STAGNATION POINT FLOW OVER A STRETCHING SHEET WITH NEWTONIAN HEATING USING LAPLACE ADOMIAN DECOMPOSITION METHOD

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Abstract: Our aim in this piece of work is to demonstrate the power of Laplace Adomian decomposition method in approximating the solution of nonlinear differential equations governing stagnation point flow over a stretching sheet with Newtonian heating.

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Key Words: stagnation point, stretching sheet, Newtonian heating, Laplace decomposition method, Adomian decomposition

1. Introduction

Fluid motion in the region of stagnation point exists on all moving bodies. The role of stagnation point is important, because the separation streamlines passing through them describe different flow regions. Also it is known that heat transfer is concerned with the exchange of thermal energy from one physical system to another. Merkin [3] pointed out four common heating processes

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specifying wall to ambient temperature distribution. These are (1) constant or prescribed wall temperature (CWT) (2) constant or prescribed heat flux (CHF) (3) conjugate conditions where heat is supplied through a bounding surface with a finite heat capacity (4) Newtonian heating (NT) where the heat transfer rate from the bounding surface with a finite capacity is proportional to the local surface temperature which is usually termed conjugate convective flow. Newtonian heating conditions (4) have been used by researchers in view of their practical applications in several engineering devices, for instance in a heat exchanger where the conduction in a solid tube wall is greatly influenced by the convection in the fluid flowing over it.

N.M. Sarif et al [6] obtained a numerical solution of flow and heat transfer over a stretching sheet with Newtonian heating using Keller Box method. Atia [1] obtained the effect of the porosity of the medium, the surface stretching velocity and heat generation/absorption coefficient on both the flow and heat transfer using numerical technique of finite difference approximations. Muhammad Khairul et al [4] have discussed the numerical solution of stagnation point flow over a stretching sheet with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature is considered. Muhammad Qasim et al [5] has solved heat transfer in a micropolar fluid over a stretching sheet with Newtonian heating by Runge Kutta Fehlberg method. M.Z. Salleh et al [7] have studied the problem of boundary layer flow and heat transfer over a stretching sheet with Newtonian heating in which the heat transfer is assumed to be proportional to the local surface temperature. They have solved the transformed boundary layer equation using finite difference method. T.R. Sivakumar and S. Baiju [8] have introduced a novel technique of handling the boundary conditions at infinity, which is a combination of shooting type Laplace Adomian Decomposition Algorithm and Pade Approximation. Newtonian heating in stagnation point flow of Burgers fluid has been studied by T.hayat et al [2].

It is usually noted that perturbation approximations are valid only for nonlinear problems with weak nonlinearity, but when nonlinearity is strong, then perturbation approximations of the nonlinear problems often break down. In such cases, Laplace Decomposition Method provides a simple way to ensure the convergence of the series solution so that one can obtain accurate enough approximations even in the nonlinear problems. In the present work, the stagnation point flow over a stretching sheet with Newtonian heating has been solved using Laplace Adomian Decomposition method.

2. Formulation

Consider a steady two dimensional stagnation point flow over a stretching plate immersed in an incompressible viscous fluid of ambient temperature T_∞ . It is assumed that the external velocity u_e and stretching velocity $u_w(x)$ are of the form $u_e(x) = ax$ and $u_w(x) = bx$ where a and b are constants. It is further assumed that the plate is subjected to Newtonian heating proposed by Muhammad Khairul et.al. The boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = u_e \frac{du_e}{dx} + v \frac{\partial^2 u}{\partial x^2}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

Subject to the boundary conditions

$$u = u_w(x), \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_s T \text{ at } y = 0, \quad (4)$$

$$u = u_e(x), \quad T \rightarrow T_\infty, \quad y \rightarrow \infty, \quad (5)$$

where h_s is the heat transfer coefficient and α is the thermal diffusivity.

Introducing similarity variables

$$\eta = \sqrt{\frac{a}{\vartheta}} y, \quad \psi = \sqrt{a\vartheta} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}, \quad (6)$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (7)$$

We obtain

$$u = ax f'(\eta) \text{ and } v = -\sqrt{a\vartheta} f(\eta). \quad (8)$$

Substituting in equations (2) and (3), we get

$$f'' + ff' - f^2 + 1 = 0, \quad (9)$$

$$\frac{1}{Pr} \theta'' + f\theta' = 0. \quad (10)$$

The corresponding boundary conditions are

$$f(0) = 0, \quad f'(0) = \epsilon, \quad \theta'(0) = -\gamma(1 + \theta(0)), \quad (11)$$

$$f(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad (12)$$

where $\epsilon = \frac{b}{a}$ is the stretching parameter. Further $\gamma = h_s \sqrt{\frac{\vartheta}{a}}$ is the conjugate parameter for Newtonian heating. It is noticed that $\gamma = 0$ for insulated plate and $\gamma \rightarrow \infty$ when the surface temperature remains constant.

