**COMMON RANDOM FIXED POINT THEOREMS OF
COMPATIBLE MAPPINGS FOR RANDOM MAPPINGS
IN MULTIPLICATIVE METRIC SPACES**

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Abstract: In this paper, we introduce the notions of compatible mappings in framework of random fixed points in multiplicative metric spaces and using these notions we prove common random fixed point theorems for these mappings.

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1. Introduction and Preliminaries

It is well know that the set of positive real numbers $\mathbb{R}_+$ is not complete according to the usual metric. To overcome this problem, in 2008, Bashirov et al. [1]

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studied the multiplicative calculus and defined a new distance so called multiplicative distance. By using this idea, Özavsar and Çevikel [5] introduced the concept of multiplicative metric spaces and studied some topological properties in such spaces.

Definition 1.1. ([1]) Let X be a nonempty set. A multiplicative metric is a mapping $d : X \times X \rightarrow \mathbb{R}_+$ satisfying the following conditions:

- (i) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$;
- (ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;
- (iii) $d(x, y) \leq d(x, z) \cdot d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Then the mapping d together with X , that is, (X, d) is a multiplicative metric space.

Example 1.2. ([5]) Let $\mathbb{R}_+^n$ be the collection of all n -tuples of positive real numbers. Let $d^* : \mathbb{R}_+^n \times \mathbb{R}_+^n \rightarrow \mathbb{R}$ be defined as follows:

$$d^*(x, y) = \left| \frac{x_1}{y_1} \right|^* \cdot \left| \frac{x_2}{y_2} \right|^* \cdots \left| \frac{x_n}{y_n} \right|^*,$$

where $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n) \in \mathbb{R}_+^n$ and $|\cdot|^* : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined by

$$|a|^* = \begin{cases} a & \text{if } a \geq 1, \\ \frac{1}{a} & \text{if } a < 1. \end{cases}$$

Then it is obvious that all conditions of a multiplicative metric are satisfied. Therefore $(\mathbb{R}_+^n, d^*)$ is a multiplicative metric space.

Remark 1.3. ([7]) We note that multiplicative metrics and metric spaces are independent.

One can refer to [5] for detailed multiplicative metric topology.

Definition 1.4. Let (X, d) be a multiplicative metric space. Then a sequence $\{x_n\}$ in X said to be

(1) a *multiplicative convergent* to x if for every multiplicative open ball $B_\epsilon(x) = \{y \mid d(x, y) < \epsilon\}$, $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $x_n \in B_\epsilon(x)$ for all $n \geq N$, that is, $d(x_n, x) \rightarrow 1$ as $n \rightarrow \infty$.

(2) a *multiplicative Cauchy sequence* if for all $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \epsilon$ for all $m, n \geq N$, that is, $d(x_n, x_m) \rightarrow 1$ as $n, m \rightarrow \infty$.

(3) We call a multiplicative metric space *complete* if every multiplicative Cauchy sequence in it is multiplicative convergent to $x \in X$.

In 2012, Özavsar and Çevikel [5] gave the concept of multiplicative contraction mapping and proved some fixed point theorem of such mappings in a complete multiplicative metric spaces.

Definition 1.5. Let f be a mapping of a multiplicative metric space (X, d) into itself. Then f is said to be a *multiplicative contraction* if there exists a real number $\lambda \in [0, 1)$ such that

$$d(fx, fy) \leq d^\lambda(x, y) \quad \text{for all } x, y \in X.$$

2. Properties of Compatible Mappings for Random Mappings

Now we introduce the following.

Definition 2.1. Let (X, d) be a multiplicative metric space and $F : \mathbb{R} \times X \rightarrow X$ be a mapping, where X is a nonempty set. Then a mapping $g : \mathbb{R} \rightarrow X$ is said to be a *random fixed point* of the function F if $F(t, g(t)) = g(t)$ for all $t \in \mathbb{R}$.

Example 2.2. Let $X = \mathbb{R}$. Define $T : \mathbb{R} \times X \rightarrow X$ by $T(t, x) = \frac{t+5x}{6}$ and a mapping $g : \mathbb{R} \rightarrow X$ defined by $g(t) = t$ for every $t \in \mathbb{R}$. Then T has a unique random fixed point in X

Next we introduce the notions of (weakly) commuting and compatible mappings for random mappings in multiplicative metric spaces as follows:

Definition 2.3. Let (X, d) be a multiplicative metric space. Two mappings A and $B : \mathbb{R} \times X \rightarrow X$ are called

(1) *commuting* for each $t \in \mathbb{R}$ if

$$A(t, B(t, g(t))) = B(t, A(t, g(t)))$$

for all $t \in \mathbb{R}$, where $g : \mathbb{R} \rightarrow X$ is a mapping;

(2) *weakly commuting* for each $t \in \mathbb{R}$ if

$$d(A(t, B(t, g(t))), B(t, A(t, g(t)))) \leq d(A(t, g(t)), B(t, g(t)))$$

for all $t \in \mathbb{R}$, where $g : \mathbb{R} \rightarrow X$ is a mapping;

(3) *compatible* for each $t \in \mathbb{R}$ ([6]) if

$$\lim_{n \rightarrow \infty} d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) = 1,$$

whenever $\lim_{n \rightarrow \infty} A(t, g_n(t)) = \lim_{n \rightarrow \infty} B(t, g_n(t))$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings.

