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L- FUZZY (K,E)-SOFT QUASI UNIFORM SPACES AND L-FUZZY (K,E)-SOFT TOPOGENOUS SPACES

Ju-Mok Oh[1], Yong Chan Kim[21], A.A. Ramadan[3]
[1,2]Department of Mathematics
Gangneung-Wonju University
Gangneung, Gangwondo, 210-702, KOREA
[3]Mathematics Department
Faculty of Science
Beni-Suef University
Beni-Suef, EGYPT

Abstract: The goal of this paper is to focus on the relationships between L-fuzzy (K,E)-soft quasi uniformities and L-fuzzy (K,E)- soft topogenous orders in complete residuated lattices. As main results, we investigate the L-fuzzy (K,E)- soft quasi uniformities induced by L-fuzzy (K,E)- soft topogenous orders. Moreover, we study the L-fuzzy (K,E)- soft topogenous orders induced by L-fuzzy (K,E)- soft uniformities. We give their examples.

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1. Introduction

Molodtsov [18] introduced a completely new concept called soft set theory to model uncertainty, which associates a set with a set of parameters. Pei and Miao [19] showed that soft sets are a class of special information systems. Later, Maji et al. [15] introduced the concept of a fuzzy soft set which combines a fuzzy set and a soft set. Presently, the soft set theory is making progress rapidly [1,2,6,15-19,26,28,31,32]. The topological structures of soft sets have been developed by

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many researchers [3,5,8,23,27,29,30,33].

Hájek [9] introduced a complete residuated lattice which is an algebraic structure for many valued logic. Bělohlávek [4] investigated information systems and decision rules in complete residuated lattices. Höhle [10] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). Uniformities in fuzzy sets, have the following approach of Lowen [14] based on powersets of the form $L^{X\times X}$ as a viewpoint of the enourage approach, the uniform covering approach of Kotzé [13], the uniform operator approach of Rodabaugh [25] as a generalization of Hutton [11] based on powersets of the form $(L^X)^{(L^X)}$, the unification approach of Gutiérrez García [7]. Recently, Gutiérrez García introduced L-valued Hutton uniformity where a quadruple $(L,\leq,\otimes,\star)$ is defined by a GL-monoid $(L,\star)$ as an extension of a completely distrbutive lattice L. Kim [12] introduced the notion of L-fuzzy uniformities as an extension of Lowen in a strictly two-sided, commutative quantale. Moreover, he investigated the relations between L- fuzzy topological spaces and L- fuzzy uniform spaces. Ramadan et.al [23] introduced the notion of L-fuzzy (K,E)-soft topogenous orders and L-fuzzy (K,E)- soft quasi uniformities in complete residuated lattices.

The goal of this paper is to focus on the relationships between L-fuzzy (K,E)- soft quasi uniformities and L-fuzzy (K,E)- soft topogenous orders in complete residuated lattices. As main results, we investigate the L-fuzzy (K,E)- soft quasi uniformities induced by L-fuzzy (K,E)- soft topogenous orders. Moreover, we study the L-fuzzy (K,E)- soft topogenous orders induced by L-fuzzy (K,E)- soft uniformities. We give their examples.

2. Preliminaries

Let $L=(L,\leq,\vee,\wedge,0,1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 2.1. [4,9,11] An algebra $(L,\wedge,\vee,\odot,\to,0,1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L=(L,\leq,\vee,\wedge,1,0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L,\odot,1)$ is a commutative monoid;

(C3) $x\odot y\leq z$ iff $x\leq y\to z$ for $x,y,z\in L$.

Remark 2.2. Every completely distributive lattice $(L,\leq,\wedge,\vee,^*)$ with order reversing involution * is a complete residuated lattice $(L,\leq,\odot,\oplus,^*)$ with a strong negation * where $\odot = \wedge$, $\oplus = \vee$ and

$$x \to y = \begin{cases} 1, & \text{if } x \leq y, \\ y, & \text{otherwise.} \end{cases}$$

In this paper, we assume that $(L, \leq, \odot, \to, \oplus, ^*)$ is a complete residuated lattice with an order reversing involution $x^* = x \to 0$ which is defined by $x \oplus y = (x^* \odot y^*)^*$.

Lemma 2.3. [4,9,11] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \leq y$ iff $x \to y = 1$.

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot y = (x \to y^*)^*$ and $x \oplus y = x^* \to y$,

(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(13) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$.

(14) $x \to y = y^* \to x^*$.

(15) $(x \vee y) \odot (z \vee w) \leq (x \vee z) \vee (y \odot w) \leq (x \oplus z) \vee (y \odot w)$.

(16) $\bigvee_{i \in \Gamma} x_i \to \bigvee_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$ and $\bigwedge_{i \in \Gamma} x_i \to \bigwedge_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$,

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.4. [3,5,23] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) An L- fuzzy soft set $h = f \oplus g$ is defined as $h_e = f_e \oplus g_e$, for each $e \in E$.

(6) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(7) f is called a null L- fuzzy soft set and is denoted by 0_X , if $f_e(x) = 0$, for each $e \in E$, $x \in X$.

(8) f is called an absolute L- fuzzy soft set and is denoted by 1_X , if $f_e(x) = 1$, for each $e \in E$, $x \in X$.

Definition 2.5. [3,5,23] A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(O1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(O2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \ \ \forall\, f, g \in (L^X)^E$,

(O3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i \in I} \mathcal{T}_k(f_i) \ \forall\, f_i \in (L^X)^E$, $i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.

Definition 2.6. [23] An L- fuzzy (K, E)-soft quasi uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X \times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X \times X})^E$ such that $\mathcal{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.

(SU3) For every $u, v \in (L^{X \times X})^E$,$\mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.

(SU4) If $\mathcal{U}_k(u) \neq 0$ then $\top_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(\top_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \leq \bigvee \{\mathcal{U}_k(v) \odot \mathcal{U}_k(w) \mid v \circ w \sqsubseteq u\}$.

The pair $(X,\mathcal{U})$ is called an L-fuzzy (K, E)-soft quasi-uniform space.

An L-fuzzy (K, E)-soft quasi-uniform space $(X,\mathcal{U})$ is said to be an L-fuzzy (K, E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x, y) = u_e(y, x)$ for each $k \in K$ and $u \in (L^{X \times X})^E$.

Remark 2.7. Let $(X,\mathcal{U})$ be an L-fuzzy (K, E)-soft uniform space.