2.1. Solution

Equations (9) and (10) are solved by using Modified Laplace Adomian Decomposition method.

Applying Laplace transform on equation (9)

$$L(f) = -\frac{1}{s} + L(f^2 - ff).$$

Applying the boundary conditions (10) and taking inverse Laplace transform

$$f(\eta) = \epsilon\eta + \frac{\alpha\eta^2}{2} - \frac{\eta^3}{6} + L^{-1} \left(\frac{1}{s^3} L(f^2 - ff) \right).$$

Assume that $f_0(\eta) = \epsilon\eta + \frac{\alpha\eta^2}{2}$ and $f_1(\eta) = -\frac{\eta^3}{6}$.

The general term is given by

$$f_{n+1}(\eta) = L^{-1} \left(\frac{1}{s^3} L(A_n - B_n) \right),$$

where A_n and B_n are the Adomian polynomials given by

$$\begin{aligned} A_0 &= f_0'^2, & B_0 &= f_0 f_0'', \\ A_1 &= 2f_0' f_1', & B_1 &= f_0 f_1'' + f_1 f_0'', \\ A_2 &= 2f_0' f_2' + f_0'^2, & B_2 &= f_0 f_2'' + f_1 f_1'' + f_2 f_0'' \cdots, \end{aligned}$$

etc., and

$$f(\eta) = f_1(\eta) + f_2(\eta) + f_3(\eta) + \cdots,$$

$$\begin{aligned} f(\eta) &= \epsilon\eta + \frac{\alpha\eta^2}{2!} + (\epsilon^2 - 1)\frac{\eta^3}{3!} + \alpha\epsilon\frac{\eta^4}{4!} + \alpha^2\frac{\eta^5}{5!} \\ &\quad + (\epsilon^2\alpha - 2\alpha)(\epsilon^2 - 1)^2\frac{\eta^6}{6!} + (2(\epsilon^2 - 1)^2 - \alpha^2\epsilon)\frac{\eta^7}{7!} \\ &\quad + (2\epsilon^3\alpha + \epsilon\alpha - \alpha^3)\frac{\eta^8}{8!} + (9\alpha^2\epsilon^2 + 4\alpha^2 - 8\epsilon(\epsilon^2 - 1)^2)\frac{\eta^9}{9!} + \cdots. \end{aligned} \quad (13)$$

3. Determination of the Free Parameter α

It is noted that the infinity condition for $f(\eta)$ cannot be applied directly as the series is divergent at ∞ . To overcome this difficulty and improve the convergence of $f(\eta)$, the $[M, M]$ Pade Approximant is used which is most suitable for expressing series solution as rational functions. We start by differentiating f , then determine the $[M, M]$ Pade approximants of the resulting series and finally apply the boundary condition $f(\infty) = 1$ by equating the coefficient of the highest power of η in the numerator to 1. Solving the resulting polynomial for α gives the average value of the free parameter $\alpha = f'(0)$. Using various values of ϵ , the value of $\alpha = f'(0)$ for first and higher order Pade Approximations are obtained and are given in the following table.

Pade Approximation order	ϵ	α
[1, 1]	0	0.7071
	1	0
	1/2	0.4330
	2	-1.2247
Pade Approximation order	ϵ	α
[2, 2]	0	1.6296
	1	0
	1/2	0.8169
	2	-2.0958
Pade Approximation order	ϵ	α
[3, 3]	0	complex
	1	0
	1/2	complex
	2	-1.6919
Pade Approximation order	ϵ	α
[4, 4]	0	1.1962
	1	0
	1/2	0.7706
	2	-2.1721

The value obtained by Wang for α for $\epsilon = 0$ is 1.232588. The value obtained for the present work for a [4,4] Pade Approximation is $\alpha = 1.1962$. Sivakumar [8] had proved in his paper that the convergence to find the value of α is faster when final value theorem on Laplace transform is used than by direct application using Pade Approximation. So Following [8], Using $[n,n]$ Pade

Approximation for the Final value theorem on Laplace transform, $\alpha = f'(0)$ is calculated.

