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THE MAHALANOBIS DISTANCE BETWEEN THE HURST COEFFICIENT AND THE ALPHA-STABLE PARAMETER: AN EARLY WARNING INDICATOR OF CRISES

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Abstract: The Hurst coefficient and the alpha-stable parameter are useful indicators in the analysis of time series to detect normality and absence of self-similarity. In particular, when these two features met simultaneously the series is driven by white noise. This paper is aimed at developing an index to measure the degree to which a time series departs from white noise. The proposed index is built by using the principal component analysis of the Mahalanobis distances between the Hurst coefficient and the alpha-stable parameter from theoretical values of normality and absence of self-similarity. The proposed index is applied to examine the Mexican Peso exchange rate against the US Dollar. The distinctive characteristic of the index is that it can be used as an early warning indicator of crises, as it is shown for the Mexican case.

AMS Subject Classification: G15, F31, C43 and C53
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1. Introduction

Reality can barely be described by any mathematical model because of the necessary oversimplification. Most models in economics and finance usually rely on two idealistic assumptions, normality and independence. Regrettably, none of these two assumptions are ever entirely fulfilled. In fact, the existence of heavy tails, volatility clusters, and other persistence phenomena becomes the rule more than the exception[1]. Maybe the most outrageous fact in analysis of financial time series is that they do not seem to be identically distributed along time. In fact, they do not seem to present any predictable structure since they are influenced by many external uncertainty sources.

On the other hand, it is well known that the Hurst coefficient and the alpha-stable parameter allow us to identify the self-similarity and the degree of impulsivity in a time series. It is worth noticing that both phenomena are related but they are not the same. In fact, we can see a single heavy tail realization associated to a single event or a volatility cluster contained on a certain range. This seems to be the case when a crisis is not declared but the market remains on the lookout.[2] A worthy microeconomic explanation of this issue is given in Lux and Marchesi (2000), in which the excess of kurtosis and volatility clusters arise as a response to the heterogeneity of economic agents in simulated artificial markets. Also, Kirchler and Huber (2007) found that the asymmetry in information is a major source of excess kurtosis in their simulated artificial market. Moreover, a financial explanation to this matter is given in Thurner et al. (2012) elucidating that the fat tails phenomena is a response to the urgent margin calls to close leveraged positions. As it can be perceived, even when both phenomena are related, they describe different ways of departing from normality. This is why it is desirable to have a single measure that indicates how far the actual time series is from the white noise assumption. Furthermore, detecting how far a time series (in particular the exchange rate) is from being white noise it can be useful to detect patterns of crisis preludes, which is an important subject in finance and economics. In this regard, the econometric based methods as proposed by Oh et al. (2006) interpret some predetermined volatility patterns as crisis preludes. Also, Al-Anaswah and Wilfling (2011) propose a Markov switching model of the present value of the stock prices to detect the bubble burst states. Meanwhile, Grkaynak (2008)

[1]Very well-known surveys on this non white noise characteristics can be found on Mandelbrot (1963), Mandelbrot and Taylor (1967) and Mandelbrot (1971).

[2]Additional Issues related with a non declared exchange rate crisis for the Mexican case can be seen in Venegas-Martnez (1999).

collects and reviews the most used econometric techniques for bubble detecting, and Lim et al. (2008) use a bicorrelation test in order to prove market efficiency during the Asian crises on seven selected Asian markets. Finally, we mention the work of Harvey et al. (2013) that use a unit root test approximation to detect the bubble burst. In most of these studies, the proposed methodology relies on the existence of the second moment of the distribution, which is not always feasible when there are fat tails. Even more, if the data is not normal, there may be some important information on the higher order sample moments that is not being considered only in the variance.

Other crisis detection models are based on non linear assumptions, as the jump diffusion detection method proposed by Andersen and Sornette (2004), the entropy based measure as suggested by Garca-Ruiz et al. (2014), and the oscillator methodology as proposed by Chian (2000). In all these cases, the models are able to capture some of the higher order dependence (depending on the model), but they are not giving a direct measurement of how far is the time series from the white noise assumption.

Our proposal can be classified as non-linear and non-normal approach. The use of the principal component analysis responds to the necessity of cleaning the overlapping explanation between both elements. As we state previously, the existence of fat tails is not necessarily associated with the volatility clusters and *vice versa*. The non-normal characteristic of our model depends on the estimation of the alpha parameter[3]. This feature remains even when the data is sub-sampled quarterly or yearly because the extreme value existence makes that the variance of a single observation dominates the variance of the sample, thereby preventing the fulfillment of the central limit theorem. Here, we need to elucidate that the consolidation of the data into low frequency series, by using averages or other aggregation method, decreases the effect of the outliers.

On the other hand, the non linearity characteristic (associated to long memory properties) in our model is provided by the Hurst (1951) coefficient. This measurement is related to the work of Cannon et al. (1997), Cheung and Lai (1995), and Cajueiro and Tabak (2004). In their papers, they found the long memory feature only for the short run or in variance. As in our proposal, the long memory effect seems to fade as the data became of low frequency. The study of this fractal measurement and the long memory effects is widely spread in economics and finance; see, for instance, Kulik and Soulier (2011) describing the limiting behavior of tail empirical processes associated with long memory stochastic volatility models. In this regard, Mensi et al. (2014) analyze the dual

[3]For a deeper insight of the alpha stable distributions see Zolotarev (1986) o Samorodnitsky and Taqqu (1994).

long memory properties of four major foreign exchange rate markets associated to the world oil exporter Saudi Arabia using the ARFIMAFIGARCH model. Other example is that of Parthasarathy (2013) examining the long memory or long range dependence in the Indian stock market vis--vis market efficiency; these are examples of the current research on long memory associated with the Hurst coefficient or a related measurement.

Thus, the proposed index will take into account the Hurst coefficient (memory component) and the alpha parameter (heavy tail component). In fact, both concepts were related in Jia et al. (2012) showing multi-fractal effects in the Shangai Stock Exchange Composite Index, with longer memory associated to volatility clusters. Also, Barany et al. (2012) relate a truncated Lévy flight distribution with the Hurst coefficient. Likewise, Shrivastava and Kapoor (2013) relate a FARIMA process to the existence of long memory by using the Hurst coefficient.

