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FIXED POINTS FOR COMPATIBLE MAPPINGS IN MULTIPLICATIVE METRIC SPACES

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Abstract: In this paper, we proved the common fixed point result for compatible mappings in multiplicative metric spaces.

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1. Introduction and Preliminaries

It is well know that the set of positive real numbers $\mathbb{R}_+$ is not complete according to the usual metric. To overcome this problem, in 2008, Bashirov et al. [1] introduced the concept of multiplicative metric spaces as follows:

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Definition 1.1. Let X be a nonempty set. A multiplicative metric is a mapping $d : X \times X \to \mathbb{R}_+$ satisfying the following conditions:

(i) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(iii) $d(x, y) \leq d(x, z) \cdot d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Then the mapping d together with X, that is, (X, d) is a multiplicative metric space.

Example 1.2. ([3]) Let $\mathbb{R}^n_+$ be the collection of all n-tuples of positive real numbers. Let $d^* : \mathbb{R}^n_+ \times \mathbb{R}^n_+ \to \mathbb{R}$ be defined as follows:

$$d^*(x, y) = \left|\frac{x_1}{y_1}\right|^* \cdot \left|\frac{x_2}{y_2}\right|^* \cdots \left|\frac{x_n}{y_n}\right|^*,$$

where $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in \mathbb{R}^n_+$ and $|\cdot|^* : \mathbb{R}_+ \to \mathbb{R}_+$ is defined by

$$|a|^* = \begin{cases} a & \text{if } a \geq 1, \\ \frac{1}{a} & \text{if } a < 1. \end{cases}$$

Then it is obvious that all conditions of a multiplicative metric are satisfied. Therefore $(\mathbb{R}^n_+, d^*)$ is a multiplicative metric space.

One can refer to [3] for detailed a multiplicative metric topology.

Definition 1.3. Let (X, d) be a multiplicative metric space. Then a sequence $\{x_n\}$ in X said to be

(1) a *multiplicative convergent* to x if for every multiplicative open ball $B_\epsilon(x) = \{y \mid d(x, y) < \epsilon\}$, $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $x_n \in B_\epsilon(x)$ for all $n \geq N$, that is, $d(x_n, x) \to 1$ as $n \to \infty$.

(2) a *multiplicative Cauchy sequence* if for all $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \epsilon$ for all $m, n \geq N$, that is, $d(x_n, x_m) \to 1$ as $n, m \to \infty$.

(3) We call a multiplicative metric space *complete* if every multiplicative Cauchy sequence in it is multiplicative convergent to $x \in X$.

In 2012, Özavsar and Çevikel [3] gave the concept of multiplicative contractive mappings and proved some fixed point theorem of such mappings in a multiplicative metric space.

Definition 1.4. Let f be a mapping of a multiplicative metric space (X, d) into itself. Then f is said to be a *multiplicative contraction* if there exists a real number $\lambda \in [0, 1)$ such that

$$d(fx, fy) \leq d^\lambda(x, y) \quad \text{for all } x, y \in X.$$

In 2015, Kang et al. [2] introduced the notion of compatible mappings as follows:

Definition 1.5. Let f and g be mappings of a multiplicative metric space (X, d) Then f and g are called *compatible* if

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 1,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$,

Now we give some properties related to compatible mappings and its variants in a multiplicative metric space, see [2].

Proposition 1.6. *Let f and g be compatible mappings of a multiplicative metric space (X, d) into itself. If $ft = gt$ for some $t \in X$, then $fgt = fft = ggt = gft$.*

Proposition 1.7. *Let f and g be compatible mappings of a multiplicative metric space (X, d) itself. Suppose that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$. Then*

(i) $\lim_{n\to\infty} gfx_n = ft$ *if f is continuous at t.*

(ii) $\lim_{n\to\infty} fgx_n = gt$ *if g is continuous at t.*

(iii) $fgt = gft$ *and* $ft = gt$ *if f and g are continuous at t.*

2. Main Results

Now we give the following theorem for compatible mappings.

