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# ON SEVERAL TYPES OF CONTINUOUS FUNCTIONS INDUCED BY GENERALIZED $w$-OPEN SETS IN ASSOCIATED $w$-SPACES

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**Abstract:** The purpose of this paper is to introduce the notions of $gw_\tau$-continuous, $gw^*_\tau$-continuous, $gw_\tau$-irresolute, and $gw^*_\tau$-irresolute functions by using $gw_\tau$-open sets between an associated $w_\tau$-spaces, and to study its characterizations and the relationships among them.

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**Key Words:** associated $w_\tau$-space, generalized $w_\tau$-open($gw_\tau$-open), $gw_\tau$-continuous, $gw^*_\tau$-continuous, $gw_\tau$-irresolute, $gw^*_\tau$-irresolute

## 1. Introduction

Siwiec [13] introduced the notions of weak neighborhoods and weak base in a topological space. We introduced the weak neighborhood systems defined by using the notion of weak neighborhoods in [8]. The weak neighborhood system induces a weak neighborhood space which is independent of neighborhood spaces [2] and general topological spaces [1].

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The notions of weak structure, $w$-space, $W$-continuity and $W^{\square}$-continuity were investigated in [9]. In fact, the set of all $g$-closed subsets [3] in a topological space is a kind of weak structure. Moreover, In [10], we introduced the notion of an associated weak space (simply, associated $w_{\tau}$-space) containing a given topology $\tau$. In the similar way introduced by Levine [3] in topological spaces, we introduced the notion of generalize $w_{\tau}$-open sets (generalize $w_{\tau}$-closed sets) [11] in an associated weak space $w_{\tau}$ and studied its properties. In this paper, we are going to introduce the notions of $gw_{\tau}$-continuous, $gw_{\tau}^{\square}$-continuous, $gw_{\tau}$-irresolute and $gw_{\tau}^{\square}$-irresolute functions between an associated $w_{\tau}$-spaces by using generalize $w_{\tau}$-open sets ($gw_{\tau}$-open sets), and to study its characterizations and the relationships among them.

## 2. Preliminaries

**Definition 2.1** ([9]). Let $X$ be a nonempty set. A subfamily $w_X$ of the power set $P(X)$ is called a *weak structure* on $X$ if it satisfies the following:

(1) $\emptyset \in w_X$ and $X \in w_X$.

(2) For $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$.

Then the pair $(X, w_X)$ is called a *w-space* on $X$. Then $V \in w_X$ is called a *w-open* set and the complement of a $w$-open set is a *w-closed* set.

The collection of all $w$-open sets (resp., $w$-closed sets) in a $w$-space $X$ will be denoted by $WO(X)$ (resp., $WC(X)$). We set $W(x) = \{U \in WO(X) : x \in U\}$.

Let $S$ be a subset of a topological space $X$. The closure (resp., interior) of $S$ will be denoted by $clS$ (resp., $intS$). A subset $S$ of $X$ is called a *preopen* set [6] (resp., $\alpha$-open set [12], *semi-open* [4]) if $S \subset int(cl(S))$ (resp., $S \subset int(cl(int(S))), S \subset cl(int(S))$). The complement of a preopen set (resp., $\alpha$-open set, *semi-open*) is called a *preclosed* set (resp., $\alpha$-closed set, *semi-closed*). The family of all preopen sets (resp., $\alpha$-open sets, semi-open sets) in $X$ will be denoted by $PO(X)$ (resp., $\alpha(X), SO(X)$). We know the family $\alpha(X)$ is a topology finer than the given topology on $X$.

Moreover, a subset $S$ of $X$ is said to be $g$-closed [3] if $cl(A) \subset U$ whenever $A \subset U$ and $U$ is open in $X$.

Then the family $GO(X) = \{U \subseteq X : U \text{ is } g\text{-open }\}$, $O(X) = \{U \subseteq X : U \text{ is open }\}$ and $CL(X) = \{F \subseteq X : F \text{ is closed }\}$ are all weak structures on $X$. But $PO(X)$, $GPO(X)$ and $SO(X)$ are not weak structures on $X$. A subfamily $m_X$ of the power set $P(X)$ of a nonempty set $X$ is called a *minimal structure* on $X$ [5] if $\emptyset \in w_X$ and $X \in w_X$. Thus clearly every weak structure is a minimal structure.