As it can be seen, our proposal incorporates the two main forms of deviation of a time series from white noise through a non-arbitrary measure, the Mahanalobis distance, which incorporates the relationship between the orthogonal components of impulsivity (alpha coefficient component) and persistence (Hurst coefficient component) of the analyzed time series as a way of detecting unusual market movements, in a certain period, indicating the formation of a market disruption that may be a sign of the prelude to a crisis. It is important to point out that when examining the dynamics of the exchange rate, one distinctive characteristic of the proposed Index is that it can be used as an early warning indicator of crises, as it will be shown for the Mexican case.

The fact of choosing the exchange rate as the crisis indicating variable is not arbitrary. As a high frequency variable its realizations can be used to feed a data consuming tool as our proposal[4].It is also a difficult variable to be manipulated in the long run by the monetary authorities, even with the use of derivatives on the basis of an exchange rate defense scheme; see, for instance, Neely (2001), Meaning and Zhu (2011) and Fatum et al. (2013). The long run independence of the exchange rate makes it the escape valve for the most of the imbalances between the local and rest of the world economies. In addition to those properties, the exchange rate has proved to be a crucial variable in the crisis analysis since the first crisis generation[5]. In fact, the exchange rate is a

[4]Later, we will see that the Hurst coefficient calculations relies in the use of a LS regression that needs at least 25 observation points in order to avoid the micro-numerosity problem. On the other hand, the alpha stable distribution adjustment is made by means of a maximum likelihood, which requires a similar amount of data.

[5]For more details on the crisis generations see, for instance, Belke and Schnalb (2013) or Kaminsky (2013).

suitable indicator of how desirable and trustable is the local economy to both foreign[6] and local investors[7], which can be turned into a predictor variable to the investment GDP component analysis. On the consumption side, the exchange rate can be understood as competitiveness and consumer's confidence indicator as people buy local goods if they are as good as their foreign competitors. Also, the public will buy capital or durable goods (local and foreign) if they foresee future economic stability[8].

The paper is organized as follows: in Sections 2 and 3, we provide the theoretical basis of the use of Hurst coefficient and alpha stable distributions, respectively; through Section 4, we show the usefulness of both parameters, separately, for characterizing the impulsivity and memory of a financial time series, and the necessity of incorporating both features in our early warning crisis index; in Section 5, we develop our Global Index of Dissimilarity by using the Mahanalobis distance of Hurst coefficient and the parameter α from a theoretical white noise for different periodicity (monthly, quarterly and annual), with this information at hand, we compareour index with the actual realizations of the Mexican Peso exchange rte against the US Dollar to identify and analyze the crisis periods; finally, in Section 6, we present conclusions and acknowledge limitations of this research.

2. Fractional Brownian Motion and the Hurst Coefficient

Brownian motion or geometric Brownian motion is a typical assumption made in modern finance in order to describe the behavior of financial returns or an increasing trend in the long run. The standard Brownian motion is a stochastic process with continuous paths starting at zero, and whose increments are independent and normally distributed[9]. Roughly speaking, this assumptions denies the existence of fat tails, It also rejects the persistence of the series because the independence of its increments. Despite of their shortcomings, the Brownian motion assumption is widely used due to its simplicity and because

[6]Here we are making a reference to both, the capital account (short and medium run investment on financial instruments) and the foreign direct investment account (long run investment on physical capital).

[7]If local investors are not willing to invest in their own country, there is a lack of confidence that will create a GDP or financial crisis.

[8]An insight on this and other foreign exchange mechanism into the economy can be reviewed in Frenkel and Johnson (2013).

[9]See Revuz and Yor (1999), Glasserman (2004). Closely related to the Brownian assumption is the geometric Brownian that can be viewed in Oksendal (2002), and Lamberton and Lapeyre (2007).

its tractability to obtain closed-form solutions when pricing assets (derivatives and bonds)[10]. Unfortunately, the behavior of financial time series does not necessarily fulfill the Brownian or geometric Brownian motion assumptions[11]. In general, financial time series are non stationary and they present: discontinuities (jumps), volatility clusters, kurtosis excess (heavy tails), bias, and long term dependency due self-similarity.

As an alternative to overcome the weaknesses in the modeling assumptions involved in the geometric Brownian motion, Mandelbrot (1997) proposed a model called multi-fractal. This is based on the fractional Brownian motion. It was first considered by Kolmogorov (1940), and its use became popular after Mandelbrot's (1965) work. The fractional Brownian motion of index $H, \{BH(t); t \geq 0\}, 0 \leq H \leq$, is a stochastic process satisfying:

- $BH(0) = 0$ almost everywhere
- $E[BH(t)] = 0$ for all $t \in R$ (1)
- The covariance of the process for two times $s, t \in R$ is: $CH(t,s) = E[BH(s)BH(t)] = \frac{1}{2}[|s|2H + |t|2H - |s-t|2H]$

The index H is called Hurst parameter[12] , which is a measure to characterize fractal sets. Note that the Brownian motion can be obtained from standard fractional Brownian motion if $H = \frac{1}{2}$.

The fractional Brownian motion has no periodic cyclical variance in all time scales, and takes into account the statistical dependence in the long term. In addition to the typical characteristics of fractional Brownian Motion (FBM) mentioned above, we shall use two features of fractal sets that give to the FBM a greater variability of behavior[13]:

(A) The self-affinity or statistical self-similarity. By reducing the time scale to represent trajectories of the process. The appearance of the series is similar to the number in the original scale.

(B) Non integer value of the dimension. In characterizing the process is related to the size variations experienced between neighboring points so that the higher the value the greater dimension variation.

[10] The shortcomings of the normality assumption are widely known; see, for instance, Lo and MacKinlay (2002).

[11] See, for instance Tankov (2004), Rogers (1997), and Doukhan et al., (2003).

[12] Referring to the British scientist Harold Edwin Hurst (1880-1978).

[13] See, for instance, Mandelbrot and van Ness (1965), Beran(1994) and Comte and Renault (1996).

Accordingly, if a time series has a high dependency property, it could be modeled by means of a fractional Brownian motion that, unlike traditional Brownian motion, incorporates the features of dependence of financial series, starting with practically the same assumptions of the traditional Brownian motion.

In order to relate the FBM used in many economic and financial applications, with its fractal properties, the literature proposes the rescaled range $(R/S)n$, which is used to determine the Hurst coefficient, H, associated to a time series. Hurst (1951) developed a methodology[14] applicable to time series that are not necessarily Brownian motion, but presents long memory:

$$(R/S)n = cn^H \tag{2}$$

Where (R/S) Notation used for the rescaled range statistic, c is a proportionality constant, n is the number of interval data, and H: is the Hurst coefficient. Here (R/S) is a statistic with zero mean. The rescaled range starts with the series in the original order, and subsequently a new series is generated by taking the log difference returns of the original data.