Theorem 2.1. *Let A, B, S and T be mappings of a complete multiplicative metric space (X, d) satisfying the following conditions*

$$(C_1) \qquad SX \subset BX, \quad TX \subset AX;$$

$$(C_2) \qquad d(Sx, Ty) \leq M^{\lambda}(x, y)$$

for each $x, y \in X$ *and* $\lambda \in (0, 1/2)$, *where*

$$\begin{aligned} &M(x,y) \\ &= \max\Big\{d(Ax,Sx), d(By,Ty), d(By,Ax), (d(Ax,Ty)\cdot d(By,Sx))^{1/2}, \\ &\quad \min\Big\{\frac{d(Ax,Sx)\cdot d(By,Ty)}{d(Ax,By)}, \frac{d(Ax,Ty)\cdot d(By,Sx)}{d(Ax,By)}, \\ &\quad \frac{d(Ax,Ty)\cdot d(By,Sx)}{d(Sx,Ty)}\Big\}\Big\}; \end{aligned}$$

(C_3) *one of* A, B, S *and* T *is continuous.*

Assume that the pairs A, S *and* B, T *are compatible. Then* A, B, S *and* T *have a unique common fixed point.*

Proof. Let $x_0 \in X$ be an arbitrary point. Since $SX \subset BX$ and $TX \subset AX$, there exists $x_1 \in X$ such that $Sx_0 = Bx_1 = y_0$ and for this point x_1, there exists $x_2 \in X$ such that $Tx_1 = Ax_2 = y_1$. Continuing in this way, we can construct a sequence $\{y_n\}$ such that

$$y_{2n+1} = Tx_{2n+1} = Ax_{2n+2}; \quad y_{2n} = Sx_{2n} = Bx_{2n+1}.$$

From (C_2) by putting $x = x_{2n}$ and $y = x_{2n+1}$, we have

$$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1}) \leq M^{\lambda}(x_{2n}, x_{2n+1}),$$

where

$$\begin{aligned} M(x_{2n},x_{2n+1}) &= \max\Big\{d(Ax_{2n},Sx_{2n}), d(Bx_{2n+1},Tx_{2n+1}), d(Bx_{2n+1},Ax_{2n}), \\ &\quad (d(Ax_{2n},Tx_{2n+1})\cdot d(Bx_{2n+1},Sx_{2n}))^{1/2}, \\ &\quad \min\Big\{\frac{d(Ax_{2n},Sx_{2n})\cdot d(Bx_{2n+1},Tx_{2n+1})}{d(Ax_{2n},Bx_{2n+1})}, \\ &\quad \frac{d(Ax_{2n},Tx_{2n+1})\cdot d(Bx_{2n+1},Sx_{2n})}{d(Ax_{2n},Bx_{2n+1})}, \\ &\quad \frac{d(Ax_{2n},Tx_{2n+1})\cdot d(Bx_{2n+1},Sx_{2n})}{d(Sx_{2n},Tx_{2n+1})}\Big\}\Big\} \\ &= \max\Big\{d(y_{2n},y_{2n-1}), d(y_{2n+1},y_{2n}), d(y_{2n},y_{2n-1}), \\ &\quad (d(y_{2n},y_{2n})\cdot d(y_{2n+1},y_{2n-1}))^{1/2}, \min\Big\{\frac{d(y_{2n},y_{2n-1})\cdot d(y_{2n},y_{2n+1})}{d(y_{2n},y_{2n-1})}, \end{aligned}$$

$$\frac{d(y_{2n}, y_{2n}) \cdot d(y_{2n+1}, y_{2n-1})}{d(y_{2n}, y_{2n-1})}, \frac{d(y_{2n}y_{2n}) \cdot d(y_{2n+1}, y_{2n-1})}{d(y_{2n}, y_{2n+1})}\Big\}\Big\}$$
$$= \max\{d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n})\}.$$

