NEW TYPE OF STRONGER FORM OF IG CLOSED SETS

A. Thiripuram
Department of Mathematics
Jeppiaar Engineering College
Chennai, Tamil Nadu, INDIA

Abstract: An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. In this paper, we have introduced stronger forms of g-closed sets via ideal topological space. Also we have studied the properties of the stronger forms of g-closed sets with respect to an ideal.

AMS Subject Classification: 54C10
Key Words: Topological spaces, generalized closed set, strongly generalized closed set and Ideal.

1. Introduction

Nowadays ideals are playing very important role in General Topology. It was the works of Newcomb[10], Rancin[13], Samuels[14] and Hamlet and Jankovic ([2,3,4,5,6]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology. A nonempty collection I of subsets on a topological space (X,τ) is called a topological ideal if it satisfies the following two conditions:

(i) If A ∈ I and B ⊂ A implies B ∈ I.(heredity).
(ii) If A ∈ I and B ∈ I then A ∪ B ∈ I.(finite additivity)
Throughout this paper (X,τ) will denote topological space. For a subset A of a topological space (X,τ). The closure of A (denoted as cl(A)) is defined as the intersection of all closed sets containing A and the interior of A (denoted as int(A)) is defined as the union of all open sets contained in A. Let A ⊂ B ⊂ X. Jafari [7] introduced the concept Generalized closed set with respect to an Ideal. In this paper, we introduce and study the concept of g*-closed sets with respect to an ideal, which is the extension of the concept of Ig-closed sets.

2. Preliminaries

Definition 2.1. A subset of a topological space (X,τ) is called a generalized closed set (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).

Definition 2.2. Let (X,τ) be a topological space and A is a subset of X, the generalized closure operator (briefly cl*)[1] is defined by the intersection of all g-closed sets containing A. The interior operator (briefly int*) is defined by union of all g-open sets contained in A.

Definition 2.3. A subset of a topological space (X,τ) is called a strongly generalized closed set (briefly g*-closed) if cl(A) ⊆ U whenever A ⊆ U where U is g-open in (X,τ).

Definition 2.4. Let (X,τ) be a topological space and I be a ideal on X. A subset A subset of X is said to be generalized closed with respect to an ideal (briefly Ig-closed) [7] if and only if cl(A)-B ∈ I whenever A ⊂ B and B is g-open.

3. Strong form of Ig* closed sets

Definition 3.1. Let (X,τ) be a topological space and I be a ideal on X. A subset A subset of X is said to be strongly g-closed with respect to an ideal (briefly Ig*-closed) if and only if cl*(A) − B ∈ I whenever A ⊂ B and B is g-open.

Remark 3.2. Every g-closed set is Ig*-closed, but the converse need not be true.

Example 3.3. Let X = {a,b,c} with topology τ = {∅, {a}, X} and I = {∅, {b}, {c}, {b,c}} clearly the set {a} is Ig*-closed but not g-closed in (X,τ).
Theorem 3.4. A set $A$ is $Ig^*$-closed in $(X,\tau)$ if and only if $F \subset cl^*(A) - A$ and $F$ is $g$-closed in $X$ implies $F \in I$.

Proof. Assume that $A$ is $Ig^*$-closed. Let $F \subset cl^*(A) - A$. Suppose $F$ is $g$-closed. Then, $A \subset X-F$. By assumption, $cl^*(A) - (X-F) \in I$.

Conversely, assume that $F \subset cl^*(A) - A$ and $F$ is $g$-closed in $X$ implies that $F \in I$. Suppose $A \subset U$ and $U$ is $g$-open. Then $cl^*(A) - U = cl^*(A) \cap (X - U)$ is a $g$-closed set in $X$, that is contained in $cl^*(A) - A$. By assumption, $cl^*(A) - U \in I$. This implies that $A$ is $Ig^*$-closed.

Theorem 3.5. If $A$ and $B$ are $Ig^*$-closed sets of $(X,\tau)$ then their union $A \cup B$ is also $Ig^*$-closed.

Remark 3.6. The intersection of $Ig^*$-closed sets need not be an $Ig^*$-closed.

Example 3.7. Let $X = \{a,b,c\}$ with topology $\tau = \{\emptyset, \{b\}, \{b,c\}, X\}$. If $A = \{b,c\}, B = \{a,b\}$ and $I = \{\emptyset, \{a\}, \{c\}, \{a,c\}\}$ then $A$ and $B$ are $Ig^*$-closed but their intersection $A \cap B = \{b\}$ is not $Ig^*$-closed.