(a) The series is divided into intervals of equal returns data number, the number of intervals will be called, hereinafter, partitions. So that the number of partitions for the number of data is equal to the size range of the number of returns. By varying the number of partitions we obtain each time series divided into intervals of equal number of data.

(b) In each partition for each of the ranges:

    (A) Calculate the mean and standard deviation.

    (B) Determine the variation of each data with respect to the mean and differences.

    (C) Establish the range of the data by subtracting the lowest to higher.

    (D) Divided the range is by the standard deviation, obtaining the standardized range.

    (E) Use the identity[15].

[14]There are, of course, other methodologies to obtain the Hurst coefficient: Rescaled Range Analysis, Detrended fluctuation Analysis and Periodogram Regression; see, for instance, Trinidad-Segovia and Garca-Pérez (2008).

[15]For further reference see the seminal Hurst, Black and Simaika (1965) paper. Moreover, some Hurst coefficient calculations issues may be reviewed in the Mielniczuk and Wojdyłło (2007) paper

$$ln\frac{R}{S}n = ln(c) + Hln(n) \tag{3}$$

If the above formula is applied to all the possible partitions, then a MCO regression is carried out to compute the Hurst coefficient, H, which is given by the slope of the regression line, while the intercept, c, is the proportionality constant.

(1) If $0 < H < \frac{1}{2}$, the series presents anti-persistency and mean reversion characteristics. That is, if the series has been up to a certain value that serves as the long-term average in the previous period is more likely to be down in the next period and *vice versa*; this is named a pink noise.

(2) If $H = \frac{1}{2}$. , the data are independent and do not have memory. This is a random walk and it is called white noise, which is consistent with the Brownian motion.

(3) If $\frac{1}{2} < H < 1$, the series is persistent. This means that it reinforces the trend and so it presents long-term cyclical behavior[16] ; this process is called black noise.

(4) If $H = 1$, the series are deterministic.

Thus, the Hurst coefficient, H, is a parameter obtained by means of a MCO regression. The parameter is a conditional mean with the usual asymptotic and normality characteristics. As stated in Greene (2003), the distribution of the estimated vector of parameters, b, conditioned on the regressor matrix, X, with a sample size, n. can be obtained form:

$$\sqrt{n}(b-\beta) = \left(\frac{X'X}{n}\right)^{-1}\left(\frac{1}{\sqrt{n}}\right)X'\varepsilon,$$

where β is the vector of population parameters, and ε is the error vector. It can be seen that given uncorrelatedness and independence on errors ε, meeting the Grenader conditions and using the central limit theorem[17], the MCO parameters, b, are normally distributed as:

$$db \sim N\left[\beta, \frac{\sigma^2}{n}\left(\frac{X'X}{n}\right)^{-1}\right] \tag{4}$$

[16]The color of the noise is given as a characteristic of its power spectrum, for more details see Kosko (2006).

[17]See, for instance, Greene (2003).

This means that if the previous conditions are met, the obtained Hurst coefficient is a normal variable that can be tested using the Annis and Lloyd (1976) test for the Hurst coefficient of a white noise process. The test is described by:

H0: The process is random and independent ($H = 0.5$).

H1: The process correlates ($H \neq 0.5$), and the test statistic given by:

$$Z_E = \frac{\hat{H} - E(\hat{H})}{\sqrt{Var(\hat{H})_n}} \tag{5}$$

It is important to point out that the Annis and Lloyd test (1976) is valid only under the assumption of white noise, which fulfills the requirements for the normality of the parameter vector of the MCO regression. There is a problem when the Hurst coefficient is relatively near from the normality case, H= 0.5. In this case, there is a biasing problem that can be large enough to invalidate a hypothesis testing but not so large to reject the mean test specification because the test statistic is not in the tail of the assumed distribution. This gives as a result a poor performance of the Annis and Lloyd test for any other Hurst coefficient value distinct form H=0.5; this problem is analyzed in Koutsoyiannis (2003) or Cajueiro and Tabak (2005).

3. Alpha-Stable Distributions

It is well known that financial time series present extreme value observations (booms and falls), which characterizes the instability of a time series and denotes the presence of heavy tails (impulsivity), which does not fulfill the requirements of the Central Limit Theorem. In fact, they present a degree of impulsivity greater than that of the normal distribution; for instance, extreme events in Mexico, such as those in 1990, 2000, 2007 stock crashes are highly unlikely to be driven from the normal distribution[18].

A very useful way of characterizing this impulsivity is the stable distributions theory. It was first developed by Paul Lévy and Aleksander Khinchine, but, it was not until the work of Mandelbrot (1963) and (1968) that the α-stable distributions were popularized. Mandelbrot proposed a theory based on these distributions to explain the acute price fluctuations observed in financial time

[18] Financial crises for the Mexican case are studied, for instance, in: intance, in Mexico Venegas-Martnez and Islas Camargo (2005), Venegas-Martnez and González-Aréchiga (2000), and González-Aréchiga et al. (2001).

series. In fact, by using standard notation, it can be demonstrated that the normal[19], $f_{2,0}(\cdot|\gamma,\delta)$, Lévy, $f_{\frac{1}{2},1}(\cdot|\gamma,\delta)$,Landau, $f_{1,1}(\cdot|\gamma,\delta)$, Holtsmark, $f_{\frac{3}{2},0}(\cdot|\gamma,\delta)$ and Cauchy, $f_{1,0}(\cdot|\gamma,\delta)$ distributions are all special cases of the α-stable distributions. Most of these distributions, in general, cannot be written analytically[20] but due the recent computational advances they had been recently rediscovered by many fields of the science, for an insight on they applications see Feldman and Taqqu (1998) and Machado et al. (2011). A random variable, X, is said to be a α-stable distribution *iff* it shows the following characteristic function (Nolan, 2005):

$$\begin{aligned} \varphi \quad x = \ \exp\left\{-\left|\gamma x\right|^{a}\left[1 - sign \quad x \quad \beta tan\left(\frac{\pi\alpha}{2}\right)\right] + i\delta x\right\}, \quad \alpha \neq 1 \\ \varphi \quad x = \ \exp\left\{-\left|\gamma x\right|\left[1 + sign \quad x \quad \beta\frac{2}{\pi}log\left|w\right|\right] + i\delta x\right\}, \quad \alpha = 1 \end{aligned} \tag{6}$$

where sign $(x) = \frac{x}{|x|}$ and $\alpha \in (0,2]$. In the above characteristic function,α controls the degree of impulsivity of the random variable X while $\beta \in [-1,+1]$ manages the symmetry of distribution[21], $\gamma > 0$ is a scale parameter (dispersion), and δ is the position parameter.

