NEW TYPE OF STRONGER FORM OF IG CLOSED SETS

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Abstract: An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. In this paper, we have introduced stronger forms of g -closed sets via ideal topological space. Also we have studied the properties of the stronger forms of g -closed sets with respect to an ideal.

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1. Introduction

Nowadays ideals are playing very important role in General Topology. It was the works of Newcomb[10], Rancin[13], Samuels[14] and Hamlet and Jankovic ([2,3,4,5,6]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology. A nonempty collection I of subsets on a topological space (X, τ) is called a topological ideal if it satisfies the following two conditions:

- (i) If $A \in I$ and $B \subset A$ implies $B \in I$.(heredity).
- (ii) If $A \in I$ and $B \in I$ then $A \cup B \in I$.(finite additivity)

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Throughout this paper (X, τ) will denote topological space. For a subset A of a topological space (X, τ) . The closure of A (denoted as $\text{cl}(A)$) is defined as the intersection of all closed sets containing A and the interior of A (denoted as $\text{int}(A)$) is defined as the union of all open sets contained in A . Let $A \subset B \subset X$. Jafari [7] introduced the concept Generalized closed set with respect to an Ideal. In this paper, we introduce and study the concept of g^* -closed sets with respect to an ideal, which is the extension of the concept of Ig-closed sets.

2. Preliminaries

Definition 2.1. A subset of a topological space (X, τ) is called a generalized closed set (briefly g -closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.2. Let (X, τ) be a topological space and A is a subset of X , the generalized closure operator (briefly cl^*)[1] is defined by the intersection of all g -closed sets containing A . The interior operator (briefly int^*) is defined by union of all g -open sets contained in A .

Definition 2.3. A subset of a topological space (X, τ) is called a strongly generalized closed set (briefly g^* -closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ where U is g -open in (X, τ) .

Definition 2.4. Let (X, τ) be a topological space and I be an ideal on X . A subset A subset of X is said to be generalized closed with respect to an ideal (briefly Ig-closed) [7] if and only if $\text{cl}(A) - B \in I$ whenever $A \subset B$ and B is g -open.

3. Strong form of Ig^* closed sets

Definition 3.1. Let (X, τ) be a topological space and I be an ideal on X . A subset A subset of X is said to be strongly g -closed with respect to an ideal (briefly Ig^* -closed) if and only if $cl^*(A) - B \in I$ whenever $A \subset B$ and B is g -open.

Remark 3.2. Every g -closed set is Ig^* -closed, but the converse need not be true.

Example 3.3. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a\}, X\}$ and $I = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$ clearly the set $\{a\}$ is Ig^* -closed but not g -closed in (X, τ) .

Theorem 3.4. *A set A is Ig^* -closed in (X, τ) if and only if $F \subset cl^*(A) - A$ and F is g -closed in X implies $F \in I$.*

Proof. Assume that A is Ig^* -closed. Let $F \subset cl^*(A) - A$. Suppose F is g -closed. Then, $A \subset X - F$. By assumption, $cl^*(A) - (X - F) \in I$. Conversely, assume that $F \subset cl^*(A) - A$ and F is g -closed in X implies that $F \in I$. Suppose $A \subset U$ and U is g -open. Then $cl^*(A) - U = cl^*(A) \cap (X - U)$ is a g -closed set in X , that is contained in $cl^*(A) - A$. By assumption, $cl^*(A) - U \in I$. This implies that A is Ig^* -closed. $\square$

Theorem 3.5. *If A and B are Ig^* -closed sets of (X, τ) then their union $A \cup B$ is also Ig^* -closed.*

Remark 3.6. The intersection of Ig^* -closed sets need not be an Ig^* -closed.

Example 3.7. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{b\}, \{b, c\}, X\}$. If $A = \{b, c\}$, $B = \{a, b\}$ and $I = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ then A and B are Ig^* -closed but their intersection $A \cap B = \{b\}$ is not Ig^* -closed.

Theorem 3.8. *If A is Ig^* -closed and $A \subset B \subset cl^*(A)$ and in (X, τ) , then B is Ig^* -closed in (X, τ) .*

Proof. Suppose A is Ig^* -closed and $A \subset B \subset cl^*(A)$ in (X, τ) . Suppose $B \subset U$ and U is g -open. Then $A \subset U$. Since A is Ig^* -closed, we have $cl^*(A) - U \in I$. $B \subset cl^*(A)$. This implies that $cl^*(B) - U \subset cl^*(A) - U \in I$. Hence B is Ig^* -closed in (X, τ) . $\square$

Theorem 3.9. *Let $A \subset Y \subset X$ and suppose that A is Ig^* -closed in (X, τ) . A is Ig^* -closed relative to the subspace Y of X , with respect to the ideal $I_Y = \{F \subset Y : F \in I\}$.*