Now if $d(y_{2n+1}, y_{2n}) > d(y_{2n-1}, y_{2n})$, then $M(x_{2n}, x_{2n+1}) = d(y_{2n+1}, y_{2n})$. Then we have,

$$d(y_{2n}, y_{2n+1}) \leq d^{\lambda}(y_{2n}, y_{2n+1}),$$

which is a contradiction as $\lambda \in (0, 1/2)$. So, $d(y_{2n}, y_{2n-1}) \geq d(y_{2n+1}, y_{2n})$, which implies that $M(x_{2n}, x_{2n+1}) = d(y_{2n-1}, y_{2n})$ and hence

$$d(y_{2n}, y_{2n+1}) \leq d^{\lambda}(y_{2n-1}, y_{2n}).$$

Similarly, we have

$$d(y_{2n-1}, y_{2n}) \leq d^{\lambda}(y_{2n-2}, y_{2n-1}).$$

Hence, in general we get

$$\begin{aligned} d(y_n, y_{n+1}) &\leq d^{\lambda}(y_{n-1}, y_n) \\ &\leq d^{\lambda^2}(y_{n-2}, y_{n-1}) \\ &\leq \cdots \leq d^{\lambda^n}(y_0, y_1). \end{aligned}$$

Let $m, n \in \mathbb{N}$ with $m > n$. Then

$$\begin{aligned} d(y_m, y_n) &\leq d(y_m, y_{m-1}) \cdot d(y_{m-1}, y_{m-2}) \cdots d(y_{n+1}, y_n) \\ &\leq d^{\lambda^{m-1}}(y_0, y_1)) \cdot d^{\lambda^{m-2}}(y_0, y_1) \cdots d^{\lambda^n}(y_0, y_1) \\ &\leq d^{\frac{\lambda^n}{1-\lambda}}(y_0, y_1). \end{aligned}$$

Taking $n \to \infty$, we have $d(y_m, y_n) \leq 1$ and hence $\lim_{n\to\infty} d(y_m, y_n) = 1$. Thus $\{y_n\}$ is a multiplicative Cauchy sequence in X and hence it converges to some point $z \in X$. Consequently, the subsequence $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Tx_{2n+1}\}$ and $\{Sx_{2n}\}$ of the sequence $\{y_n\}$ also converges to z.

Now suppose that A is continuous. Since A and S are compatible on X, it follows from Proposition 1.7 that

$$AAx_{2n} \to Az, \quad SAx_{2n} \to Az \quad \text{as } n \to \infty.$$

Now putting $x = Ax_{2n}$ and $y = x_{2n+1}$ in (C_2), we have

$$d(SAx_{2n}, Tx_{2n+1}) \leq M^{\lambda}(Ax_{2n}, x_{2n+1}),$$

where

$$\begin{aligned}&M(Ax_{2n}, x_{2n+1})\\&= \max\Big\{d(AAx_{2n}, SAx_{2n}), d(Bx_{2n+1}, Tx_{2n+1}), d(Bx_{2n+1}, AAx_{2n}),\\&\quad (d(AAx_{2n}, Tx_{2n+1}) \cdot d(Bx_{2n+1}, SAx_{2n}))^{1/2},\\&\quad \min\Big\{\frac{d(AAx_{2n}, SAx_{2n}) \cdot d(Bx_{2n+1}, Tx_{2n+1})}{d(AAx_{2n}, Bx_{2n+1})},\\&\quad \frac{d(AAx_{2n}, Tx_{2n+1}) \cdot d(Bx_{2n+1}, SAx_{2n})}{d(AAx_{2n}, Bx_{2n+1})},\\&\quad \frac{d(AAx_{2n}, Tx_{2n+1}) \cdot d(Bx_{2n+1}, SAx_{2n})}{d(SAx_{2n}, Tx_{2n+1})}\Big\}\Big\}.\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}&\lim_{n\to\infty} M(Ax_{2n}, x_{2n+1})\\&= \max\Big\{d(Az, Az), d(z, z), d(z, Az), (d(Az, z) \cdot d(z, Az))^{1/2},\\&\quad \min\Big\{\frac{d(Az, Az) \cdot d(z, z)}{d(Az, z)}, d(z, Az), d(Az, z)\Big\}\Big\}\\&= d(z, Az).\end{aligned}$$

Hence

$$d(z, Az) \leq d^{\lambda}(z, Az),$$

which a contradiction, we get $z = Az$.

