ELZAKI TRANSFORM FOR ROTATING FLUID

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Abstract: In this article, the flow of an incompressible viscous fluid bounded by an infinite flat plate of the rotating fluid is considered. Elzaki transform method is employed to transform the governing partial differential equations into ordinary ones, which are then solved analytically using Laplace transform method by duality principle. Comparative study have done for Laplace transform method and perturbation technique through skin friction results. We find that better results are obtained by Elzaki transform method.

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Key Words: rotation parameter, Elzaki transform, skin friction, perturbation technique

1. Introduction

Rotating fluid flows have been encountered in many areas, such as aerospace science, lubrication and the flow in turbo machinery. Many investigators have discussed this rotating fluid phenomenon from different feature. Gupta et al[1] analysed the incompressible fluid by an infinite flat with uniform angular velocity by Laplace transform. Vennila [2] studied the rotating fluid using

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perturbation technique. Nazar [3] studied the boundary layer flow due to a stretching surface in a rotating fluid. Cellular convection with finite amplitude in a rotating fluid have been studied by veronis. Most of the methods mentioned above are based on some analytical method and numerical analysis. There still seems to be a need for the development of analytical method to study the behaviour of the rotating fluid. Prem Kiran et al [5] analysed solution of one dimensional ground water recharge through spreading through Elzaki transform. Applications of Elzaki transform to ordinary differential equations and partial differential equations was discussed by Pallavi et al [6]. Tarig et al [7] studied the connection between Laplace and Elzaki transforms. Time shifting theorem and the convolution for Elzaki transform was discussed by Joon Kin [8]. Elzaki transform applications to partial differential equations was presented by Tarig et al [9]. Tarig et al [10] investigated some fundamental properties of Elzaki transform the boundary layer equations are solved by Elzaki transform method and the inverse can be obtained from Laplace transform method by the property of duality. This paper deals with skin friction results obtained in both techniques.

2. Mathematical Analysis

The equations of motion for the unsteady flow in the usual notation with respect to the rotating frame are given by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega v, \quad (1)$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial z^2} + 2\Omega u, \quad (2)$$

where u, v are the velocity components and the last term in equations (1) and (2) represent the Coriolis force.

The initial and boundary conditions are given by

$$\begin{cases} u = v = 0 & \text{at } t \leq 0, \\ u = ct, v = 0 & \text{at } z = 0 \text{ for } t > 0, \\ u \rightarrow 0, v \rightarrow 0 & \text{as } z \rightarrow \infty \text{ for } t > 0, \end{cases} \quad (3)$$

where $c(> 0)$ is a constant. Introducing the dimensionless quantities

$$\left. \begin{aligned} U &= \frac{u}{(\nu c)^{1/3}} \quad V = \frac{v}{(\nu c)^{1/3}} \quad Z = z \left(\frac{c}{\nu^2} \right)^{1/3}, \\ T &= t \left(\frac{c^2}{\nu} \right)^{1/3}, \quad \Omega_1 = \Omega \left(\frac{c}{\nu^2} \right)^{1/3}. \end{aligned} \right\} \quad (4)$$

The equations (1) and (2) take the following form

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Z^2} + 2\Omega_1 V, \quad (5)$$

$$\frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial Z^2} - 2\Omega_1 U. \quad (6)$$

subject to the initial and boundary conditions

$$\left. \begin{aligned} U = V = 0 & \quad \text{at } t \leq 0, \\ U = T, V = 0 & \quad \text{at } Z = 0 \text{ for } T > 0, \\ U \rightarrow 0, V \rightarrow 0 & \quad \text{as } Z \rightarrow \infty \text{ for } T > 0. \end{aligned} \right\} \quad (7)$$

Equations (5) and (6) can be combined to give

$$\frac{\partial W}{\partial T} = \frac{\partial^2 W}{\partial Z^2} - 2i\Omega_1 W, \quad (8)$$

where $W(Z, T) = U + iV$.

Let $\bar{T}(z, v)$ be the Elzaki transform of $W(Z, T)$, i.e. $E[W(Z, T)] = \bar{T}(z, v)$.