(1) By (SU1) and (SU2), we have $\mathcal{U}_k(1_{X \times X}) = 1$ because $u \sqsubseteq 1_{X \times X}$ for all $u \in (L^{X \times X})^E$.

(2) Since $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1}) \leq \mathcal{U}_k((u^{-1})^{-1}) = \mathcal{U}_k(u)$, then $\mathcal{U}_k(u) = \mathcal{U}_k(u^{-1})$.

The proof of the next lemma is similar to Lemma 3.6 [12].

Lemma 2.8. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. For each $u \in (L^{X\times X})^E$ and $f \in (L^X)^E$, the image $u[f]$ of f with respect to u is the fuzzy soft subset of X defined by

$$u_e[f_e](x) = \bigvee_{y\in X} (f_e(y) \odot u_e(y,x)), \quad \forall\, x \in X, e \in E.$$

For each $u, v, u_1, u_2 \in (L^{X\times X})^E$ and $f, f_1, f_2, f_i \in (L^X)^E$, we have

(1) $f \sqsubseteq u[f]$, for each $\mathcal{U}(u) > 0$,

(2) $u \sqsubseteq u \circ u$, for each $\mathcal{U}(u) > 0$,

(3) $(v \circ u)[f] = v[u[f]]$,

(4) $u[\bigvee_i f_i] = \bigvee_i u[f_i]$,

(5) $(u_1 \odot u_2)[f_1 \odot f_2] \sqsubseteq u_1[f_1] \odot u_2[f_2]$,

(6) $(u_1 \odot u_2)[f_1 \oplus f_2] \sqsubseteq u_1[f_1] \oplus u_2[f_2]$.

Definition 2.9. [5,23] A mapping $\xi : K \to L^{(L^X)^E\times(L^X)^E}$ is called an L-fuzzy (K,E)-soft semi-topogenous order on X if it satisfies the following axioms.

(ST1) $\xi_k(1_X, 1_X) = \xi_k(0_X, 0_X) = 1$,

(ST2) $\xi_k(f, g) \leq \bigwedge_{x\in X} \bigwedge_{e\in E}(f_e(x) \to g_e(x))$,

(ST3) If $f_1 \sqsubseteq f$, $g \sqsubseteq g_1$, then $\xi_k(f, g) \leq \xi_k(f_1, g_1)$.

An L-fuzzy (K,E)-soft semi-topogenous order ξ is called: for every f_1, f_2, $g_1, g_2 \in (L^X)^E$,

(1) topogenous if

(T) $\xi_k(f_1 \odot f_2, g_1 \odot g_2) \geq \xi_k(f_1, g_1) \odot \xi_k(f_2, g_2)$.

(2) cotopogenous if

(CT) $\xi_k(f_1 \oplus f_2, g_1 \oplus g_2) \geq \xi_k(f_1, g_1) \odot \xi_k(f_2, g_2)$,

(3) bitopogenous if ξ are L-fuzzy (K,E)-soft topogenous and L-fuzzy (K,E)-soft cotopogenous.

An L-fuzzy (K,E)-soft topogenous (resp. cotopogenous) order ξ on X is said to be L-fuzzy (K,E)-topogenous (resp. cotopogenous) space if

(TS) $\xi \circ \xi \geq \xi$, where

$$(\xi_k \circ \xi_k)(f, g) = \bigvee_{h\in(L^X)^E} (\xi_k(f, h) \odot \xi_k(h, g)).$$

Let ξ be an L-fuzzy (K,E)-soft semi-topogenous order on X and let the mapping $\xi^s : K \to L^{(L^X)^E\times(L^X)^E}$ defined by $\xi^s_k(f, g) = \xi_k(g^*, f^*)$. Then ξ^s is an L-fuzzy semi-topogenous order on X. An L-fuzzy (K,E)-soft semi-topogenous order ξ on X is called symmetric if

(ST4) $\xi = \xi^s$.

Remark 2.10. If ξ is an L-fuzzy (K,E)-soft semi-topogenous order on X.
(1) If $\xi_k(f,g) = 1$, then $f \sqsubseteq g$.
(2) $\xi_k(1_X, f) \leq \bigwedge_{x\in X}\bigwedge_{e\in E} f_e(x)$ and $\xi_k(f, 0_X) \leq \bigwedge_{x\in X}\bigwedge_{e\in E} f^*_e(x)$.
(3) If the parameters sets E, K are both one-pointed sets, then L-fuzzy (K,E)-soft semi-topogenous order is the concept of Ramadan et al.[22].

3. L-Fuzzy (K, E)-Soft Quasi-Uniform Spaces and L-Fuzzy (K, E)-Soft Topogenous Spaces

Theorem 3.1. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a mapping $\xi^{\mathcal{U}} : K \to L^{(L^X \times L^X)^E}$ by

$$\xi^{\mathcal{U}}_k(f,g) = \bigvee\{\mathcal{U}_k(u) \mid u[f] \sqsubseteq g\}.$$

Then $(X, \xi^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft topogenous space.

Proof. (ST1) Since $u[0_X] = 0_X$ and $u[1_X] = 1_X$, for $\mathcal{U}_k(u) = 1$, we have $\xi^{\mathcal{U}}_k(1_X, 1_X) = \xi^{\mathcal{U}}_k(0_X, 0_X) = 1$.
(ST2) Since for all $\mathcal{U}_k(u) > 0$, we have $f \sqsubseteq u[f]$. Then if $\xi^{\mathcal{U}}_k(f,g) = 1$, we have $f \sqsubseteq g$.
(ST3) If $f_1 \sqsubseteq f$, $g \sqsubseteq g_1$, then

$$\begin{aligned}\xi^{\mathcal{U}}_k(f,g) &= \bigvee\{\mathcal{U}_k(u) \mid u[f] \sqsubseteq g\} \leq \bigvee\{\mathcal{U}_k(u) \mid u[f] \sqsubseteq g_1\} \\ &\leq \bigvee\{\mathcal{U}_k(u) \mid u[f_1] \sqsubseteq g_1\} = \xi^{\mathcal{U}}_k(f_1, g_1).\end{aligned}$$