Next putting $x = z$ and $y = x_{2n+1}$ in (C_2), we have

$$d(Sz, Tx_{2n+1}) \leq M^{\lambda}(z, x_{2n+1}),$$

where

$$\begin{aligned}&M(z, x_{2n+1})\\&= \max\Big\{d(Az, Sz), d(Bx_{2n+1}, Tx_{2n+1}), d(Bx_{2n+1}, Az),\\&\quad (d(Az, Tx_{2n+1}) \cdot d(Bx_{2n+1}, Sz))^{1/2},\\&\quad \min\Big\{\frac{d(Az, Sz) \cdot d(Bx_{2n+1}, Tx_{2n+1})}{d(Az, Bx_{2n+1})},\\&\quad \frac{d(Az, Tx_{2n+1}) \cdot d(Bx_{2n+1}, Sz)}{d(Az, Bx_{2n+1})}, \frac{d(Az, Tx_{2n+1}) \cdot d(Bx_{2n+1}, Sz)}{d(Sz, Tx_{2n+1})}\Big\}\Big\}.\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}
&\lim_{n\to\infty} M(z, x_{2n+1}) \\
&= \max\{d(z, Sz), d(z, z), d(z, z), (d(z, z) \cdot d(z, Sz))^{1/2}, \\
&\quad \min\{d(z, Sz), d(z, Sz), d(z, z)\}\} \\
&= d(Sz, z).
\end{aligned}$$

Then we have

$$d(Sz, z) \leq d^{\lambda}(Sz, z),$$

which a contradiction, we get $Sz = z$. Since $SX \subset BX$, there exists a point $u \in X$ such that $z = Sz = Bu$.

Again putting $x = z$ and $y = u$ in (C_2), we have

$$d(z, Tu) = d(Sz, Tu) \leq M^{\lambda}(z, u),$$

where

$$\begin{aligned}
M(z, u) = \max\Bigg\{& d(Az, Sz), d(Bu, Tu), d(Bu, Az), \\
& (d(Az, Tu) \cdot d(Bu, Sz))^{1/2}, \min\left\{\frac{d(Az, Sz) \cdot d(Bu, Tu)}{d(Az, Bu)}, \right. \\
& \left. \frac{d(Az, Tu), d(Bu, Sz)}{d(Az, Bu)}, \frac{d(Az, Tu) \cdot d(Bu, Sz)}{d(Sz, Tu)}\right\}\Bigg\} \\
= & \max\{1, d(z, Tu), 1, d^{1/2}(z, Tu), 1\} \\
= & d(z, Tu).
\end{aligned}$$

Then

$$d(z, Tu) \leq d^{\lambda}(z, Tu),$$

which a contradiction. This implies that $z = Tu$. Since B and T are compatible on X and $Bu = Tu = z$, by Proposition 1.6, $BTu = TBu$ and hence $Bz = BTu = TBu = Tz$.

Also, we have

$$d(z, Bz) = d(Sz, Tz) \leq M^{\lambda}(z, z),$$

where

$$\begin{aligned}
&M(z,z)\\
&= \max\Big\{d(Az,Sz), d(Bz,Tz), d(Bz,Az),\\
&\qquad (d(Az,Tz)\cdot d(Bz,Sz))^{1/2}, \min\Big\{\frac{d(Az,Sz)\cdot d(Bz,Tz)}{d(Az,Bz)},\\
&\qquad \frac{d(Az,Tz), d(Bz,Sz)}{d(Az,Bz)}, \frac{d(Az,Tz)\cdot d(Bz,Sz)}{d(Sz,Tz)}\Big\}\Big\}\\
&= \max\Big\{1, 1, d(Bz,z), d(Bz,z), \min\Big\{\frac{1}{d(z,Bz)}, d(Bz,z), d(Bz,z)\Big\}\Big\}\\
&= d(Bz,z).
\end{aligned}$$

This implies that $z = Bz$. Hence, $z = Bz = Tz = Az = Sz$. Therefore, z is a common fixed point of A, B, S and T.

