TOTAL CO-INDEPENDENT DOMINATION OF JUMP GRAPH

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Abstract: A set $D \subseteq V[J(G)]$ is dominating set of jump graph, if every vertex not in D is adjacent to vertex in D . The domination number of the jump graph is the minimum cardinality of dominating set of jump graph $J(G)$. A total dominating set S of a graph G is called Co-independent dominating set if the induced subgraph $\langle G \rangle$ has no edge and has at least one vertex. In this paper, we introduce the total Co-independent domination of jump graph, and its exact values for some standard graphs and some results are established.

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Key Words: dominating set, total dominating set, total co-independent dominating set, jump graph

1. Introduction

Let $G(p, q)$ be a graph with $p = |V|$ and $q = |E|$ denote the number of vertices and edges of a graph G , respectively. All the graphs considered here are finite, non-trivial, undirected and connected without loops or multiple edges.

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In general, we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X . A vertex of degree one is called a pendent vertex. A vertex adjacent to pendent vertex is called the support vertex. The maximum $d(u, v)$ for all u in G is eccentricity of v and the maximum eccentricity is the diameter $diam(G)$. The circumference of a graph G with at least one cycle is the length of a longest cycle in G and it is denoted by $circum(G)$. As usual P_n , C_n and K_n are respectively, the path, cycle and complete graph of order n , $K_{r,s}$ is the complete bipartite graph with two partite sets containing r and s vertices. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D . The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G . For terminology and notations not specifically defined here we refer reader to [1][2]. For more details about domination number and its related parameters, we refer to [3][4][5].

A dominating set S of G is called a connect dominating set if the induced subgraph $\langle S \rangle$ is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$ [6]. A dominating set S of G is called total dominating set if the induced subgraph $\langle S \rangle$ has no isolated vertices. The minimum cardinality of a total dominating set of G is called the total domination number of G and is denoted by $\gamma_t(G)$ [3].

Definition 1. The line graph $L(G)$ of G has the edges of G as its vertices which are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G . We call the complement of line graph $L(G)$ as the jump graph $J(G)$ of G , found in [7]. The jump graph $J(G)$ of a graph G is the graph defined on $E(G)$ and in which two vertices are adjacent if and only if they are not adjacent in G . Since both $L(G)$ and $J(G)$ are defined on the edge set of a graph G .

Remark 2. The isolated vertices of G (if G has) play no role in line graph and jump graph transformation. Here we assume that the graph G under consideration is non-empty and has no isolated vertices found in [7].

$\{2, 3\}$ is the minimum total dominating set. Hence $\gamma_t(G) = 2$, clearly $\{2, 3\}$ is not total Co-independent dominating set of G . The minimum total Co-independent dominating sets are $\{1, 2, 3, 4\}$, $\{1, 2, 3, 5\}$, $\{2, 3, 4, 9\}$, $\{2, 3, 5, 9\}$. Therefore $\gamma_{t,coi}(G) = 4$.

As in the standard dominating set, any minimum total Co-independent dominating set is minimal, but the converse is not true as seen in figure 1. The set $\{1, 2, 4, 5, 6, 7\}$ is minimal total Co-independent dominating set but not minimum total Co-independent dominating set.

Definition 3. A total dominating set S of a $J(G)$ is called total Co-

Figure 1

independent dominating set if the induced subgraph $\langle V - S \rangle$ has no edge and has at least one vertex. The minimum cardinality of a total Co-independent dominating set of $J(G)$ is called the total Co-independent domination number of $J(G)$ and is denoted by $\gamma_{t,coi}J(G)$.

2. Main Results

Observation. For any Cycle C_p , with $p \geq 5$, $\gamma_{t,coi}[J(C_p)] = 4$.

Observation. For any path P_p , with $p \geq 5$, $\gamma_{t,coi}[J(P_p)] = 4$.

Observation. For any complete graph K_p , with $p \geq 5$, $\gamma_{t,coi}[J(K_p)] = 6$.

Observation. For any connected (p, q) graph G , $\gamma_{t,coi}[J(G)] = q$.

Observation. For any complete bipartite graph $K_{m,n}$

$$\gamma_{t,coi}[J(K_{m,n})] = \begin{cases} 4 & \text{for } k_{2,n} \text{ where } n \geq 2 \\ 6 & \text{for } k_{m,n} \text{ where } m, n \geq 3 \end{cases}$$

Observation. For any wheel W_p ,

$$\gamma_{t,coi}[J(W_p)] = \begin{cases} 6 & \text{for } p = 5, 6 \\ 4 & \text{for } p \geq 7 \end{cases}$$

Theorem 4. For any connected graph G With diameter, $diam(G) \geq 2$, $\gamma_{t,coi}J(G) \geq 4$.

Proof. Let uv be a path of maximum distance in G . Then $d(u,v) = diam(G)$. We can prove the theorem with the following cases.

Case (i): For $diam(G) = 2$, choose a vertex V_1 of eccentricity 2 with maximum degree among others. Let $V_1 = \{v_1, v_2, \dots\}$ corresponding to the elements of $\{e_1, e_2, \dots\}$ forming a total Co-independent dominating set in jump

graph $J(G)$. Every vertex $u \notin V_1$ is adjacent to a vertex in V_1 . Hence V_1 is a minimum total Co-independent dominating set. So the total Co-independent domination number of the jump graph will be equal to the degree of V_1 we say $\gamma_{t,coi}[J(G)] > 4$.