Proof. Suppose $A \subset U \cap Y$ and U is g -open in (X, τ) then $A \subset U$. Since A is Ig^* -closed in (X, τ) we have $cl^*(A) - U \in I$. Now $(cl^*(A) \cap U) - (U \cap Y) = (cl^*(A) \cap Y) \in I$, whenever $A \subset U \cap Y$ and U is g -open. Hence then $A \subset U$ and $B \subset U$. By definition of Ig^* -closed $cl^*(A) - U \in I$ and $cl^*(B) - U \in I$. Hence A is Ig^* -closed relative to the subspace Y . $\square$

Theorem 3.10. *Let A be an Ig^* -closed and F be a g -closed set in (X, τ) , then $A \cap F$ is an Ig^* -closed in (X, τ) .*

Proof. Let $A \cap F \subset U$ and U is g -open. Then $A \subset U \cup (X - F)$. Since A is Ig^* -closed, we have $cl^*(A) - (U \cup (X - F)) \in I$. Now $cl^*(A \cap F) - U \subset cl^*(A \cap F) - (X - F) \subset cl^*(A) - (U \cup (X - F)) \in I$. Hence $A \cap F$ is an Ig^* -closed in (X, τ) . $\square$

4. Stronger form of Ig^* - open sets

Definition 4.1. Let (X, τ) be a topological space and I be an ideal on X . A subset of A of X is said to be strongly g -open with respect to an ideal (briefly Ig^* - open) if and only if $X-A$ is Ig^* -closed.

Theorem 4.2. A set A in Ig^* -closed in (X, τ) if and only if $F-U \subset \text{int}^*(A)$ for some $U \in I$ whenever $F \subset A$ and F is g -closed.

Proof. Assume that A is Ig^* - open. Let $F \subset \text{cl}^*(A) - A$. Suppose F is g -closed. Then, $X-A \subset X-F$. By our assumption, $\text{cl}^*(A) - (X-A) \subset (X-F) \cup U$ for some $U \in I$. This implies $X-(X-F) \cup U \subset X - \text{cl}^*(X - A)$. Conversely, assume that $F \subset A$ and F is g -closed in X implies $F - U \subset \text{int}^*(A)$ for some $U \in I$, whenever $F \subset A$ and F is g -closed. Consider an g -open set G such that $X - A \subset G$. Then $X - G \subset A$. By our assumption, $(X-G) - U \subset \text{int}^*(A) = X - \text{cl}^*(X) - A$. This gives that $(X-(G \cup V)) \subset X - \text{cl}^*(X - A) \subset G \cup U$, for some $U \in I$. This shows that $\text{cl}^*(X - A) - G \in I$. Hence $X-A$ is Ig^* -closed. $\square$

Theorem 4.3. If A and B are separated Ig^* - open of (X, τ) then their union $A \cup B$ is also Ig^* - open.

Proof. Suppose A and B are separated Ig^* - open of (X, τ) F be a g -closed subset of $A \cup B$. Then $F \cap \text{cl}^*(A) \subset A$ and $F \cap \text{cl}^*(B) \subset B$. By assumption, $(F \cap \text{cl}^*(A)) - U_1 \subset \text{int}^*(A)$ and $(F \cap \text{cl}^*(B)) - U_2 \subset \text{int}^*(B)$, for some $U_1, U_2 \in I$. This means $F \cap \text{cl}^*(A) - \text{int}^*(A) \in I$ and $F \cap \text{cl}^*(B) - \text{int}^*(B) \in I$. Hence $F \cap \text{cl}^*(A) \cup \text{cl}^*(B) - \text{int}^*(A) \cup \text{int}^*(B) \in I$ and we have $F - \text{int}^*(A) \cup B) (\text{cl}^*(B) \subset B)$. By definition n Ig^* -closed, $\text{cl}^*(A) - U \in I$ and $\text{cl}^*(B) - U \in I$. Hence $\text{cl}^*(A \cup B) - U = (\text{cl}^*(A) - U) \cup (\text{cl}^*(B) - U) \in I$. Therefore, $A \cup B$ is Ig^* -open. $\square$

Corollary 4.4. If A and B are Ig^* -closed sets and suppose $X-A$ and $X-B$ are separated in (X, τ) . Then their intersection $A \cap B$ is Ig^* -closed.

Corollary 4.5. If A and B are Ig^* -open sets in (X, τ) . Then $A \cap B$ is Ig^* -open.

Theorem 4.6. If A is Ig^* -closed and $A \subset B \subset X$ is Ig^* -open relative to B is Ig^* -open relative to X , then A is Ig^* -open relative to X .