In this research, we will use the Nolan (2005) algorithm to estimate the alpha stable distribution parameters. Nolan (1997) software is based on the Zotolarev (1986) representation of the characteristic function. The idea of his software is to perform the numerical integration of a set of splits of the random variable domain. These splits are based on the sign change of the trigonometric functions (sinus and cosines) that result of the transformation of Zotolarev's (1986) representation[22] into the complex plane.

It is important to point out that there are other methodologies for estimating the parameters of an alpha-stable distribution, like those presented in Belov (2005) proposing a combination of Gaussian and Laguerre quadratures; and Mittnik et al. (1999) presenting an algorithm that applies the fast Fourier transform. Finally, DuMouchel (1973) by using a Bergstrm series expansion on the Zotolarev's characteristic function representation approximates an alpha stable cumulative distribution. After obtaining the α-stable distribution parameters, we will use the Anderson-Darling test in order to verify the fitting

[19]Since $\beta tan\pi = 0$.

[20]The integral with respect to w of the alpha-stable characteristic function (6) only have an analytical solution for the described cases (Nolan, 2005).

[21]A value $\beta = 0$ represents a symmetrical α-stable distribution, while the values $\beta = 1$ and $\beta = -1$ represent positive and negative skewed α-stable distribution, respectively.

[22]The Zotolarev's representation is based on a succession of Meijer G-Functions.

parameters. This test is used due to its efficiency in the case of heavy tailed series; see, for instance, Kabasinskas and Sakalauskas (2006) or Barbulescu and Bautu (2012).

4. The Mexican exchange crisis

By analyzing the Mexican Peso exchange rate against the US Dollar series in five cross sections characterized by financial crises, the Hurst coefficient will be estimated and tested whether there is independence of the time series or else if there is persistence or antipersistence. The results are compared with the estimate of the parameter α of an alpha-stable distribution, which identifies the impulsivity in the series. We expect that in cases far from white noise, we will find fractal characteristics. This means some degree of impulsivity, which is higher when there are extreme values and volatility clusters.

Hurst coefficient and alpha parameter are estimated for the Mexican Peso exchange rate against the US Dollar for different intervals of time. Time windows are constructed based on the distinction of high volatility periods characterized by financial crises. The methodology to calculate the Hurst coefficient is the rescaled range applied to the logarithmic returns of the Fix Mexican Peso exchange rate against the US Dollar during the period 1992-2012. In this case, 180 partitions were conducted on average for each period taking into account the number of observations in each cut; this is the maximum number of partitions with the least possible loss of data, see Table 1. For the adjustment of the alpha-stable distribution, it is considered a $S1$ parameterization, usually used for modeling financial data[23]. By setting data into five considered periods, we obtain the parameters characterizing the distribution by the three generally methods used, *i.e.*, quantiles, maximum likelihood, and regression, as in Nolan (2005). The estimated parameters by the regression method are presented with a goodness of fit criterion to the data in accordance with the Anderson-Darling test. Note that in all three methods used for five periods it is not rejected the hypothesis test H_0: the series follows the α-stable distribution. However, the regression method is mostly used in the analysis of financial series providing a best fit of the tails of the distribution.

In what follows, parameters are compared statistically. For the case of Hurst coefficient it is used the Anis-Loyd statistic, and for the case of α-stable it is used the distribution Anderson-Darling test; both at 95% confidence. The obtained results show that the Hurst coefficient was statistically different from

[23]For more details see Samorodnitsky and Taqqu (1994)

Table 1: Hurst coefficient and parameter α /1

Period	**Years**	**Hurst coefficient**	**Statistical Anis-Loyd**	**P-value 5%**	**Parameter α**	**Stastistical Anderson Darling**	**P-value 5%**
I	1992-1995	0.790	-3.994	0.000	0.789	0.777	0.497
II	1996-1999	0.683	-0.609	0.542	1.680	0.193	0.991
III	2000-2003	0.465	6.292	0.000	1.924	0.471	0.776
IV	2004-2007	0.374	9.186	0.000	1.943	0.793	0.485
V	2008-2012	0.579	2.681	0.007	1.612	0.472	0.774
Total	1992-2012	0.656	-2.046	0.040	1.503	2.340	0.060

1 / To both the power of the statistical test used is affected by the number of data.
The order of potency of the test is 34% to 250 data, 63% for 500 data and 83% for at least 1000 observations.
Data from Banxico

0.5, with the exception of period II, which shows that the series is not independent over time. In the case of the α-stable distribution, in all periods, we cannot reject the hypothesis that the series is α-stable distributed. It is important to detect if the series presents impulsivity and time dependence. The existence of those two characteristics may lead to errors in the estimates; this is one reason why we cannot use normality and independence assumptions indiscriminately. Both indicators are related in their implications towards determining a range analysis. In the case of presence a fractal behavior, it is convenient to use the fractional Brownian motion. The characterization of a series based on these two parameters will determine the correct methodological analysis and will help to determine how much does a time series is far from both common hypothetical assumptions, namely, independence and normality.

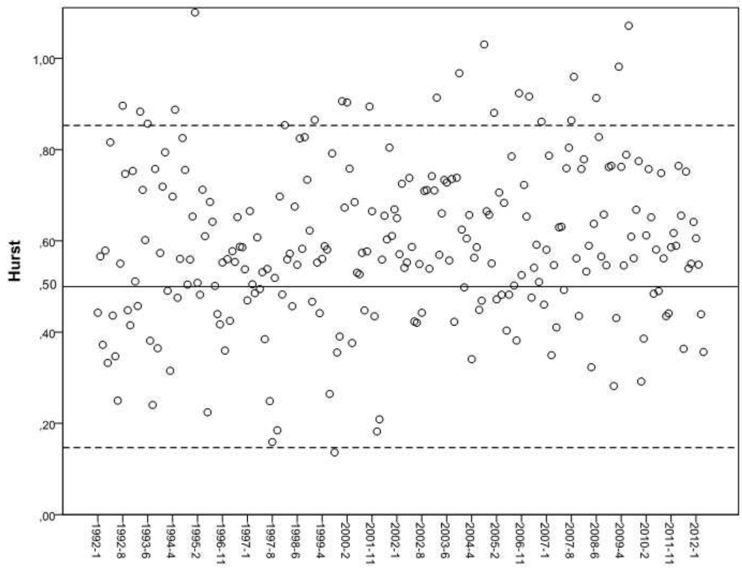
5. A crisis Early Warning Indicator

In this section, the Hurst coefficient and the parameter α will be calculated monthly, quarterly and annually for the daily exchange rates during the period 1992-2012. First, we estimate the Hurst coefficient for different time intervals. Next, we estimate the parameter α and, finally, we discuss the differences between the estimates. With both indicators, we will construct an index of early warning crises by using a multidimensional scaling.