Remark 2.4. Notice that commuting mappings are obviously weakly commuting. However, converse need not be true, in general, as follows.

Example 2.5. Let $X = \mathbb{R}$ be the usual multiplicative metric d defined as $d(x, y) = 3^{|x-y|}$. Define mappings A and $B : \mathbb{R} \times X \rightarrow X$ by

$$A(t, x) = t^2 - \frac{1}{3}x^2 \quad \text{and} \quad B(t, x) = t^2 - \frac{1}{3}x$$

for all $x \in X$, $t \in \mathbb{R}$ and $g : \mathbb{R} \rightarrow X$ defined by $g(t) = \sqrt{t}$. Then A and B are weakly commuting but not commuting.

Remark 2.6. Every weakly commuting mappings is compatible but converse need not be true, in general, as follows.

Example 2.7. Let $X = \mathbb{R}$ be the usual multiplicative metric d defined as $d(x, y) = 3^{|x-y|}$. Define mappings A and $B : \mathbb{R} \times X \rightarrow X$ by

$$A(t, x) = \frac{1}{3}(1 - t^2 + x) \quad \text{and} \quad B(t, x) = \frac{1}{3}(1 - t^2 + 2x)$$

for all $x \in X$, $t \in \mathbb{R}$ and $g_n : \mathbb{R} \rightarrow X$ defined by $g_n(t) = (1 - t^2)^{1+\frac{1}{n}}$. Then A and B are compatible but not weakly commuting mappings.

Definition 2.8. Let (X, d) be a multiplicative metric space and A and $B : \mathbb{R} \times X \rightarrow X$ be mappings. Then the mapping A and B are called *jointly continuous* for each $t \in \mathbb{R}$ if

$$\lim_{n \rightarrow \infty} A(t, A(t, g_n(t))) = \lim_{n \rightarrow \infty} A(t, B(t, g_n(t))) = A(t, g(t)),$$

whenever $\lim_{n \rightarrow \infty} A(t, g_n(t)) = \lim_{n \rightarrow \infty} B(t, g_n(t)) = g(t) \in X$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings.

Next we give the properties of compatible mappings for random mappings.

Proposition 2.9. Let (X, d) be a multiplicative metric space and A and $B : \mathbb{R} \times X \rightarrow X$ be mappings. Assume that A and B are compatible mappings. If $A(t, g(t)) = B(t, g(t))$ for some $g(t) \in X$, then

$$A(t, B(t, g(t))) = A(t, A(t, g(t))) = B(t, B(t, g(t))) = B(t, A(t, g(t))).$$

Proof. Suppose that $\{g_n\}$ is a sequence in X defined by $g_n(t) = g(t)$, $n = 1, 2, \dots$, for each $t \in X$ and $A(t, g(t)) = B(t, g(t))$. Then we have $A(t, g_n(t))$ and $B(t, g_n(t)) \rightarrow A(t, g(t))$ as $n \rightarrow \infty$. Since A and B are compatible, we have

$$\begin{aligned} d(A(t, B(t, g(t))), B(t, A(t, g(t)))) &= d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) \\ &= 1. \end{aligned}$$

Hence we have $A(t, B(t, g(t))) = B(t, A(t, g(t)))$. Therefore, we have

$$A(t, B(t, g(t))) = A(t, A(t, g(t))) = B(t, B(t, g(t))) = B(t, A(t, g(t))).$$

This completes the proof. $\square$

From Proposition 2.9 we have

Proposition 2.10. *Let (X, d) be a multiplicative metric space and A and $B : \mathbb{R} \times X \rightarrow X$ be mappings.*

Suppose that $\lim_{n \rightarrow \infty} A(t, g_n(t)) = \lim_{n \rightarrow \infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$. Then

(a) $\lim_{n \rightarrow \infty} B(t, A(t, g_n(t))) = A(t, g(t))$ *if A is jointly continuous.*

(b) $\lim_{n \rightarrow \infty} A(t, B(t, g_n(t))) = B(t, g(t))$ *if B is jointly continuous.*

(c) $A(t, B(t, g(t))) = B(t, A(t, g(t)))$ *and $A(t, g(t)) = B(t, g(t))$ if A and B are jointly continuous.*