Let $X$ be a nonempty set and let $(X, \tau)$ be a topological space. A subfamily $w$ of the power set $P(X)$ is called *an associated weak structure* (simply, $w_\tau$) [10] on $X$ if $\tau \subseteq w$ and $w$ is a weak structure. Then the pair $(X, w_\tau)$ is called *an associated $w$-space* with $\tau$.

Let $(X, w_\tau)$ be an associated $w$-space with a topology $\tau$ and $A \subseteq X$. Then $A$ is called *a generalized $w_\tau$-closed set* (simply, $gw_\tau$-closed set) [11] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $w$-open. Then the union of two $gw_\tau$-closed sets is a $gw_\tau$-closed set, but the intersection of two $gw_\tau$-closed sets is not always $gw_\tau$-closed. And $A$ is called *a generalized $w_\tau$-open set* (simply, $gw_\tau$-open set) if $X - A$ is $gw_\tau$-closed.

Then $A$ is $gw_\tau$-open if and only if $F \subseteq int(A)$ whenever $F \subseteq A$ and $F$ is $w$-closed.

For a subset $A$ of $X$, *$gw_\tau$-closure* of $A$ and *$gw_\tau$-interior* of $A$ are defined as the following:

(1) $gw_\tau C(A) = \cap\{F : A \subseteq F, F \text{ is } gw_\tau\text{-closed}\}$.

(2) $gw_\tau I(A) = \cup\{U : U \subseteq A, U \text{ is } gw_\tau\text{-open}\}$.

**Theorem 2.2** ([11]). *Let $(X, w_\tau)$ be an associated $w$-space with a topology $\tau$ and $A \subseteq X$.*

*(1) If $A$ is $gw_\tau$-open ($gw_\tau$-closed), then $gw_\tau I(A) = A$ ($gw_\tau C(A) = A$).*

*(2) If $A \subseteq B$, then $gw_\tau I(A) \subseteq gw_\tau I(B)$; $gw_\tau C(A) \subseteq gw_\tau C(B)$.*

*(3) $gw_\tau C(X - A) = X - gw_\tau I(A)$; $gw_\tau I(X - A) = X - gw_\tau C(A)$.*

*(4) $x \in gw_\tau I(A)$ if and only if there exists a $gw_\tau$-open set $U$ containing $x$ such that $U \subseteq A$.*

*(5) $x \in gw_\tau C(A)$ if and only if $A \cap V \neq \emptyset$ for all $gw_\tau$-open set $V$ containing $x$.*

*(6) $\emptyset = gw_\tau C(\emptyset)$; $X = gw_\tau I(X)$.*

*(7) $gw_\tau I(A) \subseteq A \subseteq gw_\tau C(A)$.*

*(8) $gw_\tau C(A \cup B) = gw_\tau C(A) \cup gw_\tau C(B)$; $gw_\tau I(A \cap B) = gw_\tau I(A) \cap gw_\tau I(B)$.*

*(9) $gw_\tau C(gw_\tau C(A)) = gw_\tau C(A)$; $gw_\tau I(gw_\tau I(A)) = gw_\tau I(A)$.*

## 3. $gw_\tau$-Continuity; $gw^*_\tau$-Continuity

**Definition 3.1.** Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a function in two associated $w$-spaces with $\tau$ and $\mu$. Then $f$ is said to be

(1) *$gw_\tau$-continuous* if for $x \in X$ and for each open set $V$ containing $f(x)$, there is a $gw_\tau$-open set $U$ containing $x$ such that $f(U) \subseteq V$:

(2) *$gw^{\square}_{\tau}$-continuous* if for every open set $V$ in $Y$, $f^{-1}(V)$ is a $gw_{\tau}$-open set in $X$.

Obviously we obtain the following theorem:

**Theorem 3.2.** *Every $gw^{\square}_{\tau}$-continuous function is $gw_{\tau}$-continuous.*

The following example supports that the converse of the above theorem is not true in general.