Case (ii): For $diam(G) > 2$. Let e_1 be any edge adjacent to u and e_2 be any edge adjacent to v . Let $\{e_1, e_2\} \subseteq E(G)$ form a corresponding vertex set $v_1, v_2 \subseteq V(J(G))$. These two vertices form total Co-independent dominating set in jump graph. Since these vertices v_1, v_2 are adjacent to all other vertices of $V(J(G))$, it follows that v_1, v_2 becomes a minimum total Co-independent dominating set. Hence $\gamma_{t,coi}[J(G)] = 4$. In view of above cases, we can conclude that for any connected graph G , $\gamma_{t,coi}[J(G)] \geq 4$. □

Theorem 5. For any connected graph G with circumference $circum(G) \geq 4$, $\gamma_{t,coi}[J(G)] \geq 4$.

Proof. Let $u - v - u$ be a longest cycle in G . Then $C(G) = circum(G)$. We can prove the theorem with the following cases.

Case i: For $Circum(G) = 4$, choose a vertex V_1 that have longest cycle among others. Let $V_1 = \{v_1, v_2, \dots\}$ corresponding to the elements of $\{e_1, e_2, \dots\}$ forming a total Co-independent dominating set in jump graph $J(G)$. Every vertex $u \notin V_1$ is adjacent to a vertex in V_1 . Hence V_1 is a minimum total Co-independent dominating set. So the total Co-independent domination number of the jump graph will be equal to the longest cycle. We say $\gamma_{t,coi}[J(G)] > 4$.

Case ii: For $Circum(G) > 4$. Let e_1 be any edge adjacent to u and e_2 be any edge adjacent to v . Let $\{e_1, e_2\} \subseteq E(G)$ form a corresponding vertex set $\{v_1, v_2\} \subseteq V(J(G))$. These two vertices form total Co-independent dominating set in jump graph. Since these vertices $\{v_1, v_2\}$ are adjacent to all other vertices of $V(J(G))$, it follows that $\{v_1, v_2\}$ becomes a minimum total Co-independent dominating set. Hence $\gamma_{t,coi}[J(G)] = 4$. In view of above cases, we can conclude that for any connected graph G , $\gamma_{t,coi}[J(G)] \geq 4$. □

Theorem 6. For any tree T with diameter greater than 3, $\gamma_{t,coi}[J(T)] = q$.

Proof. If the diameter is less than or equal to 3, then the jump graph will be disconnected. Let uv be a path of maximum length in a tree T where diameter is greater than 3. Let e_i be the pendant edge adjacent to u and e_k be the pendant edge adjacent to v . The vertex set $\{v_i, v_k\}$ of $J(T)$ corresponding to the edges of $\{e_i, e_k\}$ in T will form total Co-independent dominating set in

$J(T)$. Since all the other vertices of $V[J(T)]$ are adjacent with $\{v_i, v_k\}$, it forms a minimum total Co-independent dominating set. Hence $\gamma_{t,coi}[J(T)] = q$. $\square$

Theorem 7. *For any jump graph $J(G) = (V, E)$ with no isolated vertices and $|V| \geq 3$, $\gamma(J(G)) \leq \gamma_t(J(G)) \leq \gamma_{t,coi}(J(G))$.*

Proof. Let $J(G) = (V, E)$ be a graph with no isolated vertices. Suppose that $S \subseteq V$ is any minimum total Co-independent dominating set of $J(G)$. Since for any jump graph $J(G)$ any total Co-independent dominating set S is also total dominating set and every total dominating set is also dominating set. Hence $\gamma(J(G)) \leq \gamma_t(J(G)) \leq \gamma_{t,coi}(J(G))$. $\square$

Theorem 8. *let $J(G)$ be a jump graph with D as minimal total Co-independent dominating set, then $V - D$ is independent dominating set of $J(G)$.*

Proof. Let D be minimal total Co-independent dominating set of $J(G)$. Suppose that $V - D$ is not independent dominating set of $J(G)$, since D is total Co-independent dominating set of G then $V - D$ is independent set, that means if we suppose that $V - D$ is not independent dominating set of $J(G)$, then there exists a vertex u such that u is not dominated by any vertex in $V - D$. Since $J(G)$ has total Co-independent dominating set, then $J(G)$ has no isolated vertices, therefore u is dominated by at least one vertex in $D - \{u\}$. Thus $D - \{u\}$ is total Co-independent dominating set of $J(G)$, which contradicts the minimality of D . Thus every vertex in D is adjacent with at least one vertex in $V - D$ and $V - D$ is independent set. Hence $V - D$ is independent dominating set of $J(G)$. $\square$

Theorem 9. *For any $J(G)$ any total Co-independent dominating set of $J(G)$ contains all the support vertices.*

Proof. Suppose the graph $J(G)$ has a total Co-independent dominating set D and let V be support vertex does not belongs to D then clearly the pendent vertex which adjacent to V cannot belong to D from the definition of total Co-independent dominating set. Hence D is not a dominating set, which is contradiction. $\square$

3. Conclusion

Thus we conclude the total Co-independent domination number and the parameters of total Co-independent domination number of jump graphs like k_p , C_p, P_p , W_p and $K_{m,n}$.

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