5.1. The Hurst coefficient estimation

The Hurst coefficient is estimated for the series of the Fix Mexican Peso exchange rate against the US Dollar during 1992-2012 with daily data. In particular, we analyze the data independence with the rescaled range methodology using the maximum number of possible partitions with less data loss. The Hurst coefficient shows a persistence and antipersistence in episodes of high volatility. We can observe that the coefficient estimation using monthly partitions is close to 1, suggesting a kind of determinism in the time series; however, there is some evidence pointing out that using the technique of rescaled range in small series, the parameter estimates may be overstated (Hastings and Sugira, 1993). The observed values for monthly data indicates the presence of few values where the Hurst coefficient is consistent with the random walk, H = 0.5, denoted by the solid line. In fact, a lot of them show long memory (70% of data are above the solid line) with little episodes of short memory (30% of values below the solid line).

Figure 1: Hurst coefficient for monthly data (1992-2012)



Source: Author's own elaboration with Banxico data.

For the case of quarterly estimates, the Hurst coefficient values rarely are observed near of the top limit (1). In fact, they are clustered around the white noise solid line, (H = 0.5) but there remains some long memory observations and other observations below the solid line (70% of data shows persistence and 30% antipersistence, it remains the same proportion as in the monthly case).

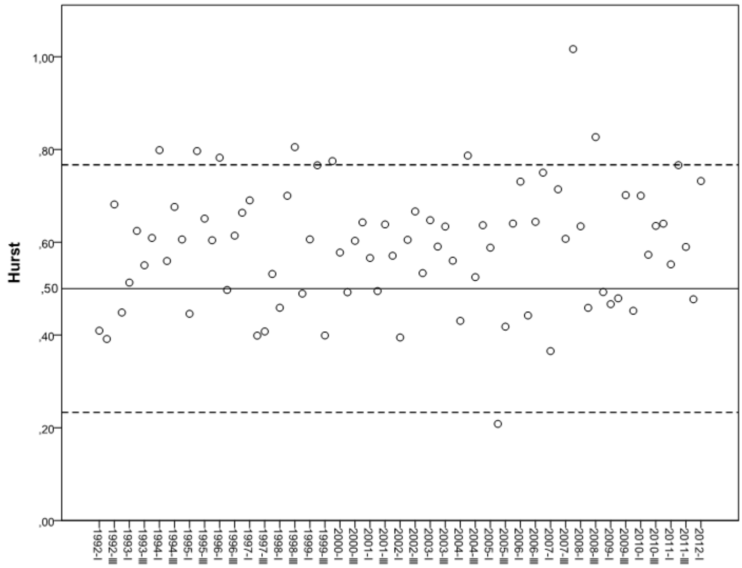
For annual data, Hurst coefficient reflects the varying results in the same sense as in the two previous periodicities but with higher clustering effect. It is clear that the more data is used to estimate the coefficient, the higher will be the robustness of the parameter.

Only in two years, 1999 and 2006, the Hurst coefficient has values close to the white noise. In other periods show persistence and anti-persistence; especially in those years identified as periods of crisis.

5.2. Estimation of the Parameter α

The parameter α is estimated based on the software developed by J. P. Nolan *estable.exe*. The adjusted α-stable distribution parameters were fitted to the Mexican Peso exchange rate against the US Dollar. Particularly the parameter

Figure 2: Hurst coefficient for quarterly data (1992-2012)



Source: Author's own elaboration with Banxico data.

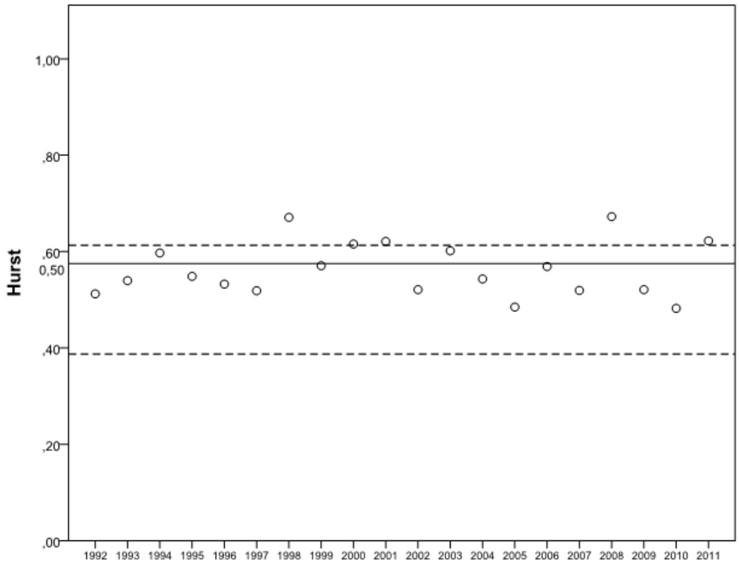
α, will notify if a series is impulsive. It also indicates the presence of outliers or heavy tails; this phenomenon is associated in financial series with high volatility or stress periods. We emphasize that we used the S_1 parameterization [24]. Adjustment parameters of the data are obtained by characterizing the distribution with the three more commonly used methods, namely: quantile, maximum likelihood, and regression (Nolan, 2005). In this case, the regression method is preferred due to its better adjustment of the tails of the distribution.

As it can be seen in Figure 4, there are several observations near to the normal case, which are showed with the solid line on $\alpha = 2$. In this case, the series does not present a high degree of impulsiveness and, therefore, there are not heavy tails on the distribution. It is noteworthy that there are periods where the parameter α is close to the normal case. This allows the differentiation of cases depending on the behavior of the parameter α for the construction of an early warning index. This can be clearly seen in the observations where the alpha parameter is visibly far from 2. Finally, is important to mention that in all cases, there is a stress period in the exchange market as in 1994.

In consistency with the estimated Hurst coefficient, the less volatile periods

[24]More details can be seen in Samorodnitsky and Taqqu, (1994) and Zotolarev (1986).