Theorem 3.8. If $A$ is $Ig^*$-closed and $A \subset B \subset cl^*(A)$ and in $(X,\tau)$, then $B$ is $Ig^*$-closed in $(X,\tau)$.

Proof. Suppose $A$ is $Ig^*$-closed and $A \subset B \subset cl^*(A)$ in $(X,\tau)$. Suppose $B \subset U$ and $U$ is $g$-open. Then $A \subset U$. Since $A$ is $Ig^*$-closed, we have $cl^*(A) - U \in I$. This implies that $cl^*(B) - U \subset cl^*(A) - U \in I$. Hence $B$ is $Ig^*$-closed in $(X,\tau)$.

Theorem 3.9. Let $A \subset Y \subset X$ and suppose that $A$ is $Ig^*$-closed in $(X,\tau)$. $A$ is $Ig^*$-closed relative to the subspace $Y$ of $X$, with respect to the ideal $I_y = \{F \subset Y : F \in I\}$.

Proof. Suppose $A \subset U \cap Y$ and $U$ is $g$-open in $(X,\tau)$ then $A \subset U$. Since $A$ is $Ig^*$-closed in $(X,\tau)$ we have $cl^*(A) - U \in I$. Now $(cl^*(A) \cap Y) - (U \cap Y) = (cl^*(A) \cap Y) \in I$, whenever $A \subset U \cap Y$ and $U$ is $g$-open. Hence then $A \subset U$ and $B \subset U$. By definition $n$ of $Ig^*$-closed $cl^*(A) - U \in I$ and $cl^*(B) - U \in I$. Hence $A$ is $Ig^*$-closed relative to the subspace $Y$.

Theorem 3.10. Let $A$ be an $Ig^*$-closed and $F$ be a $g$-closed set in $(X,\tau)$, then $A \cap F$ is an $Ig^*$-closed in $(X,\tau)$.

Proof. Let $A \cap F \subset U$ and $U$ is $g$-open. Then $A \subset U \cup (X-F)$. Since $A$ is $Ig^*$-closed, we have $cl^*(A) - (U \cup (X-F)) \in I$. Now $cl^*(A \cap F) - U \subset cl^*(A \cap F) - (X-F) \subset cl^*(A) - (U \cup (X-F)) \in I$. Hence $A \cap F$ is an $Ig^*$-closed in $(X,\tau)$.
4. Stronger form of $Ig^*$-open sets

**Definition 4.1.** Let $(X,\tau)$ be a topological space and $I$ be an ideal on $X$. A subset of $A$ of $X$ is said to be strongly g-open with respect to an ideal (briefly $Ig^*$-open) if and only if $X-A$ is $Ig^*$-closed.

**Theorem 4.2.** A set $A$ in $Ig^*$-closed in $(X,\tau)$ if and only if $F-U \subset \text{int}^*(A)$ for some $U \in I$ whenever $F \subset A$ and $F$ is g-closed.

**Proof.** Assume that $A$ is $Ig^*$-open. Let $F \subset cl^*(A) - A$. Suppose $F$ is g-closed. Then, $X-A \subset X-F$. By our assumption, $cl^*(A) - (X-A) \subset (X-F) \cup U$ for some $U \in I$. This implies $X-(X-F) \cup U \subset X - cl^*(X - A)$. Conversely, assume that $F \subset A$ and $F$ is g-closed. Consider an g-open set $G$ such that $X - A \subset G$. Then $X - G \subset A$. By our assumption, $(X-G) - U \subset \text{int}^*(A) = X - cl^*(X - A)$. This gives that $(X-(G \cup U)) \subset X - cl^*(X - A) \subset G \cup U$, for some $U \in I$. This shows that $cl^*(X - A) - G \in I$. Hence $X-A$ is $Ig^*$-closed.