(ST5)

$$\begin{aligned}&\xi^{\mathcal{U}}_k(f_1,g_1) \odot \xi^{\mathcal{U}}_k(f_2,g_2) \\ &= \bigvee\{\mathcal{U}_k(u) \mid u[f_1] \sqsubseteq g_1\} \odot \bigvee\{\mathcal{U}(v) \mid v[f_2] \sqsubseteq g_2\} \\ &\leq \bigvee\{\mathcal{U}_k(u) \odot \mathcal{U}(v) \mid u[f_1] \odot v[f_2] \sqsubseteq g_1 \odot g_2\} \\ &\leq \bigvee\{\mathcal{U}_k(u \odot v) \mid (u \odot v)[f_1 \odot f_2] \sqsubseteq g_1 \odot g_2\} \\ &\leq \bigvee\{\mathcal{U}_k(w) \mid w[f_1 \odot f_2] \sqsubseteq g_1 \odot g_2\} = \xi^{\mathcal{U}}_k(f_1 \odot f_2, g_1 \odot g_2).\end{aligned}$$

(ST6)

$$\begin{aligned}
&\xi_k^{\mathcal{U}}(f_1,g_1)\odot \xi_k^{\mathcal{U}}(f_2,g_2)\\
&=\bigvee\{\mathcal{U}_k(u)\mid u[f_1]\sqsubseteq g_1\}\odot\bigvee\{\mathcal{U}_k(v)\mid v[f_2]\sqsubseteq g_2\}\\
&\le\bigvee\{\mathcal{U}_k(u)\odot\mathcal{U}(v)\mid u[f_1]\oplus v[f_2]\sqsubseteq g_1\oplus g_2\}\\
&\le\bigvee\{\mathcal{U}_k(u\odot v)\mid u\odot v[f_1\oplus f_2]\sqsubseteq g_1\oplus g_2\}\\
&\le \xi_k^{\mathcal{U}}(f_1\oplus f_2, g_1\oplus g_2).
\end{aligned}$$

(TS) For each $u\in (L^{X\times X})^E$ such that $u[f]\sqsubseteq g$, by (SU5), we have

$$\mathcal{U}_k(u)=\bigvee\{\mathcal{U}_k(v)\odot\mathcal{U}_k(w)\mid v\circ w\sqsubseteq u\}.$$

Thus,

$$\begin{aligned}
&\bigvee\{\mathcal{U}_k(u)\mid u[f]\sqsubseteq g\}\\
&\le\bigvee\{\mathcal{U}_k(v)\odot\mathcal{U}_k(w)\mid v\circ w[f]=v[w[f]]\sqsubseteq g\}\\
&\le\bigvee_{h\in(L^X)^E}\{\bigvee\{\mathcal{U}_k(v)\odot\mathcal{U}(w)\mid w[f]\sqsubseteq h,\ v[h]\sqsubseteq g\}\}\\
&\le\bigvee_{h\in(L^X)^E}\{\bigvee\{\mathcal{U}_k(v)\mid v[h]\sqsubseteq g\}\odot\bigvee\{\mathcal{U}_k(w)\mid w[f]\sqsubseteq h\}\}\\
&=\bigvee_{\gamma\in L^X}\xi_k^{\mathcal{U}}(f,h)\odot\xi_k^{\mathcal{U}}(h,g).
\end{aligned}$$

Lemma 3.2. For every $f,g\in(L^X)^E$, we define $u_{f,g},u_{f,g}^{-1}\in(L^{X\times X})^E$ by

$$(u_{f,g})_e(x,y)=f_e(x)\to g_e(y)\qquad \forall e\in E,$$
$$(u_{f,g}^{-1})_e(x,y)=(u_{f,g})_e(y,x).$$

Then we have the following statements
(1) $1_{X\times X}=u_{0_X,0_X}=u_{1_X,1_X}$,
(2) If $f_1\sqsubseteq f_2$ and $g_1\sqsubseteq g_2$, then $u_{f_2,g_1}\sqsubseteq u_{f_1,g_2}$,
(3) If $f\sqsubseteq g$, then $1_\triangle\sqsubseteq u_{f,g}$,
(4) For $u_{g,h}\in(L^{X\times X})^E$ and $f\in(L^X)^E$, $u_{h,g}\circ u_{f,h}\sqsubseteq u_{f,g}$,
(5) $u_{f_1,g_1}\odot u_{f_2,g_2}\sqsubseteq u_{f_1\odot f_2,g_1\odot g_2}$,
(6) $u_{f_1,g_1}\odot u_{f_2,g_2}\sqsubseteq u_{f_1\oplus f_2,g_1\oplus g_2}$,
(7)$u_{f,g}^{-1}=u_{g^*,f^*}$,
(8)$u^{-1}_{f_1\odot f_2,g_1\odot g_2}=u_{g_1^*\oplus g_2^*,f_1^*\oplus f_2^*}$,
(9)$u^{-1}_{f_1\oplus f_2,g_1\oplus g_2}=u_{g_1^*\odot g_2^*,f_1^*\odot f_2^*}$.
(10) $u[f]\sqsubseteq g$ iff $u\sqsubseteq u_{f,g}$.
(11) $u_{f,g}=\bigvee\{u\in(L^{X\times X})^E|u[f]\sqsubseteq g\}$.
(12) $u_{f,g}[f]\sqsubseteq g$ and $u_{f,f}[f]=f$.

Proof. (1) $(1_{X\times X})_e(x,y)=1=(u_{0_X,0_X})_e(x,y)=(0_X)_e(x)\to(0_X)_e(y)=(1_X)_e(x)\to(1_X)_e(y)=(u_{1_X,1_X})_e(x,y)$.

(2) Let $f_1 \sqsubseteq f_2$ and $g_1 \sqsubseteq g_2$, then

$$\begin{array}{l}(u_{f_2,g_1})_e(x,y) = (f_2)_e(x) \rightarrow (g_1)_e(y)\\ \leq (f_1)_e(x) \rightarrow (g_2)_e(y) = (u_{f_1,g_2})_e(x,y).\end{array}$$

(3) Since $1_\triangle[f] = f \sqsubseteq g$, then $1_\triangle \sqsubseteq u_{f,g}$.