Proof. (a) Suppose that A is jointly continuous. Since $\lim_{n \rightarrow \infty} A(t, g_n(t)) = \lim_{n \rightarrow \infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$, we have $A(t, B(t, g_n(t))) \rightarrow A(t, g(t))$ as $n \rightarrow \infty$. Since A and B are compatible, we have

$$\begin{aligned} & \lim_{n \rightarrow \infty} d(B(t, A(t, g_n(t))), A(t, g(t))) \\ & \leq \lim_{n \rightarrow \infty} [d(B(t, A(t, g_n(t))), A(t, B(t, g_n(t)))) \cdot d(A(t, B(t, g_n(t))), A(t, g(t)))] \\ & = 1. \end{aligned}$$

Therefore,

$$\lim_{n \rightarrow \infty} B(t, A(t, g_n(t))) = A(t, g(t)).$$

This completes the proof.

(b) The proof of $\lim_{n \rightarrow \infty} A(t, B(t, g_n(t))) = B(t, g(t))$ follows by similar arguments as in (a).

(c) Suppose that A and B are jointly continuous. Since $B(t, g_n(t)) \rightarrow g(t)$ as $n \rightarrow \infty$ and A is jointly continuous, by (a), $B(t, A(t, g_n(t))) \rightarrow A(t, g(t))$ as $n \rightarrow \infty$. On the other hand, B is also jointly continuous, $B(t, B(t, g_n(t))) \rightarrow B(t, g(t))$. Thus, we have $A(t, g(t)) = B(t, g(t))$ by the uniqueness of limit and so by Proposition 2.9, $A(t, B(t, g(t))) = B(t, A(t, g(t)))$. This completes the proof. $\square$

3. Main Results

Now, we prove the random fixed point theorems of compatible mappings for random mappings in a multiplicative metric space.

Theorem 3.1. *Let (X, d) be a complete multiplicative metric space and A, B, S and $T : \mathbb{R} \times X \rightarrow X$ be mappings satisfying the following conditions;*

$$(C_1) \quad S(t, X) \subset B(t, X) \quad \text{and} \quad T(t, X) \subset A(t, X);$$

$$(C_2) \quad \begin{aligned} & d(S(t, x(t)), T(t, y(t))) \\ & \leq [\max\{d(A(t, x(t)), B(t, y(t))), d(A(t, x(t)), S(t, x(t))), \\ & \quad d(B(t, y(t)), T(t, y(t))), d(S(t, x(t)), B(t, y(t))), \\ & \quad d(A(t, x(t)), T(t, y(t)))\}]^\lambda \end{aligned}$$

for all $x, y \in X$ and $t \in \mathbb{R}$, where $\lambda \in (0, 1/2)$;

$$(C_3) \quad \text{one of } A, B, S \text{ and } T \text{ is jointly continuous.}$$

Assume that the pairs A, S and B, T are compatible. Then A, B, S and T have a unique common random fixed point.

Proof. Let $g_0 : \mathbb{R} \rightarrow X$ be an arbitrary mapping. By (C_1) , there exists $g_1 : \mathbb{R} \rightarrow X$ such that $B(t, g_1(t)) = S(t, g_0(t))$, $t \in \mathbb{R}$ and for this $g_1 : \mathbb{R} \rightarrow X$ we can choose $g_2 : \mathbb{R} \rightarrow X$ such that $T(t, g_1(t)) = A(t, g_2(t))$, $t \in \mathbb{R}$, and so on. By the method of induction we can define a sequence $\{y_n(t)\}$, $t \in \mathbb{R}$, of mappings as follows:

$$\begin{aligned} y_{2n+1}(t) &= B(t, g_{2n+1}(t)) = S(t, g_{2n}(t)), \\ y_{2n}(t) &= A(t, g_{2n}(t)) = T(t, g_{2n-1}(t)), \quad t \in \mathbb{R}, \end{aligned} \tag{3.1}$$

$n = 0, 1, 2, \dots$

On putting $x(t) = g_{2n}(t)$ and $y(t) = g_{2n-1}(t)$ in (C_2) and using (3.1), we

have

$$\begin{aligned}
 & d(y_{2n+1}(t), y_{2n}(t)) \\
 &= d(S(t, g_{2n}(t)), T(t, g_{2n-1}(t))) \\
 &\leq [\max\{d(A(t, g_{2n}(t)), B(t, g_{2n-1}(t))), d(A(t, g_{2n}(t)), S(t, g_{2n}(t))), \\
 &\quad d(B(t, g_{2n-1}(t)), T(t, g_{2n-1}(t))), d(S(t, g_{2n}(t)), B(t, g_{2n-1}(t))), \\
 &\quad d(A(t, g_{2n}(t)), T(t, g_{2n-1}(t)))\}]^\lambda \\
 &= [\max\{d(y_{2n}(t), y_{2n-1}(t)), d(y_{2n}(t), y_{2n+1}(t)), d(y_{2n}(t), y_{2n-1}(t)), \\
 &\quad d(y_{2n+1}(t), y_{2n-1}(t)), d(y_{2n}(t), y_{2n}(t))\}]^\lambda \\
 &\leq [\max\{d(y_{2n}(t), y_{2n-1}(t)), d(y_{2n}(t), y_{2n+1}(t)), \\
 &\quad d(y_{2n+1}(t), y_{2n}(t)) \cdot d(y_{2n}(t), y_{2n-1}(t))\}]^\lambda \\
 &= [d(y_{2n+1}(t), y_{2n}(t)) \cdot d(y_{2n}(t), y_{2n-1}(t))]^\lambda.
 \end{aligned}$$