**Theorem 4.3.** If $A$ and $B$ are separated $Ig^*$-open of $(X,\tau)$ then their union $A \cup B$ is also $Ig^*$-open.

**Proof.** Suppose $A$ and $B$ are separated $Ig^*$-open of $(X,\tau)$ $F$ be a g-closed subset of $A \cup B$. Then $F \cap cl^*(A) \subset A$ and $F \cap cl^*(B) \subset B$. By assumption, $(F \cap cl^*(A)) - U_1 \subset \text{int}^*(A)$ and $(F \cap cl^*(B)) - U_2 \subset \text{int}^*(B)$, for some $U_1, U_2 \in I$. This means $F \cap cl^*(A) - \text{int}^*(A) \in I$ and $F \cap cl^*(B) - \text{int}^*(B) \in I$. Hence $F \cap cl^*(A) \cup cl^*(B) - \text{int}^*(A) \cup \text{int}^*(B) \in I$ and we have $F - \text{int}^*(A) \cup B(cl^*(B) \subset B)$. By definition n $Ig^*$-closed, $cl^*(A) - U \in I$ and $cl^*(B) - U \in I$. Hence $cl^*(A \cup B) - U = (cl^*(A) - U) \cup (cl^*(B) - U) \in I$. Therefore, $A \cup B$ is $Ig^*$-open.

**Corollary 4.4.** If $A$ and $B$ are $Ig^*$-closed sets and suppose $X-A$ and $X-B$ are separated in $(X,\tau)$. Then their intersection $A \cap B$ is $Ig^*$-closed.

**Corollary 4.5.** If $A$ and $B$ are $Ig^*$-open sets in $(X,\tau)$. Then $A \cap B$ is $Ig^*$-open.

**Theorem 4.6.** If $A$ is $Ig^*$-closed and $A \subset B \subset X$ is $Ig^*$-open relative to $B$ is $Ig^*$-open relative to $X$, then $A$ is $Ig^*$-open relative to $X$.

**Proof.** Suppose $A \subset B \subset X$, $A$ is $Ig^*$-closed and in $(X,\tau)$. Suppose $A$ is $Ig^*$-open relative to $B$ and $B$ is $Ig^*$-open relative to $X$. Suppose $F \subset A$ and $F$ is g-closed. Since $A$ is $Ig^*$-open relative to $B$, by theorem 4.2, $F - U \subset \text{int}^*(A)$ for some $U \in I$, whenever $F \subset A$ and $F$ is g-closed. Consider an g-open set $G$.
such that $X - A \subset G$. Then $X - G \subset A$. By our assumption, $(X-G)-U \subset int^*(A)$ = $X - cl^*(X-A)$. This gives that $(X-(G \cup V)) \subset X - cl^*(X-A)$. Then $cl^*(X-A) \subset G \cup U$, for some $U \in I$. This shows that $cl^*(X-A) - G \in I$. Hence $X - A$ is $Ig^*$-closed.

Now $F - U \subset int^*(A)$ for some $U \in I$, whenever $F \subset A$ and $F$ is $g$-closed. Consider an $g$-open set $G$ such that $X - A \subset G$. Then $X - G \subset A$.

**Theorem 4.7.** Let $int^*(A) \subset B \subset A$ and suppose $A$ is $Ig^*$-open in $(X,\tau)$, $A$ is $Ig^*$-open in $X$.

**Proof.** Suppose $int^*(A) \subset B \subset A$ and $A$ is $Ig^*$-open. Then $X-A-B \subset X-B \subset cl^*(X-A)$ and $X-A$ is $Ig^*$-closed in $(X,\tau)$, theorem 3.8, $X-A$ is $Ig^*$-closed and hence $A$ is $Ig^*$-open.

**Theorem 4.8.** A set $A$ is $Ig^*$-closed in $(X,\tau)$, if and only if $cl^*(A)-A$ is $Ig^*$-open.

**Proof.** Necessity: Suppose $F \subset cl^*(A)-A$ and $F$ be $g$-closed. Then $F \in I$. This implies that $F-U = \emptyset$ for some $U \in I$. Clearly, $F-U \subset int^*(cl^*(A)-A)$. By theorem 4.2 $cl^*(A)-A$ is $Ig^*$-open.

Sufficiency: Suppose $A \subset G$ and $G$ is open in $X$. Let $A \cap F \subset U$ and $U$ is $g$-open. Then $A \subset U \cup (X-F)$. Since $A$ is $Ig^*$-closed, we have $cl^*(A) - (U \cup (X-F)) \in I$. Now $cl^*(A \cap F) \subset cl^*(A \cap F) - (X-F)$. Therefore, $cl^*(A \cap F) - U \subset cl^*(A \cap F) - (X-F) \subset cl^*(A) - (U \cup (X-F)) \in I$. Hence $A \cap F$ is $Ig^*$-closed in $(X,\tau)$. Thus $A$ is $Ig^*$-closed.

**References**