(4)

$$\begin{array}{l}(u_{h,g})_e(x,z) \circ (u_{f,h})_e(x,z)\\ = \bigvee_{y\in X}((h_e(y) \rightarrow g_e(z)) \odot (f_e(x) \rightarrow h_e(y)))\\ \leq f_e(x) \rightarrow g_e(z) = (u_{f,g})_e(x,z).\end{array}$$

(5)

$$\begin{array}{l}(u_{f_1,g_1} \odot u_{f_2,g_2})_e(x,y) = (u_{f_1,g_1})_e(x,y) \odot (u_{f_2,g_2})_e(x,y)\\ = ((f_1)_e(x) \rightarrow (g_1)_e(y)) \odot ((f_2)_e(x) \rightarrow (g_2)_e(y))\\ \leq (f_1)_e(x) \odot (f_2)_e(x) \rightarrow (g_1)_e(y) \odot (g_2)_e(y)\\ = (u_{f_1 \odot f_2}, u_{g_1 \odot g_2})_e(x,y).\end{array}$$

(6)

$$\begin{array}{l}(u_{f_1,g_1} \odot u_{f_2,g_2})_e(x,y) = (u_{f_1,g_1})_e(x,y) \odot (u_{f_2,g_2})_e(x,y)\\ \leq ((f_1)_e(x) \rightarrow (g_1)_e(y)) \odot ((f_2)_e(x) \rightarrow (g_2)_e(y))\\ \leq (f_1)_e(x) \oplus (f_2)_e(x) \rightarrow (g_1)_e(y) \oplus (g_2)_e(y)\\ = (u_{f_1 \oplus f_2}, u_{g_1 \oplus g_2})_e(x,y).\end{array}$$

(7)

$$\begin{array}{l}(u^{-1}_{f,g})_e(x,y) = (u_{f,g})_e(y,x) = f_e(y) \rightarrow g_e(x)\\ = g^*_e(x) \rightarrow f^*_e(y) = (u_{g^*,f^*})_e(x,y).\end{array}$$

(8),(9) are similarly proved

(10)

$$\begin{array}{l}u_e[f_e](x) = \bigvee_{y\in X}(f_e(y) \odot u_e(y,x)) \leq g_e(x)\\ \text{iff } u_e(y,x) \leq f_e(y) \rightarrow g_e(x) = (u_{f,g})_e(y,x).\end{array}$$

(11) Since $u_e[f_e](x) = \bigvee_{y\in X}(f_e(y) \odot u_e(y,x)) \leq g_e(x)$, then $u_e(y,x) \leq f_e(y) \rightarrow g_e(x) = (u_{f,g})_e(y,x)$. Moreover, $(u_{f,g})_e(y,x) \odot f_e(y) = (f_e(y) \rightarrow g_e(x)) \odot f_e(y) \leq g_e(x)$. Hence $u_{f,g} = \bigvee\{u \in (L^{X\times X})^E | u[f] \sqsubseteq g\}$.

(12) Since $(u_{f,g})_e[f_e](y) = \bigvee_{x\in X}((f_e(x) \rightarrow g_e(y)) \odot f_e(x)) \leq g_e(y)$, then $u_{f,g}[f] \leq g$. Moreover,

$$\begin{array}{l}(u_{f,f})_e[f_e](y) \geq (u_{f,f})_e(y,y) \odot f_e(y)\\ = (f_e(y) \rightarrow f_e(y)) \odot f_e(y) = f_e(y).\end{array}$$

In the following theorem, we obtain an L-fuzzy (K,E)-soft quasi uniform space from an L-fuzzy (K,E)-soft topogenous order.

Theorem 3.3. Let (X,ξ) be an L-fuzzy (K,E)-soft topogenous space. Define $\mathcal{U}^{\xi}: K \to L^{(}L^{X\times X})^{E}$ by

$$\mathcal{U}^{\xi}_k(u) = \bigvee\{\odot^{n}_{i=1}\xi_k(f_i,g_i) \mid \odot^{n}_{i=1} u_{f_i,g_i} \sqsubseteq u\},$$

where $\bigvee$ is taken over every finite family $\{u_{f_i,g_i} \mid i = 1,2,3,....,n\}$. Then

(1) $\mathcal{U}^{\xi}_k(u_{\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i}) = \xi_k(\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i)$

(2) $\mathcal{U}^{\xi}$ is an L-fuzzy (K,E)-soft quasi uniformity on X,

(3) $\xi^{\mathcal{U}^{\xi}} = \xi$.

(4) $\mathcal{U}^{\xi^{s}}(u) = \mathcal{U}^{\xi}(u^{-1})$

Proof. (1) Since $\odot^{n}_{i=1}u_{f_i,g_i} \sqsubseteq u_{\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i}$, from Lemma 2.3 (14) and by (T), $\odot^{n}_{i=1}\xi_k(f_i,g_i) \leq \xi_k(\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i)$, then

$$\mathcal{U}^{\xi}_k(u_{\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i}) = \xi_k(\odot^{n}_{i=1}f_i,\odot^{n}_{i=1}g_i).$$

(2) (SU1) Since $\xi_k(0_X,0_X) = \xi_k(1_X,1_X) = 1$, there exists $1_{X\times X} = u_{0_X,0_X} = u_{1_X,1_X} \in (L^{X\times X})^{E}$. It follows $\mathcal{U}^{\xi}_k(1_{X\times X}) = 1$.

(SU2) It is trivial from the definition of $\mathcal{U}^{\xi}$.

(SU3) For every $u,v \in (L^{X\times X})^{E}$, each two families $\{u_{f_i,g_i} \mid \odot^{n}_{i=1}u_{f_i,g_i} \sqsubseteq u\}$ and $\{u_{h_j,w_j} \mid \odot^{k}_{j=1} u_{h_j,w_j} \sqsubseteq v\}$, we have

$$\begin{aligned}
&\mathcal{U}^{\xi}_k(u) \odot \mathcal{U}^{\xi}_k(v) = \big(\bigvee\{\odot^{n}_{i=1}\xi_k(f_i,g_i) \mid \odot^{n}_{i=1} u_{f_i,g_i} \sqsubseteq u\}\big)\\
&\odot\big(\bigvee\{\odot^{k}_{j=1}\xi_k(h_i,w_i) \mid \odot^{k}_{j=1} u_{h_i,w_i} \sqsubseteq v\}\big)\\
&\leq \bigvee\{\odot^{n}_{i=1}\xi_k(f_i,g_i) \odot \odot^{k}_{j=1}\xi_k(h_i,w_i) \mid\\
&\odot^{n}_{i=1}u_{f_i,g_i} \sqsubseteq u,\ \odot^{k}_{j=1} u_{h_i,w_i} \sqsubseteq v\}\\
&\leq \bigvee\{\odot^{n}_{i=1}\xi_k(f_i,g_i) \odot \odot^{k}_{j=1}\xi_k(h_i,w_i) \mid\\
&\odot^{n}_{i=1}u_{f_i,g_i} \odot \odot^{k}_{j=1}u_{h_i,w_i} \sqsubseteq u \odot v\}\\
&\leq \mathcal{U}^{\xi}_k(u \odot v).
\end{aligned}$$

(SU4) If $\mathcal{U}_k(u) \neq 0$, there exists a family $\{u_{f_i,g_i} \mid \odot^{m}_{i=1} u_{f_i,g_i} \sqsubseteq u\}$ such that $\odot^{m}_{j=1}\xi_k(f_i,g_i) \neq 0$. Since $\xi_k(f_i,g_i) \neq 0$, for $i = 1,2,...,m$, then $f_i \sqsubseteq g_i$for $i = 1,2,...,m$, i.e. $1_{\triangle} \sqsubseteq u_{f_i,g_i}$. Thus $1_{\triangle} \sqsubseteq \odot^{m}_{i=1}u_{f_i,g_i} \sqsubseteq u$.