Differentiating equation (13)

$$\begin{aligned} f'(\eta) = & \epsilon + \alpha\eta + (\epsilon^2 - 1)\frac{\eta^2}{2!} + \alpha\epsilon\frac{\eta^3}{3!} + \alpha^2\frac{\eta^3}{3!} \\ & + (\epsilon^2\alpha - 2\alpha)(\epsilon^2 - 1)^2\frac{\eta^5}{5!} + (2(\epsilon^2 - 1)^2 - \alpha^2\epsilon)\frac{\eta^6}{6!} \\ & + (2\epsilon^3\alpha + \epsilon\alpha - \alpha^3)\frac{\eta^8}{8!} + (9\alpha^2\epsilon^2 + 4\alpha^2 - 8\epsilon(\epsilon^2 - 1)^2)\frac{\eta^9}{9!} + \dots \end{aligned}$$

Taking Laplace transform on the above function we get

$$\begin{aligned} L(f'(\eta)) = & \frac{\epsilon}{s} + \alpha\left(\frac{1}{s^2}\right) + (\epsilon^2 - 1)\left(\frac{1}{s^3}\right) + \alpha\epsilon\left(\frac{1}{s^4}\right) + \alpha^2\left(\frac{1}{s^5}\right) \\ & + (\epsilon^2\alpha - 2\alpha)(\epsilon^2 - 1)^2\left(\frac{1}{s^6}\right) + (2(\epsilon^2 - 1)^2 - \alpha^2\epsilon)\left(\frac{1}{s^7}\right) \\ & + (2\epsilon^3\alpha + \epsilon\alpha - \alpha^3)\left(\frac{1}{s^8}\right) + (9\alpha^2\epsilon^2 + 4\alpha^2 - 8\epsilon(\epsilon^2 - 1)^2)\left(\frac{1}{s^9}\right) + \dots \end{aligned}$$

Taking Pade approximation for $sL(f'(\eta))$ and assuming second order Pade approximations

$$sL(f'(\eta)) = \frac{p_0 + \frac{p_1}{s} + \frac{p_2}{s^2}}{1 + \frac{p_3}{s} + \frac{p_2}{s^2}}.$$

the values of α for different orders are tabulated below when $\epsilon = 0$.

Pade Approximation order	Value of α
[1, 1]	1
[2, 2]	1.272006289
[3, 3]	1.1892
[4, 4]	1.23285264

Following [9] and taking Laplace transform on equation (10), we get

$$\frac{1}{Pr}L(\theta) + L(f\theta) = 0.$$

Taking the corresponding inverse Laplace transform, and applying suitable boundary conditions, we get

$$\theta(\eta) = \beta_1 + \beta_2\eta - PrL^{-1}\left\{\frac{1}{s^2}L(f\theta)\right\},$$

where $\beta_1 = \theta(0)$ and $\beta_2 = \theta'(0)$.

Assuming $\theta_0(\eta) = \beta_1 + \beta_2\eta$, and using $f_0(\eta)$, we receive

$$\theta_1(\eta) = -L^{-1} \left\{ \frac{1}{s^2} L(A_0) \right\},$$

where $A_0 = f_0\theta_0$;

$$\theta_1(\eta) = -Pr\beta_2 \left(\frac{\epsilon\eta^3}{3!} + \frac{\alpha\eta^4}{4!} \right).$$

Similarly,

$$\begin{aligned} \theta_2(\eta) = & -Pr\beta_2 \left\{ [(\epsilon^2 - 1) - 3Pr\epsilon^2] \frac{\eta^5}{5!} + \alpha\epsilon(1 - 10Pr) \frac{\eta^6}{6!} \right. \\ & \left. + (\alpha^2 - 10Pr\alpha^2) \frac{\eta^7}{7!} \right\}, \\ \theta_3(\eta) = & Pr\beta_2 \left\{ [-15Pr\epsilon(\epsilon^2 - 1 + 15Pr^2\epsilon^2)] \frac{\eta^7}{7!} \right. \\ & + [-35\alpha Pr(\epsilon^2 - 1) + 45\alpha Pr^2\epsilon^2 - 15\alpha Pr\epsilon^2 \\ & - 6\alpha Pr\epsilon^2(1 - 10Pr) + \alpha\epsilon^2 - 2\alpha] \frac{\eta^8}{8!} \\ & + [-28\epsilon Pr\alpha^2(1 - 10Pr) - 56\epsilon Pr\alpha^2 - \epsilon\alpha^2] \frac{\eta^9}{9!} \\ & \left. + [-28\epsilon Pr\alpha^3(1 - 10Pr) - 56Pr\alpha^3 - Pr] \frac{\eta^{10}}{10!} \right\}. \end{aligned}$$

So,

$$\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \theta_3(\eta) + \dots$$

To overcome the difficulty of the convergence at infinity, we use Pade Approximation and find that $\beta_1 = -1$ and $\beta_2 = 0$.

4. Conclusion

The stagnation point flow over a stretching sheet with Newtonian heating has been solved theoretically using Laplace decomposition method. Increase in the value of stretching parameter, decreases the value of α . The numerical result obtained for $\alpha = 1.2328$ agreed very well with the previously published results obtained by Muhammad Khairul et.al. Also technique of handling the boundary conditions at infinity introduced by T.R.Sivakumar and S.Baiju has been used to obtain the value of $\alpha = 1.2328$. Computations are performed using MATLAB.

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