**Example 3.3.** Let $X = \{a, b, c, d\}$, a topology $\tau = \{\emptyset, \{b\}, X\}$ and a $w$-structure $w_X = \{\emptyset, \{a, c\}, \{a\}, \{b\}, \{c\}, \{a, d\}, X\}$ in $X$. Note: The set of all $gw_{\tau}$-closed sets is $\{\emptyset, X, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Now, consider a function $f : (X, w_{\tau}) \to (X, w_{\tau})$ defined by $f(a) = f(c) = a$; $f(b) = f(d) = b$. For $b, d \in X$ and for the only open set $V = \{b\}$ containing $f(b) = f(d)$, since $A = \{b\}$ and $B = \{d\}$ are $gw_{\tau}$-open sets containing $b$ and $d$, respectively, so obviously $f$ is $gw_{\tau}$-continuous. But $f^{-1}(V) = \{b, d\} = A \cup B$ is not $gw_{\tau}$-open, so $f$ is not $gw^{\square}_{\tau}$-continuous.

We recall that: Let $f : (X, w_{\tau}) \to (Y, \mu)$ be a function on an associated $w$-space with $\tau$ and a topological space $(Y, \mu)$. Then $f$ is said to be

(1) *WO-continuous* [10] if for $x \in X$ and for each open set $V$ containing $f(x)$, there is a $w$-open set $U$ containing $x$ such that $f(U) \subseteq V$;

(2) *WK-continuous* [10] if for every open set $V$ in $Y$, $f^{-1}(V)$ is a $w$-open set in $X$.

Obviously, the following things are obtained:

**Theorem 3.4.** *(1) Every $WO$-continuous function is $gw_{\tau}$-continuous.*

*(2) Every $WK$-continuous function is $gw^{\square}_{\tau}$-continuous.*

The following example supports that the converses of the above theorem are not true in general.

**Example 3.5.** (1) Consider the function $f$ defined in Example 3.3. Then the $gw_{\tau}$-continuous function $f$ is not $WO$-continuous since for $d \in X$ and for the only open set $V = \{b\}$ containing $f(d)$, there is no any $w$-open set $U$ containing $d$ such that $f(U) \subseteq V$.

(2) Let $X = \{a, b, c, d\}$, a topology $\tau = \{\emptyset, \{b\}, X\}$ and a $w$-structure $w_X = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, X\}$ in $X$. Consider a function $f : (X, w_{\tau}) \to (X, w_{\tau})$ defined by $f(a) = f(c) = a$; $f(b) = f(d) = b$. Then $f^{-1}(\{b\}) = \{b, d\}$ is $gw_{\tau}$-open, so $f$ is $gw^{\square}_{\tau}$-continuous. But since $\{b, d\}$ is not $w$-open, $f$ is not $WK$-continuous.

**Remark 3.6.** For a function from an associated $w_{\tau}$-space with $\tau$ to a topological space, we have the following diagram:

$$\begin{array}{ccccc} \text{Continuity} \Rightarrow & WK\text{-continuity} & \Rightarrow & WO\text{-continuity} \\ & \Downarrow & & \Downarrow \\ & gw_\tau^{\star}\text{-continuity} & \Rightarrow & gw_\tau\text{-continuity} \end{array}$$

**Theorem 3.7.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a function on an associated $w$-space with $\tau$ and a topological space $(Y, \mu)$. Then the following statements are equivalent:*
*(1) $f$ is $gw_\tau$-continuous.*
*(2) $f(gw_\tau C(A)) \subseteq cl(f(A))$ for $A \subseteq X$.*
*(3) $gw_\tau C(f^{-1}(V)) \subseteq f^{-1}(cl(V))$ for $V \subseteq Y$.*
*(4) $f^{-1}(int(V)) \subseteq gw_\tau I(f^{-1}(V))$ for $V \subseteq Y$*

*Proof.* (1) $\Rightarrow$ (2) Let $x \in gw_\tau C(A)$. Suppose that $f(x)$ is not in $cl(f(A))$; then there exists an open set $V$ containing $f(x)$ such that $V \cap f(A) = \emptyset$. By $gw_\tau$-continuity, there is a $gw_\tau$-open set $U$ containing $x$ such that $f(U) \subseteq V$, and so $f(U) \cap f(A) = \emptyset$. Hence $U \cap A = \emptyset$, which is a contradiction to $x \in gw_\tau C(A)$. This implies that $f(gw_\tau C(A)) \subseteq cl(f(A))$.