(SU5) Suppose there exists $u \in L^{X\times X}$ such that

$$\bigvee\{\mathcal{U}^{\xi}_k(v) \odot \mathcal{U}^{\xi}_k(w) \mid v \circ w \leq u\} \ngeq \mathcal{U}^{\xi}_k(u).$$

Put $t = \bigvee\{\mathcal{U}_k^\xi(v) \odot \mathcal{U}_k^\xi(w) \mid v \circ w \leq u\}$. From the Definition of $\mathcal{U}_k^\xi(u)$, there exists family $\{u_{f_i,g_i} \mid \odot_{i=1}^m u_{f_i,g_i} \sqsubseteq u\}$ such that

$$t \not\geq \odot_{i=1}^m \xi_k(f_i, g_i).$$

Since $\xi_k \circ \xi_k \geq \xi_k$,

$$t \not\geq \odot_{i=1}^m \xi_k \circ \xi_k(f_i, g_i) = \odot_{i=1}^m \{\bigvee_{h \in (L^X)^E} \{(\xi_k(h, g_i) \odot (\xi_k(f_i, h))\}.$$

Since L is a stsc-quantel, there exists $h_i \in (L^X)^E$ such that

$$t \not\geq \odot_{i=1}^m (\xi_k(h_i, g_i) \odot \xi_k(f_i, h_i)).$$

On the other hand, put $v_i = u_{h_i,g_i}, w_i = u_{f_i,h_i}$, then from Lemma 3.1 (4), it satisfies $v_i \circ w_i \sqsubseteq u_{h_i,g_i} \circ u_{f_i,h_i} \sqsubseteq u_{f_i,g_i}$,

$$\mathcal{U}_k^\xi(v_i) \geq \xi_k(h_i, g_i), \ \mathcal{U}_k^\xi(w_i) \geq \xi_k(f_i, h_i).$$

Let $v = \odot_{i=1}^m v_i$ and $w = \odot_{i=1}^m w_i$ be given . Since $v_i \circ w_i \sqsubseteq u_{f_i,g_i}$, for each $i = 1, 2, 3, ..., m$, we have $\big(\odot_{i=1}^m v_i\big) \circ \big(\odot_{i=1}^m v_i\big) = \odot_{i=1}^m (v_i \circ w_i) \sqsubseteq \odot_{i=1}^m u_{f_i,g_i} \sqsubseteq u$.

Then we have $v \circ w \sqsubseteq u$ and $\mathcal{U}_k^\xi(v) \geq \odot_{i=1}^m \mathcal{U}_k^\xi(v_i)$ and $\mathcal{U}_k^\xi(w) \geq \odot_{i=1}^m \mathcal{U}_k^\xi(w_i)$. Thus,

$$\begin{aligned} t &= \bigvee\{\mathcal{U}_k^\xi(v) \odot \mathcal{U}_k^\xi(w) \mid v \circ w \sqsubseteq u\} \\ &\geq \mathcal{U}_k^\xi(v) \odot \mathcal{U}_\xi(w) \geq \odot_{i=1}^m (\xi_k(h_i, g_i) \odot \xi_k(f_i, h_i)). \end{aligned}$$

It is a contradiction. Then $\mathcal{U}^\xi$ is an L-fuzzy (K, E)-soft quasi uniformity on X.

(3) Since $u[f] \sqsubseteq g$, by Lemma 3.2(10), $u \sqsubseteq u_{f,g}$. Hence, $\xi_k^{\mathcal{U}^\xi}(f, g) = \bigvee\{\mathcal{U}_k^\xi(u) \mid u[f] \sqsubseteq g\} = \mathcal{U}_k^\xi(u_{f,g}) = \xi_k(f, g)$.

(4)

$$\begin{aligned} \mathcal{U}_k^{\xi^s}(u) &= \bigvee\{\odot_{i=1}^n \xi_k^s(f_i, g_i) \mid \odot_{i=1}^n u_{f_i,g_i} \sqsubseteq u\} \\ &= \bigvee\{\odot_{i=1}^n \xi_k(g_i^*, f_i^*) \mid \odot_{i=1}^n u_{f_i,g_i}^{-1} \sqsubseteq u^{-1}\} \\ &= \bigvee\{\odot_{i=1}^n \xi_k(g_i^*, f_i^*) \mid \odot_{i=1}^n u_{g_i^*,f_i^*} \sqsubseteq u^{-1}\} = \mathcal{U}_k^\xi(u^{-1}). \end{aligned}$$

The following corollary is similarly proved as Theorem 3.3.

Corollary 3.4. Let (X, ξ) be an L-fuzzy (K, E)-soft cotopogenous space. Define $\mathcal{U}^\xi : K \to L^{(}L^{X \times X})^E$ by

$$\mathcal{U}_k^\xi(u) = \bigvee\{\odot_{i=1}^n \xi_k(f_i, g_i) \mid \odot_{i=1}^n u_{f_i,g_i} \sqsubseteq u\},$$

where $\bigvee$ is taken over every finite family $\{u_{f_i,g_i} \mid i = 1, 2, 3, ..., n\}$. Then

(1) $\mathcal{U}^{\xi}_k(u_{\oplus_{i=1}^n f_i, \oplus_{i=1}^n g_i}) = \xi_k(\oplus_{i=1}^n f_i, \oplus_{i=1}^n g_i)$

(2) $\mathcal{U}^{\xi}$ is an L-fuzzy (K, E)-soft quasi uniformity on X,

(3) $\xi^{\mathcal{U}^{\xi}} = \xi$.

(4) $\mathcal{U}^{\xi^s}(u) = \mathcal{U}^{\xi}(u^{-1})$

Definition 3.5. An L- fuzzy (K, E)-soft uniform structure $\mathcal{U}$ on X is said to be compatible with an L- fuzzy (K, E)-soft topogenous order ξ on X if $\xi^{\mathcal{U}} = \xi$. The class $\prod(\xi)$ denotes the family of all L- fuzzy (K, E)-soft uniform structure which are compatible with a given L- fuzzy (K, E)-soft topogenous structure ξ.