Then taking the Elzaki transform on both of equation (8), we have

$$\frac{\bar{T}(z, v)}{v} - vW(z, 0) = \frac{d^2 \bar{T}}{dz^2} - 2i\Omega_1 \bar{T}(z, v). \quad (9)$$

Using condition (7) in (9), we obtain

$$\frac{d^2 \bar{T}}{dz^2} - 2i\Omega_1 \bar{T}(z, v) - \frac{\bar{T}(z, v)}{v} = 0, \quad (10)$$

$$\frac{d^2 \bar{T}}{dz^2} - \left(2i\Omega_1 + \frac{1}{v}\right) \bar{T} = 0. \quad (11)$$

On solving the above equation, we get

$$\bar{T}(z, v) = Ae^{\left(\frac{1}{v} + 2i\Omega_1\right)^{1/2} Z} + Be^{-\left(\frac{1}{v} + 2i\Omega_1\right)^{1/2} Z}, \quad (12)$$

where

$$\bar{T}(z, v) = v \int_0^\infty e^{\frac{t}{v}} W(Z, T) dt$$

represents the Elzaki transform of $W(Z, T)$.

Applying Elzaki transform of the boundary conditions, we obtain

$$\bar{T}(0, v) = v^3, \quad \bar{T}(z, 0) = 0 \text{ and } \bar{T}(\infty, 0) = 0. \quad (13)$$

We apply equations (13) in (12), we have $A = 0, B = v^3$.

Substituting these values in (12), we obtain

$$\overline{T}(z, v) = v^3 \exp \left[- \left(\frac{1}{v} + 2i\Omega_1 \right)^{1/2} z \right]. \quad (14)$$

By duality property of Elzaki and Laplace transform

$$T(z, v) = vF \left(\frac{1}{v} \right), \quad F(s) = sT \left(\frac{1}{s} \right). \quad (15)$$

Applying (15) in (14), it can be converted into

$$\overline{W}(z, s) = \frac{1}{s^2} \exp \left[-(s + 2i\Omega_1)^{1/2} z \right], \quad (16)$$

where $\overline{W}(z, s)$ is the Laplace transform of $W(Z, T)$.

Using Hetnarski's algorithm for generating inverse Laplace transform, we obtain from (16),

$$W(Z, T) = \left. \begin{aligned} & \left[\frac{T}{2} - \frac{Z}{4(2i\Omega_1)^{1/2}} \right] \exp \left[-z(2i\Omega_1)^{1/2} \right] \\ & \times \operatorname{erfc} \left[\frac{Z}{2T^{1/2}} - (2i\Omega_1 T)^{1/2} \right] \\ & + \left[\frac{T}{2} - \frac{Z}{4(2i\Omega_1)^{1/2}} \right] \exp \left[z(2i\Omega_1)^{1/2} \right] \\ & \times \operatorname{erfc} \left[\frac{Z}{2T^{1/2}} + (2i\Omega_1 T)^{1/2} \right], \end{aligned} \right\} \quad (17)$$

where $\operatorname{erfc}(x)$ is the complementary error function defined by

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x).$$

Using perturbation technique, we obtain

$$W(Z, T) = \frac{T}{1 + e^{i\omega T}} \left[e^{-(2i\Omega_1)^{1/2} Z} + e^{i\omega T} e^{-(2i\Omega_1 + i\omega)^{1/2} Z} \right]. \quad (18)$$

From the Table 1, it is clean that the skin friction results increase when we increase the time and rotation parameter for both techniques. Also it is noted that the skin friction results are same in both direction.

		Laplace Transform Method		Perturbation Technique	
T	Ω_1	τ_x	τ_y	τ_x	τ_y
0.20	0.40	0.50506	0.02690	0.1269	0.1269
0.20	0.60	0.50559	0.04034	0.1552	0.1552
0.20	0.80	0.50635	0.05375	0.1792	0.1792
0.40	0.80	0.72331	0.15136	0.3583	0.3583
0.60	0.80	0.90040	0.27607	0.5375	0.5375

Table 1: Comparison of skin friction components with Ω_1 and time

3. Conclusion

The skin friction components are considered as a table values for the rotating fluid by two different techniques. It is observed that skin friction components increases when we increase time and rotation parameter. The important findings of the analysis of the results in the problem showed that the skin friction components obtained from perturbation technique is less than from Laplace transform method. Finally, we conclude that Elzaki transform method give better results than other methods.

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