(2) $\Rightarrow$ (3) Let $A = f^{-1}(B)$ for $B \subseteq Y$. From (2), it follows $f(gw_\tau C(A)) \subseteq cl(f(A)) = cl(f(f^{-1}(B))) \subseteq cl(B)$. It implies that $gw_\tau C(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

(3) $\Rightarrow$ (4) By Theorem 2.2, it is obvious.

(4) $\Rightarrow$ (1) Let $V$ be any open set containing $f(x)$ for each $x \in X$. Then since $f(x) \in int(V)$, and by (4), $x \in f^{-1}(int(V)) \subseteq gw_\tau I(f^{-1}(V))$. There exists a $gw_\tau$-open set $U$ such that $x \in U \subseteq gw_\tau I(f^{-1}(V)) \subseteq f^{-1}(V)$. Hence, $f$ is $gw_\tau$-continuous. □

**Corollary 3.8.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a function on an associated $w$-space with $\tau$ and a topological space $(Y, \mu)$. Then the following statements are equivalent:*
*(1) $f$ is $gw_\tau$-continuous.*
*(2) $f^{-1}(V) = gw_\tau I(f^{-1}(V))$ for every open set $V \in Y$.*
*(3) $f^{-1}(B) = gw_\tau C(f^{-1}(B))$ for every closed set $B \subseteq Y$.*

*Proof.* From Theorem 2.2, it is obvious. □

**Theorem 3.9.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a function on an associated $w$-space and a topological space $(Y, \mu)$. Then $f$ is $gw_\tau^{\star}$-continuous if and only if for every closed set $F$ in $Y$, $f^{-1}(F)$ is $gw_\tau$-closed in $X$.*

*Proof.* It is obvious. □

Let $(X, w_\tau)$ be an associated $w$-space with a topology $\tau$. Let $gW_\tau(x)$ denote the set of all $gw_\tau$-open set containing $x$ in $X$, and $O(x)$ denote the set of all open set containing $x$ in $X$.

A collection $\mathcal{H}$ of subsets of $X$ is called an *m-family* [7] on $X$ if $\cap\mathcal{H} \neq \emptyset$. Let $\mathcal{H}$ be an $m$-family on $X$. Then we say that an $m$-family $\mathcal{H}$ *$gw_\tau$-converges* to $x \in X$ if $\mathcal{H}$ is finer than $gW_\tau(x)$ i.e., $gW_\tau(x) \subseteq \mathcal{H}$. Let $f : X \to Y$ be a function; then it is obvious $f(\mathcal{H}) = \{f(F) : F \in \mathcal{H}\}$ is an $m$-family on $Y$. If $\mathcal{F}$ is a filter base, we denote by $< \mathcal{F} >$ the filter generated by $\mathcal{H}$.

**Theorem 3.10.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a function on an associated $w_\tau$-space and a topological space $(Y, \mu)$. Then if $f$ is $gw_\tau$-continuous, then for an $m$-family $\mathcal{H}$ $gw_\tau$-converging to $x \in X$, a filter $< f(\mathcal{H}) >$ converges to $f(x)$.*

*Proof.* Suppose $f$ is $gw_\tau$-continuous and $\mathcal{H}$ is an $m$-family $gw_\tau$-converging to $x \in X$. By $gw_\tau$-continuity, for an open set $V$ containing $f(x)$, there exists a $gw_\tau$-open set $U$ containing $x$ such that $f(U) \subseteq V$. Since $f(gW_\tau(x)) \subseteq f(\mathcal{H})$, $V \in < f(\mathcal{H}) >$ i.e., $O(f(x)) \subseteq < f(\mathcal{H}) >$. Hence the filter $< f(\mathcal{H}) >$ converges to $f(x)$. $\square$