Theorem 3.6. Let ξ be an L- fuzzy (K, E)-soft topogenous order on X and the L- fuzzy (K, E)-soft topogenous order $\xi^{\mathcal{U}^{\xi}}$ induced by $\mathcal{U}^{\xi}$. Then

(1) $\xi^{\mathcal{U}^{\xi}} = \xi$, that is $\mathcal{U}^{\xi} \in \prod(\xi)$,

(2) $\mathcal{U}^{\xi}$ is the coarsest member of $\prod(\xi)$, i.e.,$\mathcal{U}^{\xi} \leq \mathcal{U}$.

Proof. (1) is easily proved from Theorem 3.3.

(2) Let $\mathcal{U}$ be an arbitrary member of $\prod(\xi)$. We will show that $\mathcal{U}^{\xi}_k(u) \leq \mathcal{U}_k(u)$, for all $u \in (L^{X\times X})^E$.

Suppose that there exists $u \in (L^{X\times X})^E$ such that

$$\mathcal{U}^{\xi}_k(u) \not\leq \mathcal{U}_k(u).$$

There exists a family $\{u_{f_i,g_i} \mid \odot_{i=1}^m u_{f_i,g_i} \sqsubseteq u\}$ such that $\odot_{j=1}^m \xi_k(f_i, g_i) \not\leq \mathcal{U}^{\xi}_k(u)$.

Since $\mathcal{U}^{\xi} \in \prod(\xi)$, that is $\xi_k(f_i, g_i) = \xi^{\mathcal{U}}_k(f_i, g_i)$ for each $i = 1, 2, ..., m$, by the definition of $\xi^{\mathcal{U}}$, there exists $v_i \in (L^{X\times X})^E$ with $v_i[f_i] \sqsubseteq g_i$ such that

$$\odot_{j=1}^m \mathcal{U}^{\xi}_k(v_i) \not\leq \mathcal{U}_k(u)..$$

On the other hand, put $v = \odot_{i=1}^m v_i$. Since $v_i[f_i] \sqsubseteq g_i$ by the definition of u_{f_i,g_i}, we have $v_i \sqsubseteq u_{f_i,g_i}$. It follows that

$$v = \odot_{i=1}^m v_i \sqsubseteq \odot_{i=1}^m u_{f_i,g_i} \sqsubseteq u.$$

Hence, $\mathcal{U}_k(u) \geq \mathcal{U}^{\xi}_k(\odot_{i=1}^m u_{f_i,g_i}) \geq \mathcal{U}^{\xi}_k(v) \geq \odot_{i=1}^m \mathcal{U}^{\xi}_k(v_i)$. It is a contradiction.

Example 3.7. Let $X = \{h_i \mid i = \{1, 2, 3\}\}$ with h_i=house and $E = \{e, b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\}, \ x \to y = \min\{1 - x + y, 1\}$$

$$x \oplus y = \min\{1, x+y\},\ x^* = 1-x$$

Then $([0,1], \wedge, \rightarrow, 0, 1)$ is a complete residuated lattice (ref. [4,9]).

(1) Put $v, v \odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v \odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix},\ (v \odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : E \rightarrow [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_e(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v \odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_b(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

Since $v \circ v = v$, $w \circ w = w$ and $(v \odot v) \circ (v \odot v) = (v \odot v)$, $\mathcal{U}$ is a $[0,1]$-fuzzy soft (E,E)-quasi-uniformity on X.

From Theorem 3.1, we obtain a $[0,1]$-fuzzy soft (E,E)-topogenous order $\xi^{\mathcal{U}} : E \rightarrow [0,1]^{([0,1]^X)^E \times ([0,1]^X)^E}$ as follows

$$\xi^{\mathcal{U}}_e(f,g) = \begin{cases} 1, & \text{if } [1_{X\times X}](f) \sqsubseteq g \not\sqsupseteq v[f], \\ 0.6, & \text{if } v[f] \sqsubseteq g \not\sqsupseteq (v \odot v)[f], \\ 0.3, & \text{if } \ (v \odot v)[f] \sqsubseteq g \\ 0, & \text{otherwise,} \end{cases}$$

$$\xi^{\mathcal{U}}_b(f,g) = \begin{cases} 1, & \text{if } [1_{X\times X}](f) \sqsubseteq g \not\sqsupseteq w[f], \\ 0.5, & \text{if } w[f] \sqsubseteq g, \\ 0, & \text{otherwise,} \end{cases}$$

From Theorem 3.3, we obtain $\mathcal{U}^{\xi^{\mathcal{U}}} : E \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_e^{\xi^{\mathcal{U}}}(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X} \\ 0.6, & \text{if } u_{v[f],v[f]} \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } u_{(v\odot v)[f],(v\odot v)[f]} \sqsubseteq u \not\sqsupseteq u_{v[f],v[f]}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_b^{\xi^{\mathcal{U}}}(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X} \\ 0.5, & \text{if } u_{v[f],v[f]} \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

Since

$$\begin{aligned} &\bigvee_{x\in X}(v_e(x,y)\odot f_e(x)) \to \bigvee_{x\in X}(v_e(x,z)\odot f_e(x)) \\ &\geq \bigwedge_{x\in X}((v_e(x,y)\odot f_e(x)) \to (v_e(x,z)\odot f_e(x)) \\ &\geq \bigwedge_{x\in X}(v_e(x,y) \to v_e(x,z)) \geq v_e(y,z), \end{aligned}$$

$u_{v[f],v[f]} \sqsubseteq v$ and $u_{(v\odot v)[f],(v\odot v)[f]} \sqsubseteq v$. Hence $\mathcal{U}^{\xi^{\mathcal{U}}} \leq \mathcal{U}$.