Similarly, we can also complete the proof when B is continuous.

Next suppose that S is continuous. Since A and S are compatible on X, it follows that

$$S^2x_{2n} \to Sz, \quad ASx_{2n} \to Sz \quad \text{as } n \to \infty.$$

Now putting $x = Sx_{2n}$ and $y = x_{2n+1}$ in (C_2)

$$d(SSx_{2n}, Tx_{2n+1}) \leq M^{\lambda}(Sx_{2n}, x_{2n+1}),$$

where

$$\begin{aligned}
&M(Sx_{2n}, x_{2n+1})\\
&= \max\Big\{d(ASx_{2n}, SSx_{2n}), d(Bx_{2n+1}, Tx_{2n+1}), d(Bx_{2n+1}, ASx_{2n}),\\
&\qquad (d(ASx_{2n}, Tx_{2n+1})\cdot d(Bx_{2n+1}, SSx_{2n}))^{1/2},\\
&\qquad \min\Big\{\frac{d(ASx_{2n}, SSx_{2n})\cdot d(Bx_{2n+1}, Tx_{2n+1})}{d(ASx_{2n}, Bx_{2n+1})},\\
&\qquad \frac{d(ASx_{2n}, Tx_{2n+1})\cdot d(Bx_{2n+1}, SSx_{2n})}{d(ASx_{2n}, Bx_{2n+1})},\\
&\qquad \frac{d(ASx_{2n}, Tx_{2n+1})\cdot d(Bx_{2n+1}, SSx_{2n})}{d(SSx_{2n}, Tx_{2n+1})}\Big\}\Big\}.
\end{aligned}$$

Letting $n \to \infty$, we get

$$\lim_{n\to\infty} M(Sx_{2n}, x_{2n+1}) = \max\left\{d(Sz,Sz), d(z,z), d(z,Sz), d(z,Sz), \min\left\{\frac{1}{d(z,Sz)}, d(z,Sz), d(z,Sz)\right\}\right\} = d(z,Sz).$$

This implies that $Sz = z$. Since $SX \subset BX$, there exists a point $v \in X$ such that $z = Sz = Bv$.

Now putting $x = Sx_{2n}$ and $y = v$ in (C_2)

$$d(SSx_{2n}, Tv) \leq M^{\lambda}(Sx_{2n}, v),$$

where

$$M(Sx_{2n}, v) = \max\left\{d(ASx_{2n}, SSx_{2n}), d(Bv,Tv), d(Bv, ASx_{2n}), (d(ASx_{2n},Tv).d(Bv,SSx_{2n}))^{1/2}, \min\left\{\frac{d(ASx_{2n},SSx_{2n})\cdot d(Bv,Tv)}{d(ASx_{2n},Bv)}, \frac{d(ASx_{2n},Tv)\cdot d(Bv,SSx_{2n})}{d(ASx_{2n},Bv)}, \frac{d(ASx_{2n},Tv)\cdot d(Bv,SSx_{2n})}{d(SSx_{2n},Tv)}\right\}\right\}.$$

Letting $n \to \infty$, we get

$$\lim_{n\to\infty} M(Sx_{2n}, v) = \max\{1, d(z,Tv), 1, d^{1/2}(z,Tv), \min\{d(z,Tv), d(z,Tv), 1\}\} = d(z,Tv).$$

This implies that $u = Tv$. Since B and T are compatible on X and $Bv = Tv = z$, $BTv = TBv$ and hence $Bz = BTv = TBv = Tz$.