**Theorem 3.11.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a bijective function in an associated $w$-space and a topological space $(Y, \mu)$. Then $f$ is $gw^{\square}_\tau$-continuous iff for an $m$-family $\mathcal{H}$ $gw_\tau$-converging to $x \in X$, the filter $< f(\mathcal{H}) >$ converges to $f(x)$.*

*Proof.* Suppose $f$ is $gw^{\square}_\tau$-continuous and $\mathcal{H}$ is an $m$-family $gw_\tau$-converging to $x \in X$. By hypothesis and surjectivity, we get $O(f(x)) \subseteq f(gW_\tau(x)) \subseteq f(\mathcal{H})$, so that a filter $< f(\mathcal{H}) >$ converges to $f(x)$.

For the converse, let $U \in O(f(x))$ for $U \subset Y$. Since the family $gW_\tau(x)$ clearly $gw_\tau$-converges to $x$, by hypothesis, we get $O(f(x)) \subseteq < f(gW_\tau(x)) >$ for $x \in X$. From $f$ is injectivity, it follows $f^{-1}(U) \in gW_\tau(x)$. $\square$

**Corollary 3.12.** *Let $f : (X, w_\tau) \to (Y, \mu)$ be a function in an associated $w$-space and a topological space $(Y, \mu)$. If $gw_\tau = w$, then $f$ is $WK$-continuous iff for an $m$-family $\mathcal{H}$ $w$-converging to $x \in X$, the filter $< f(\mathcal{H}) >$ converges to $f(x)$.*

## 4. $gw_\tau$-Irresolute; $gw^*_\tau$-Irresolute

**Definition 4.1.** Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a function on two associated $w$-spaces with $\tau$ and $\mu$. Then $f$ is said to be *$gw^{\square}_\tau$-irresolute* if for every $gw_\tau$-open set $V$ in $Y$, $f^{-1}(V)$ is $gw_\tau$-open in $X$.

**Theorem 4.2.** *Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a function on two associated $w$-spaces with $\tau$ and $\mu$. $f$ is $gw_\tau^\star$-irresolute if and only if for every $gw_\mu$-closed set $F$ in $Y$, $f^{-1}(F)$ is $gw_\tau$-closed in $X$.*

*Proof.* It is obvious. □

**Theorem 4.3.** *Every $gw_\tau^\star$-irresolute function is $gw_\tau^\star$-continuous.*

From he following example, we can can that the converse of the above theorem is not true in general.

**Example 4.4.** Let $X = \{a, b, c, d\}$ and a topology $\tau = \{\emptyset, \{a, b\}, X\}$. Consider a $w$-structures $w = \{\emptyset, \{a, b\}, \{b\}, \{b, c\}, X\}$ in $X$. Let $f : (X, w_\tau) \to (X, w_\tau)$ be a function defined as follows $f(a) = b; f(b) = a; f(c) = d : f(d) = c$. Then $f$ is $gw_\tau^\star$-continuous by Theorem 3.9, since $f^{-1}(\{c, d\}) = \{c, d\}$ for the only closed set $\{c, d\}$ in $(X, \tau)$. But for a $gw_\tau$-closed set $\{a, d\}$ in an associated $w$-space, $f^{-1}(\{a, d\}) = \{b, c\}$ is not $gw_\tau$-closed in the associated $w$-space. So, by Theorem 4.3, $f$ is not $gw_\tau^\star$-irresolute.

**Theorem 4.5.** *Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a bijective function on two associated $w$-spaces with $\tau$ and $\mu$. Then $f$ is $gw_\tau^\star$-irresolute iff for an $m$-family $\mathcal{H}$ $gw_\tau$-converging to $x \in X$, $f(\mathcal{H})$ $gw_\mu$-converges to $f(x)$.*

*Proof.* Suppose $f$ is $gw_\tau^\star$-irresolute and $\mathcal{H}$ is an $m$-family $gw_\tau$-converging to $x \in X$. Then $f(gW_\tau(x)) \subseteq f(\mathcal{H})$ and from surjectivity of $f$, $gW_\mu(f(x)) \subseteq f(gW_\tau(x)) \subseteq f(\mathcal{H})$. So, $f(\mathcal{H})$ $gw_\mu$-converges to $f(x)$.