(2) Put $l, m \in (L^X)^E$ such that

$$\begin{aligned} &l_e(h_1) = 0.3, l_e(h_2) = 0.5, l_e(h_3) = 0.6, \\ &l_b(h_1) = 0.6, l_b(h_2) = 0.3, l_b(h_3) = 0.6, \\ &m_e(h_1) = 0.4, m_e(h_2) = 0.3, m_e(h_3) = 0.5, \\ &m_b(h_1) = 0.2, m_b(h_2) = 0.6, m_b(h_3) = 0.6. \end{aligned}$$

We define a $[0,1]$-fuzzy soft (E,E)-topogenous order $\xi : E \to [0,1]^{([0,1]^X)^E \times ([0,1]^X)^E}$ as follows

$$\xi_e(f,g) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } g = 1_X, \\ 0.6, & \text{if } f \sqsubseteq l \sqsubseteq g, \\ 0.3, & \text{if } 0_X \neq f \sqsubseteq l\odot l \sqsubseteq g, l \not\sqsubseteq g \\ 0, & \text{otherwise,} \end{cases}$$

$$\xi_b(f,g) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } g = 1_X, \\ 0.5, & \text{if } 0_X \neq f \sqsubseteq l \sqsubseteq g \neq 1_X, \\ 0, & \text{otherwise,} \end{cases}$$

We obtain $u_{l,l}, u_{l\odot l,l\odot l}, u_{m,m} \in ([0,1]^{X\times X})^E$ as follows

$$u_{l,l}(e) = \begin{pmatrix} 1 & 1 & 1 \\ 0.8 & 1 & 1 \\ 0.7 & 0.9 & 1 \end{pmatrix}, u_{l,l}(b) = \begin{pmatrix} 1 & 0.7 & 1 \\ 1 & 1 & 1 \\ 1 & 0.7 & 1 \end{pmatrix}$$

$$u_{l\odot l,l\odot l}(e) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0.8 & 0.8 & 1 \end{pmatrix}, u_{l\odot l,l\odot l}(b) = \begin{pmatrix} 1 & 0.8 & 1 \\ 1 & 1 & 1 \\ 1 & 0.8 & 1 \end{pmatrix}$$

$$u_{m,m}(e) = \begin{pmatrix} 1 & 0.9 & 1 \\ 1 & 1 & 1 \\ 0.9 & 0.8 & 1 \end{pmatrix}, u_{m,m}(b) = \begin{pmatrix} 1 & 1 & 1 \\ 0.6 & 1 & 1 \\ 0.6 & 1 & 1 \end{pmatrix}$$

By Theorem 3.3, we obtain a $[0,1]$-fuzzy soft (E,E)- quasi- uniformity $\mathcal{U}^\xi : E \to [0,1]^{([0,1]^{X\times X})^E}$ as follows

$$\mathcal{U}^\xi_e(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X}, \\ 0.6, & \text{if } u_{l,l} \sqsubseteq u \neq 1_{X\times X}, \\ 0.3, & \text{if } u_{l\odot l,l\odot l} \leq u \not\sqsupseteq u_{l,l}, \\ 0.2, & \text{if } u_{l,l}\odot u_{l,l} \sqsubseteq u \not\sqsupseteq u_{l\odot l,l\odot l}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}^\xi_b(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X}, \\ 0.5, & \text{if } u_{m,m} \sqsubseteq u \neq 1_{X\times X}, \\ 0, & \text{otherwise.} \end{cases}$$

Since $u_{f,f}[f] = f$ and $u_{f\odot f,f\odot f}[f\odot f] = f\odot f$ from Lemma 3.2 (12), by Theorems 3.1 and 3.3(3), we have $\xi^{\mathcal{U}^\xi} = \xi$, that is, $\mathcal{U}_\xi \in \Pi(\xi)$.

(3) By Definition 2.9 and (1), we obtain a $[0,1]$-fuzzy soft (E,E)-cotopogenous order $\xi^s : E \to [0,1]^{([0,1]^X)^E\times([0,1]^X)^E}$ as follows

$$\xi^s_e(f,g) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } g = 1_X, \\ 0.6, & \text{if } f \sqsubseteq l^* \sqsubseteq g, \\ 0.3, & \text{if } 0_X \neq f \sqsubseteq l^*\oplus l^* \sqsubseteq g, l^* \not\sqsubseteq g \\ 0, & \text{otherwise,} \end{cases}$$

$$\xi^s_b(f,g) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } g = 1_X, \\ 0.5, & \text{if } 0_X \neq f \sqsubseteq l^* \sqsubseteq g \neq 1_X, \\ 0, & \text{otherwise,} \end{cases}$$

We obtain $u_{l^*,l^*}, u_{l^*\oplus l^*,l^*\oplus l^*}, u_{m^*,m^*} \in ([0,1]^{X\times X})^E$ as follows

$$u_{l^*,l^*}(e) = \begin{pmatrix} 1 & 0.8 & 0.7 \\ 1 & 1 & 0.9 \\ 1 & 1 & 1 \end{pmatrix}, u_{l^*,l^*}(b) = \begin{pmatrix} 1 & 1 & 1 \\ 0.7 & 1 & 0.7 \\ 1 & 1 & 1 \end{pmatrix}$$

$$u_{l^*\oplus l^*,l^*\oplus l^*}(e) = \begin{pmatrix} 1 & 1 & 0.8 \\ 1 & 1 & 0.8 \\ 1 & 1 & 1 \end{pmatrix},$$

$$u_{l^*\oplus l^*,l^*\oplus l^*}(b) = \begin{pmatrix} 1 & 1 & 1 \\ 0.8 & 1 & 0.8 \\ 1 & 1 & 1 \end{pmatrix}$$

$$u_{m^*,m^*}(e) = \begin{pmatrix} 1 & 1 & 0.9 \\ 0.9 & 1 & 0.8 \\ 1 & 1 & 1 \end{pmatrix}, u_{m^*,m^*}(b) = \begin{pmatrix} 1 & 0.6 & 0.6 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

By Corollary 3.4, we obtain a $[0,1]$-fuzzy soft (E,E)- quasi-uniformity $\mathcal{U}^{\xi^s}$: $E \to [0,1]^{([0,1]^{X\times X})^E}$ as follows

$$\mathcal{U}_e^{\xi^s}(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X}, \\ 0.6, & \text{if } u_{l^*,l^*} \sqsubseteq u \neq 1_{X\times X}, \\ 0.3, & \text{if } u_{l^*\oplus l^*,l^*\oplus l^*} \leq u \not\sqsupseteq u_{l^*,l^*}, \\ 0.2, & \text{if } u_{l^*,l^*} \odot u_{l^*,l^*} \sqsubseteq u \not\sqsupseteq u_{l^*\oplus l^*,l^*\oplus l^*}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_b^{\xi^s}(u) = \begin{cases} 1, & \text{if } u = 1_{X\times X}, \\ 0.5, & \text{if } u_{m^*,m^*} \sqsubseteq u \neq 1_{X\times X}, \\ 0, & \text{otherwise.} \end{cases}$$

Since $u_{l^*,l^*}[l^*] = l^*$ and $u_{l^*\oplus l^*,l^*\oplus l^*}[l^* \oplus l^*] = l^* \oplus l^*$ from Lemma 3.2 (12), by Corollary 3.4, we have $\xi^{\mathcal{U}^{\xi^s}} = \xi^s$, that is, $\mathcal{U}_{\xi^s} \in \Pi(\xi^s)$.