Now putting $x = x_{2n}$ and $y = z$ in (C_2)

$$d(Sx_{2n}, Tz) \leq M^{\lambda}(x_{2n}, z),$$

where

$$\begin{aligned}
&M(x_{2n}, z)\\
&= \max\Big\{d(Ax_{2n}, Sx_{2n}), d(Bz, Tz), d(Bu, Ax_{2n}),\\
&\qquad (d(Ax_{2n}, Tz)\cdot d(Bz, Sx_{2n}))^{1/2}, \min\Big\{\frac{d(Ax_{2n}, Sx_{2n})\cdot d(Bz, Tz)}{d(Ax_{2n}, Bz)},\\
&\qquad \frac{d(Ax_{2n}, Tz)\cdot d(Bz, Sx_{2n})}{d(Ax_{2n}, Bz)}, \frac{d(Ax_{2n}, Tz)\cdot d(Bz, Sx_{2n})}{d(Sx_{2n}, Tz)}\Big\}\Big\}.
\end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned}
&\lim_{n\to\infty} M(x_{2n}, z)\\
&= \max\Big\{1, 1, d(Tz, z), d(Tz, z), \min\Big\{\frac{1}{d(z, Tz)}, d(Tz, z), d(Tz, z)\Big\}\Big\}\\
&= d(Tz, z).
\end{aligned}$$

This implies that $Tz = z$. Since $TX \subset AX$, there exists a point $w \in X$ such that $z = Tz = Aw$.

Now putting $x = w$ and $y = z$ in (C_2)

$$d(Sw, Tz) \le M^{\lambda}(w, z),$$

where

$$\begin{aligned}
&M(w, z)\\
&= \max\Big\{d(Aw, Sw), d(Bz, Tz), d(Bz, Aw),\\
&\qquad (d(Aw, Tz)\cdot d(Bz, Sw))^{1/2}, \min\Big\{\frac{d(Aw, Sw)\cdot d(Bz, Tz)}{d(Aw, Bz)},\\
&\qquad \frac{d(Aw, Tz)\cdot d(Bz, Sw)}{d(Aw, Bz)}, \frac{d(Aw, Tz)\cdot d(Bz, Sw)}{d(Sw, Tz)}\Big\}\Big\}\\
&= \max\{d(z, Sw), 1, 1, d^{1/2}(z, Sw), \min\{d(z, Sw), d(z, Sw), 1\}\}\\
&= d(z, Sw).
\end{aligned}$$

This implies that $Sw = z$. Since A and S are compatible on X and $Sw = Aw = z$, $ASw = SAw$ and hence $Az = ASw = SAw = Sz$. That is, $z = Az = Sz = Bz = Tz$. Therefore, z is a common fixed point of A, B, S and T.

Similarly, we can complete the proof when T is continuous.

Finally, suppose that z and w are two common fixed points of A, B, S and T.

Now putting $x = z$ and $y = w$ in (C_2)

$$d(z,w) = d(Sz,Tw) \leq M^{\lambda}(z,w),$$

where

$$\begin{aligned}
&M(z,w)\\
&= \max\Bigg\{d(Az,Sz), d(Bw,Tw), d(Bw,Az),\\
&\qquad (d(Az,Tw)\cdot d(Bw,Sz))^{1/2}, \min\left\{\frac{d(Az,Sz)\cdot d(Bw,Tw)}{d(Az,Bw)},\right.\\
&\qquad\quad \left.\frac{d(Az,Tw)\cdot d(Bw,Sz)}{d(Az,Bw)}, \frac{d(Az,Tw)\cdot d(Bw,Sz)}{d(Sz,Tw)}\right\}\Bigg\}\\
&= \max\left\{1, 1, d(z,w), d(z,w), \min\left\{\frac{1}{d(z,w)}, d(z,w), d(z,w)\right\}\right\}\\
&= d(z,w),
\end{aligned}$$

which implies that $z = w$. Therefore, A, B, S and T have a unique common fixed point in X. This completes the proof. $\square$

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