Conversely, let $U \in gW_\mu(f(x))$ for $U \subseteq Y$. Since $gW_\tau(x)$ $gw_\tau$-converges to $x$, by hypothesis, we get $gW_\mu(f(x)) \subseteq f(gW_\tau(x))$ for $x \in X$. From injectivity of $f$, we have $f^{-1}(U) \in gW_\tau(x)$. □

**Definition 4.6.** Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a function on two associated $w$-spaces with $\tau$ and $\mu$. Then $f$ is said to be *$gw_\tau$-irresolute* if for $x \in X$ and for each $gw_\mu$-open set $V$ containing $f(x)$, there is $gw_\tau$-open set $U$ containing $x$ such that $f(U) \subseteq V$.

**Theorem 4.7.** *(1) Every $gw_\tau^\star$-irresolute function is $gw_\tau$-irresolute.*
*(2) Every $gw_\tau$-irresolute is $gw_\tau$-continuous.*

The following example supports that the converses of the above theorem are not true in general.

**Example 4.8.** (1) In Example 3.3, we can easily explain that the function $f$ is $gw_\tau$-irresolute but not $gw_\tau^\star$-irresolute.

(2) Let $X = \{a, b, c, d\}$ and a topology $\tau = \{\emptyset, \{a, b\}, X\}$. Consider a $w$-structures $w = \{\emptyset, \{a, b\}, \{b\}, \{b, c\}, \{a, b, d\}, X\}$ in $X$. Note that the following all sets are $gw_\tau$-open: $\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{b, d\}, X$.

Let $f : (X, w_\tau) \to (X, w_\tau)$ be a function defined as follows $f(a) = b; f(b) = a; f(c) = f(d) = d$. Then obviously $f$ is $gw_\tau$-continuous. For $c \in X$ and for a $gw_\tau$-open set $V = \{d\}$ containing $f(c) = d$, there is no a $gw_\tau$-open set $U$ containing $c$ such that $f(U) \subseteq V = \{d\}$. So, $f$ is not $gw_\tau$-irresolute.

From Remark 3.6 and the above theorems, we have the following diagram:

$$\begin{array}{ccccc}
\text{continuity} \Rightarrow & WK\text{-continuity} & \Rightarrow & WO\text{-continuity} \\
& \Downarrow & & \Downarrow \\
& gw_\tau^{\square}\text{-continuity} & \Rightarrow & gw_\tau\text{-continuity} \\
& \Uparrow & & \Uparrow \\
& gw_\tau^{\square}\text{-irresolute} & \Rightarrow & gw_\tau\text{-irresolute}
\end{array}$$

**Theorem 4.9.** *Let $f : (X, w_\tau) \to (Y, w_\mu)$ be a function on two associated $w$-spaces with $\tau$ and $\mu$. Then the following statements are equivalent:*

*(1) $f$ is $gw_\tau$-irresolute.*

*(2) $f(gw_\tau C(A)) \subseteq gw_\mu C(f(A))$ for $A \subseteq X$.*

*(3) $gw_\tau C(f^{-1}(V)) \subseteq f^{-1}(gw_\mu C(V))$ for $V \subseteq Y$.*

*(4) $f^{-1}(gw_\mu I(V)) \subseteq gw_\tau I(f^{-1}(V))$ for $V \subseteq Y$*

*Proof.* It is similar to the proof of Theorem 3.7. □

**Corollary 4.10.** *Let $f : (X, w_X) \to (Y, w_Y)$ be a function between $w$-spaces. Then the following statements are equivalent:*

*(1) $f$ is $gw_\tau$-irresolute.*

*(2) $f^{-1}(V) = gw_\tau I(f^{-1}(V))$ for every $gw_\mu$-open set $V \in Y$.*

*(3) $f^{-1}(B) = gw_\tau C(f^{-1}(B))$ for every $gw_\mu$-closed set $B \subseteq Y$.*

*Proof.* From Theorem 2.2 and Theorem 4.9, it is obvious. □

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