References

[1] H. Aktas, N. Cağman, Soft sets and soft groups, *Inf. Sci.*, **177**(13)(2007),2726-2735.

[2] A. Aygünoglu, H. Aygün, Introduction to fuzzy soft groups, *Computers and Mathematics with Appl.*, **58**(2009), 1279-1286.

[3] A. Aygünoglu, V. Cetkin, H. Aygün, An introduction to fuzzy soft topological spaces, *Hacettepe Journal of Math. and Stat.*, **43**(2)(2014), 193-204.

[4] R. Bělohlávek, *Fuzzy Relational Systems*, Kluwer Academic Publishers, New York , 2002.

[5] V. Cetkin , H. Aygün , On fuzzy soft topogenous structure, *J. Intell. Fuzzy Syst.* , **27**(2014), 247-255.

[6] F. Feng, C. Li, B. Davvaz, M. Arfan Ali, *Soft sets combined with fuzzy sets and rough sets: a tentative approach* , Soft Comput. **14**(9)(2010), 899-911.

[7] J.Gutiérrez García , M. A. de Prade Vicente, Šostak A. P., *A unified approach to the concept of fuzzy L-uniform spaces*, **Chapter 3** in [15], 81-114.

[8] D.N. Georgiou, A.C. Megaritis, V.I. Petropoulos, On soft topological spaces, *Appl. Math. Inf. Sci.*, **7**(5)(2013), 1889-1901.

[9] P. Hájek, *Metamathematices of Fuzzy Logic*, Kluwer Academic Publishers, Dordrecht (1998) **doi:** 10.1007/978-94-011-5300-3.

[10] U. Höhle, S.E. Rodabaugh, *Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory*, The Handbooks of Fuzzy Sets Series 3, Kluwer Academic Publishers, Boston, 1999 **doi:** 10.1007/978-1-4615-5079-2.

[11] B. Hutton, Uniformities in fuzzy topological spaces, *J. Math. Anal. Appl.*, **58** (1977), 74-79.

[12] Y.C. Kim, A.A. Ramadan, M. A. Usama, L-fuzzy Uniform Spaces, *The Journal of Fuzzy Mathematics*, **14** (2006), 821-850.

[13] W. Kotzé , *Uniform spaces*, in: Höhle U., Rodabaugh S. E.(Eds.), Mathematics of Fuzzy Sets: Logic, Topology and Measure Theory, Handbook Series, **Chapter 8**, Vol. 3, Kluwer Academic Publishers, Boston, Dordrecht, London, 1999, 553-580.

[14] R. Lowen, Fuzzy uniform spaces, *J. Math. Anal. Appl.*, **82** (1981), 370-385, **doi:** 10.1016/0022-247x(81)90202-x.

[15] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, *Journal of Fuzzy Mathematics*, **9**(3)(2001), 589-602.

[16] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, *Computers Mathematics with Appl.*, **45**(2003), 555-562.

[17] P. Majumdar, S.K. Samanta, Generalized fuzzy soft sets, *Computers Mathematics with Appl.*, 59(2010), 1425-1432.

[18] D. Molodtsov, Soft set theory, *Computers Mathematics with Appl.*, **37**(1999), 19-31.

[19] D. Pei, D. Miao, *From soft sets to information systems*, Granular Computing,2005 IEEE,International Conferences on (2)(2005),617-621.

[20] A.A. Ramadan,Y.C. Kim, M.K. El-Gayyar, On fuzzy uniform spaces, *The Journal of Fuzzy Mathematics*, **11** (2003), 279-299.

[21] A.A. Ramadan, E.H. Elkordy and Yong Chan Kim, Relationships between L-fuzzy quasi-uniform structures and L-fuzzy topologies, *Journal of Intelligent and Fuzzy Systems*, **28** (2015), 2319-2327.

[22] A.A. Ramadan, E.H. Elkordy, Y.C. Kim , Perfect L-fuzzy topogenous space, L-fuzzy quasi-proximities and L-fuzzy quasi-uniform spaces, *Journal of Intelligent and Fuzzy Systems*, **28**(6)(2015), 2591-2604.

[23] A.A. Ramadan, Yong Chan Kim , On L-fuzzy (K, E)- soft topogenous orders and L-fuzzy (K, E)- soft topologies, *Journal of Intelligent and Fuzzy Systems*,(2016) submitted.

[24] S.E. Rodabaugh, *Categorical foundations of variable-basis fuzzy topology*, in: Höhle U., Rodabaugh S. E.(Eds.), Mathematics of Fuzzy Sets: Logic, Topology and Measure Theory, Handbook Series , Chapter 4, Kluwer Academic Publishers, 1999.

[25] S.E. Rodabaugh, E.P. Klement, *Topological and Algebraic Structures In Fuzzy Sets*, The Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Kluwer Academic Publishers, Boston, Dordrecht, London, 2003.

[26] A.R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, *International of Computational and Applied Mathematics*, 203(2007),412-418.

[27] M. Shabir, M. Naz, On soft topological spaces, *Computers and Mathematics with Appl.*, **61**(2011), 1786-1799, **doi:** 10.1016/j.camwa.2011.02.006.

[28] S.A. Solovyov, Lattice-valued soft algebras, *Soft Comput.* **17(10)**(2013),1751-1766.

[29] B. Tanay, M.B. Kandemir, Topological structures of fuzzy soft sets, *Computers and Mathematics with Appl.*, **61**(2011),412-418, **doi:** 10.1016/j.camwa.2011.03.056.

[30] B.P. Varol, H. Aygün , Fuzzy soft topology, *Hacet J. Math. Stat.* **41**(3)(2012), 407-419.

[31] G. Xuechang, L. Yongming, F. Feng, A new order relation on fuzzy soft sets and its application, *Soft Comput.*, **17**(1)(2013),63-70.

[32] J. Zhan, Y.B. Jun, Soft BL-algebras based on fuzzy sets, *Comput Math. Appl.* **59**(6)(2010), 2037-2046.

[33] L. Zhaowen, X. Tusheng, The relationship among soft sets, soft rough sets and topologies, *Soft Comput.* , **18**(4)(2014),717-728.