Figure 3: Hurst coefficient for annual data (1992-2011)



Source: Author's own elaboration with Banxico data.

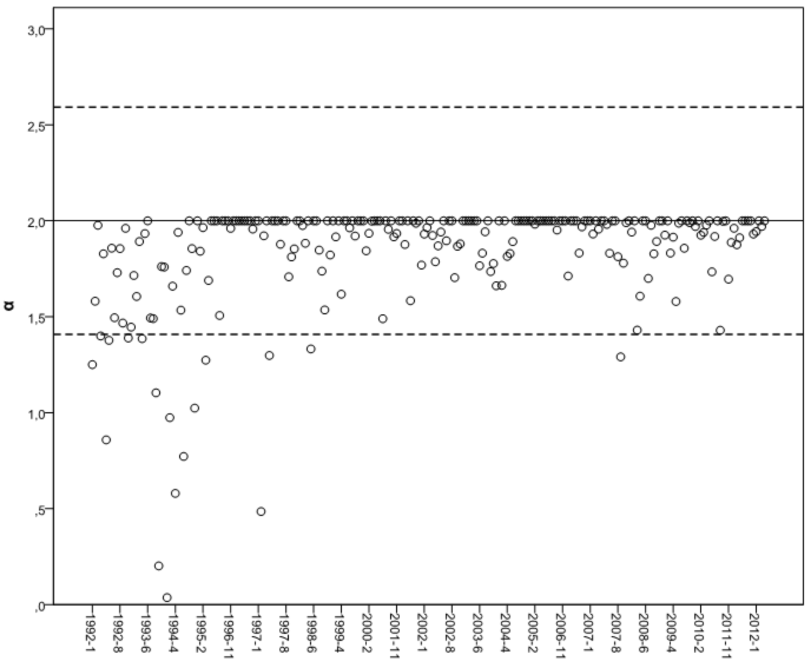
have values close to H = 0.5 and $\alpha = 2$. The cases consistent with the white noise assumption that are less impulsive are associated with these observations. As in the case of the Hurst coefficient, the estimated parameter on a quarterly basis shows less impulsivity that that of monthly observations.

Using annual frequency data, it can be seen a trend towards normality that was significantly broken by the 1994-1995 and 2008 crises when de parameter drops to 1.0 and 1.1 respectively. After that, the annual parameters remain in a range between 1.4 and 2. Finally, we wish to emphasize that the impulsivity periods are exactly the same that presents long memory behavior.

6. Early Warning Indicators

By considering that data with low impulsivity and no dependence over time satisfy, respectively, H = 0.5 and $\alpha = 2$, it is possible to quantify the contrast between these parameters by choosing the Mahalanobis distance as a measure of this dissimilarity. The Mahalanobis ranges from 0 to 1, and it is calculated as the distance between the individual observations and a parameter vector. Ma-

Figure 4: Parameter α for monthly data (1992-2012)



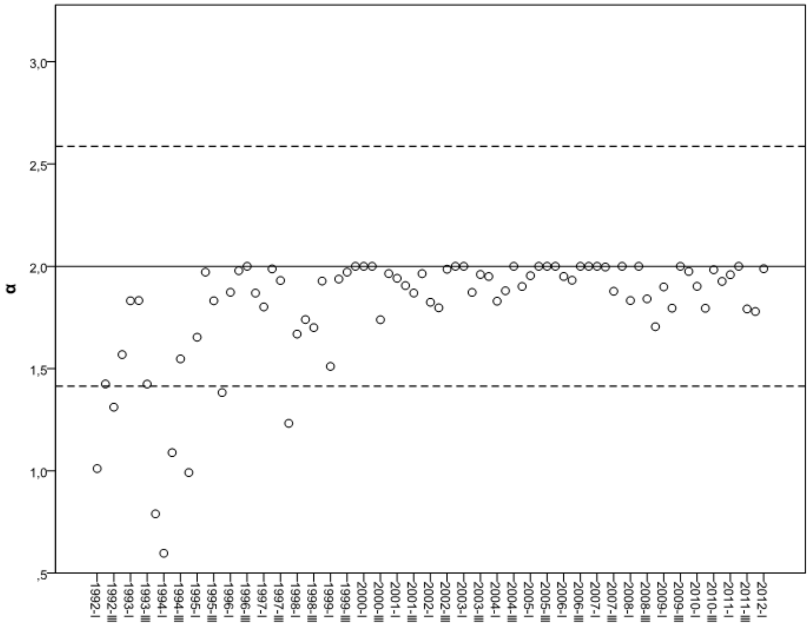
Source: Author's own elaboration with Banxico data.

halanobis dissimilarity was chosen since it meets the following essential properties (Cartera, 1998): non negativity, symmetry, and triangle inequality. It is invariant under nonsingular linear transformations of the variables. Also takes into account the correlations between variables, for two variables is greater than for a single, decreasing with increasing correlation between variables [25]. In order to have a magnitude of how far is the data in each period of the normal case and the independence of the data (or self-similarity), we construct a graph of dissimilarities for both parameters using the Mahalanobis dissimilarity measure estimated from the normal case to the actual data.

As shown in Figure 7, the dissimilarity between the parameters increases in those cases where there are periods of high volatility, which were crisis years. The parameter α is far from to 2 indicating impulsivity in the series. We can see this phenomenon in the crisis of 1995, and in every monthly observation associated to market stress moments. In all cases, the impulsivity measurement

[25]For further details see Mahalanobis (1936), Cartera, (1998), and McLachlan (2004).

Figure 5: Parameter α for quarterly data (1992-2012)



Source: Author's own elaboration with Banxico data.

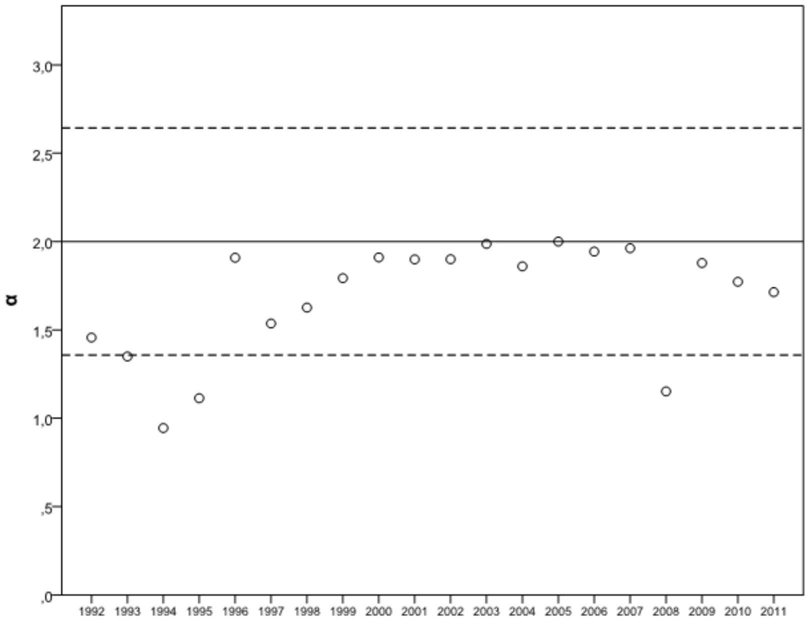
of the alpha parameter corresponds to a high Hurst coefficient.

