shabir, M. Naz, On soft topological spaces, *Computers and Mathematics with Appl.*, **61**(2011), 1786-1799, **doi:** 10.1016/j.camwa.2011.02.006.