Proof. Suppose $A \subset B \subset X$, A is Ig^* -closed and in (X, τ) . Suppose A is Ig^* -open relative to B and B is Ig^* -open relative to X . Suppose $F \subset A$ and F is g -closed. Since A is Ig^* -open relative to B , by theorem 4.2, $F - U \subset \text{int}^*(A)$ for some $U \in I$, whenever $F \subset A$ and F is g -closed. Consider an g -open set G

such that $X - A \subset G$. Then $X - G \subset A$. By our assumption, $(X-G)-U \subset \text{int}^*(A) = X - \text{cl}^*(X - A)$. This gives that $(X - (G \cup V)) \subset X - \text{cl}^*(X - A)$. Then $\text{cl}^*(X - A) \subset G \cup U$, for some $U \in I$. This shows that $\text{cl}^*(X - A) - G \in I$. Hence $X - A$ is Ig^* -closed.

Now $F - U \subset \text{int}^*(A)$ for some $U \in I$, whenever $F \subset A$ and F is g -closed. Consider an g -open set G such that $X - A \subset G$. Then $X - G \subset A$.

$F - (U_1 \cup U_2) = (F - U_1) \cap (F - U_2) \subset G \cap G_2 \subset G_1 \cap G \subset A$. This implies that $F - (U_1 \cap U_2) \subset \text{int}^*(A)$, for some $(U_1 \cap U_2) \in I$ and hence A is Ig^* -open relative to X . $\square$

Theorem 4.7. *Let $\text{int}^*(A) \subset B \subset A$ and suppose A is Ig^* -open in (X, τ) , A is Ig^* -open in X .*

Proof. Suppose $\text{int}^*(A) \subset B \subset A$ and A is Ig^* -open. Then $X - A - B \subset X - B \subset \text{cl}^*(X - A)$ and $X - A$ is Ig^* -closed in (X, τ) , theorem 3.8, $X - A$ is Ig^* -closed and hence A is Ig^* -open. $\square$

Theorem 4.8. *A set A is Ig^* -closed in (X, τ) , if and only if $\text{cl}^*(A) - A$ is Ig^* -open.*

Proof. Necessity: Suppose $F \subset \text{cl}^*(A) - A$ and F be g -closed. Then $F \in I$. This implies that $F - U = \emptyset$ for some $U \in I$. Clearly, $F - U \subset \text{int}^*(\text{cl}^*(A) - A)$. By theorem 4.2 $\text{cl}^*(A) - A$ is Ig^* -open.

Sufficiency: Suppose $A \subset G$ and G is open in X . Let $A \cap F \subset U$ and U is g -open. Then $A \subset U \cup (X - F)$. Since A is Ig^* -closed, we have $\text{cl}^*(A) - (U \cup (X - F)) \in I$. Now $\text{cl}^*(A \cap F) \subset \text{cl}^*(A \cap F) - (X - F)$. Therefore, $\text{cl}^*(A \cap F) - U \subset \text{cl}^*(A \cap F) - (X - F) \subset \text{cl}^*(A) - (U \cup (X - F)) \in I$. Hence $A \cap F$ is Ig^* -closed in (X, τ) . Thus A is Ig^* -closed. $\square$

References

- [1] W.Dunham, A new closure operator for non-T1 topologies, Kyunpook Math., J.22(1982), 55-60.
- [2] T.R.Hamlett and D.Jankovik, Compactness with respect to an ideal, Boll. Un.Mat.Ita., (7), 4-B, (1990), 849-861..
- [3] T.R. Hamlett and D.Jankovik, Ideals in topological spaces and the set operator, Boll. Un.Mat.Ita., (7), 4-B, (1990), 863-874..
- [4] T.R. Hamlett and D.Jankovik, Ideals in General topology and Applications (Middletown, CT, 1988), Lecture notes in Pure and Appl.Math.Dekker, New York, (1990), 115-125,.

- [5] T.R. Hamlett and D.Jankovik, Compatible extensions of Ideals, Boll. Un.Mat.Ita.,7,(1992),453-465.
- [6] D.Jankovik and T.R. Hamlett, New topologies from old via ideals, Amer. Math. Month., 97,(1990),295-310.
- [7] S. Jafari, N. Rajesh Generalized Closed Sets with Respect to an Ideal European Journal of Pure and Applied Mathematics vol.4, No. 2,(2011),147-151
- [8] K. Kuratowski, Topologies I, Warszawa, 1933.
- [9] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo,19(2),(1970), 89-96.
- [10] R. L. Newcomb, Topologies which are compact modulo an ideal, Ph.D. Dissertation, Univ.Cal.at Santa Barbara,(1967).
- [11] A. Pushpalatha and K.Anitha, g^* -closed sets in Topological Spaces Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 19,917-219.
- [12] A. Pushpalatha, S.Eswaran and P.Rajarubi, τ^* - Generalized Closed Sets in Topological Spaces, Proceedings on the World Congress on Engineering 2009 Vol II, WCE 2009, July 1-3, 2009.London, U.K.
- [13] D. V. Rancin, Compactness modulo an ideal, Soviet Math. Dokl., 13,(1972), 193-197.
- [14] P. Samuels, A topology from a given topology and ideal, J. London Math. Soc. (2)(10),(1975), 409-416.