The graph of dissimilarities allows us to know, in an analytical way, when the normality assumption is not met, and, as a consequence, the normal based estimations or forecasts are neither accurate nor reliable. This happens especially during market stress periods. When analyzing the quarterly estimated dissimilarities, we see that the behavior of Mahalanobis distance is consistent with the monthly estimated dissimilarities. The estimated parameters deviate from the theoretical values in periods of crisis and high volatility of the exchange rate. Again, the departures from the normality assumption can be easily viewed in both the Hurst coefficient and the alpha parameter.

For the annual event dissimilarities, the graph shows a similar trend than in the previous data frequencies. The differences between the estimated parameter and the parameters consistent with the normal and independence assumptions are larger in the years when there is high volatility in the series.

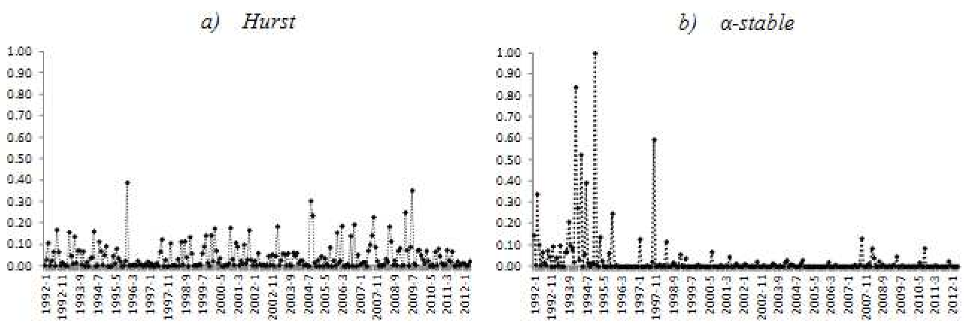
A key discussion element is that the dissimilarities charts provide a scale in which we can see how far the actual parameters are from their theoretical vales.

Figure 6: Parameter α for annual data (1992-2011)



Source: Author's own elaboration with Banxico data.

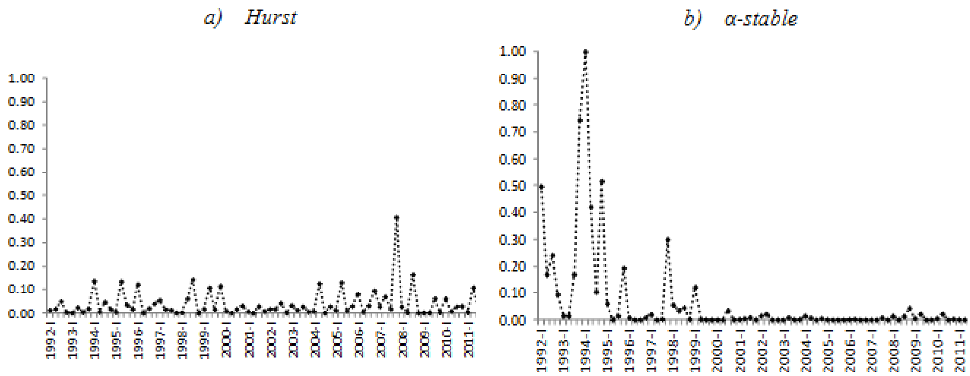
Figure 7: Dissimilarities Charts on a Monthly Basis (1992-2012)



Source: Author's own elaboration with Banxico data.

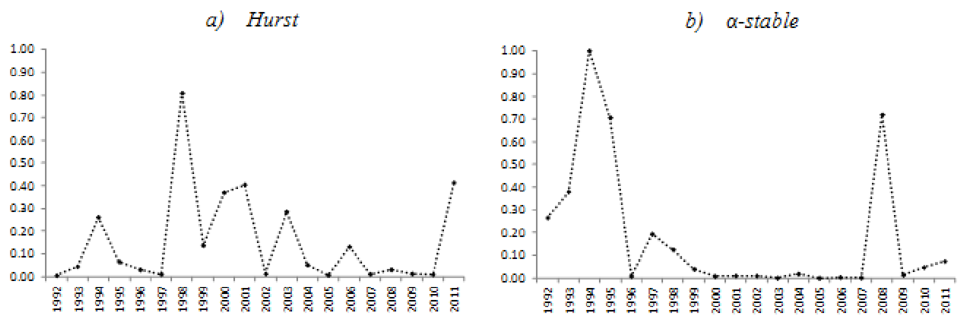
This allows us to set thresholds for financial analysis. Note that in the case of the estimation of both parameters at all frequencies we are not contrasting their statistical significance because, as mentioned in the previous section, the used tests have a sample smaller than 250 data. In this way, it has been shown

Figure 8: Dissimilarities Charts on a Quarterly Basis (1992-2012)



Source: Author's own elaboration with Banxico data.

Figure 9: Dissimilarities Charts on an Annual Basis (1992-2011)

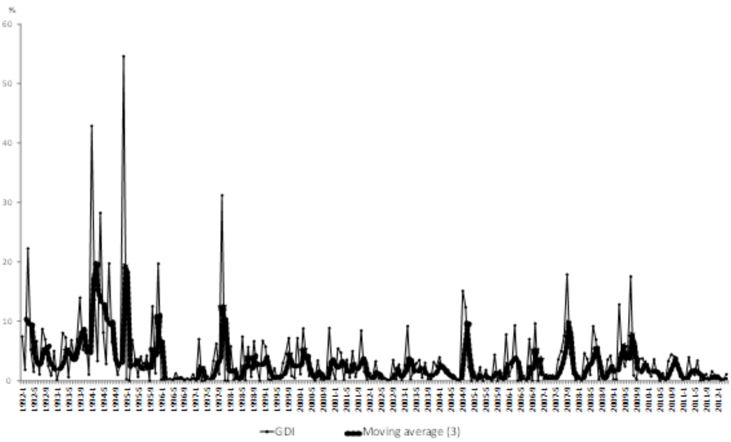


Source: Author's own elaboration with Banxico data.

that the series is characterized by a fractal behavior and that the adjustment to α-stable distribution is consistent for monthly, quarterly and annual data.

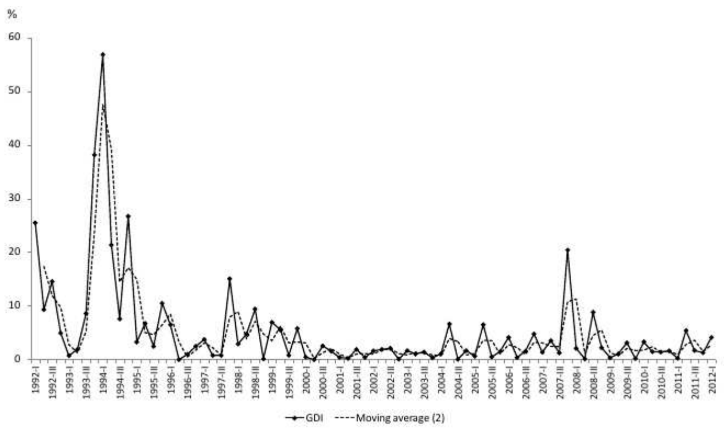
Based on the parameter dissimilarity estimations, we built a Global Index of Dissimilarity (GID). To do this, we use a multidimensional metric scaling that ranges from 0 in the case of normality to 100, the latter in the case of the maximum Mahanalobis distance. This means that we are proposing a normalized index that can be compared only with its own past realizations. This is not really a big shortcoming because if we desire to compare two different time series, we just have to make the comparison using the crude Mahalanobis distance. The use of the normalization index is due to the need of preserving some the intrinsic time series charactheristics that may be beyond normality and memory measurements. In other words, this is a way of not deleting non-analyzed

Figure 10: Monthly Global Dissimilarity Index (1992-2012)



Source: Author's own elaboration with Banxico data.

Figure 11: Quarterly Global Dissimilarity Index (1992-2012)

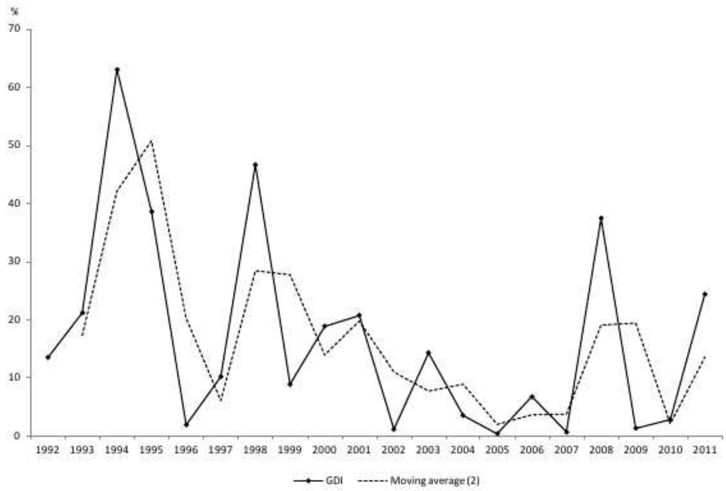


Source: Author's own elaboration with Banxico data.

characteristics of the time series under study.

After analyzing our empirical findings, we can see that the periods of high volatility in the exchange rate show dissimilarities on the Hurst coefficient and the parameter α. This means that they are far from their theoretical values (random walk H = 0.5 and α = 2). In the graph, we can see a significant amount of values corresponding to stress periods. We want to emphasize that the quarterly and yearly indexes shows the same behavior.

Figure 12: Annual Global Dissimilarity Index (1992-2011)



Source: Author's own elaboration with Banxico data.

In the case of estimating the index on a yearly basis, the previously presented behavior confirms the identification of thresholds on which the index peaks. In the years that showed high volatility in the exchange rate, the index alert is triggered.

The proposed indicator is a useful tool providing information on compliance of the data with the assumptions of white noise to the analyst. On the basis of this information every analyst should define their tolerance ranges and thresholds dissimilarity bearable before modifying the methodologies used in the financial analysis.

We calculate the GID in three frequencies, monthly, quarterly and annually in order to analyze the information dynamically. We recognize that the most robust index version is the monthly, followed by the quarterly because more data improves the estimation of the parameters. We recommend the use of this index in order to detect heavy tails and dependence on the time series, our indicator will allow the analyst to assess if his model is the best for the fitting of data, without having to force fit to the normal case. Of course, the tolerance level of the dissimilarity measure depends on the thresholds set by the analyst.

Although this analysis is only considering two major assumptions usually made on financial series, they are of great importance in determining the best method for analysis and prediction. In the case of the presence of self-similarity in series or dependence on the time data it is possible to use the fractional

Brownian motion. Similarly to the case of the assumption of the absence of the normal series that can be modeled by considering the use $\alpha - stable$ distributions to properly represent the presence of heavy tails. Our proposal is a first step towards improving the financial series modeling, taking into account the characteristics of the series itself. Most of empirical researchers are trying to look for information that allows them to leave as much as possible the normal and independence assumptions that can be misleading, usually incomplete. We have showed that the Hurst coefficient and the parameter α are useful as early warning indicators of crises, when the relevant time series is the exchange rate, because they provide a measure of market disparity compared to the theoretical values.

7. Conclusions

The existence of herd behavior, noise traders and complex microeconomic structures in the different economies produce the existence of long memory and impulsiveness of financial time series. These characteristics may lead to incorrect econometric analysis if the independence and normality assumptions are not hold. The proposed methodology gives the researchers a measurement of the accuracy of such assumptions. This paper showed empirical evidence that the proposed index can be used as an early warning alert for exchange rate crisis without making any assumption on the time series probability distribution or the causes of the crisis. This is a major advantage in an open economy that is exposed to many risk sources that works on some complex ways that the economists have not fully understood. The obtained results indicates that the independence and normality assumptions made in the mainstream financial analysis are not always fulfilled, making them of little use to decision making or prognosis, especially in high volatility crisis. Finally, the quantification of the dissimilarity between the values of the time series parameters respect to their theoretical values is important to validate the accuracy of the used analysis tools, especially when the 5% confidence interval margin is reached faster than in the normal case.

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