Hence we have

$$d(y_{2n+1}(t), y_{2n}(t)) \leq d^k(y_{2n}(t), y_{2n-1}(t)),$$

where $k = \frac{\lambda}{1-\lambda} < 1$.

Similarly, we can prove that

$$d(y_{2n+1}(t), y_{2n+2}(t)) \leq d^k(y_{2n}(t), y_{2n+1}(t)).$$

Hence, in general,

$$d(y_n(t), y_{n+1}(t)) \leq d^{k^n}(y_0(t), y_1(t)).$$

Let $m, n \in \mathbb{N}$ and $m > n$

$$\begin{aligned}
 & d(y_m(t), y_n(t)) \\
 &\leq d(y_m(t), y_{m-1}(t)) \cdot d(y_{m-1}(t), y_{m-2}(t)) \cdots d(y_{n+1}(t), y_n(t)) \\
 &\leq d^{k^{m-1}}(y_0(t), y_1(t)) \cdot d^{k^{m-2}}(y_0(t), y_1(t)) \cdots d^{k^n}(y_0(t), y_1(t)) \\
 &\leq d^{\frac{k^n}{1-k}}(y_0(t), y_1(t)).
 \end{aligned}$$

Taking $n \rightarrow \infty$, we have $\{y_n(t)\}$ is a multiplicative Cauchy sequence and since X is complete so $\{y_n(t)\}$ converges to a point $z(t) \in X$ as $n \rightarrow \infty$. Also subsequences of $\{y_n(t)\}$ also converges to a point $z(t) \in X$, that is

$$\begin{aligned}
 & B(t, g_{2n+1}(t)) \rightarrow z(t), \quad S(t, g_{2n}(t)) \rightarrow z(t), \\
 & A(t, g_{2n}(t)) \rightarrow z(t), \quad T(t, g_{2n+1}(t)) \rightarrow z(t), \quad t \in \mathbb{R}
 \end{aligned}$$

as $n \rightarrow \infty$.

Now suppose that A is jointly continuous, so

$$A(t, S(t, g_{2n}(t))) \rightarrow A(t, z(t)), \quad A(t, A(t, g_{2n}(t))) \rightarrow A(t, z(t))$$

as $n \rightarrow \infty$. Since A and S are compatible, by Proposition 2.10, we have

$$S(t, A(t, g_{2n}(t))) \rightarrow A(t, z(t)) \quad \text{as } n \rightarrow \infty.$$

Firstly, we claim that $z(t) = A(t, z(t))$.

On putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in (C_2) , we obtain

$$\begin{aligned} & d(S(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\ & \leq [\max\{d(A(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ & \quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\ & \quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ & \quad d(A(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^\lambda. \end{aligned}$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned} & d(A(t, z(t)), z(t)) \\ & \leq [\max\{d(A(t, z(t)), z(t)), d(A(t, z(t)), A(t, z(t))), d(z(t), z(t)), \\ & \quad d(A(t, z(t)), z(t)), d(A(t, z(t)), z(t))\}]^\lambda \\ & = [d(A(t, z(t)), z(t))]^\lambda. \end{aligned}$$

This implies that $d(A(t, z(t)), z(t)) = 1$ and hence $A(t, z(t)) = z(t)$.

Next we claim that $z(t) = S(t, z(t))$.

On putting $x(t) = z(t)$ and $y(t) = g_{2n+1}(t)$ in (C_2) , we have

$$\begin{aligned} & d(S(t, z(t)), T(t, g_{2n+1}(t))) \\ & \leq [\max\{d(A(t, z(t)), B(t, g_{2n+1}(t))), d(A(t, z(t)), S(t, z(t))), \\ & \quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, z(t)), B(t, g_{2n+1}(t))), \\ & \quad d(A(t, z(t)), T(t, g_{2n+1}(t)))\}]^\lambda. \end{aligned}$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned} & d(S(t, z(t)), z(t)) \\ & \leq [\max\{d(z(t), z(t)), d(z(t), S(t, z(t))), d(z(t), z(t)), \\ & \quad d(S(t, z(t)), z(t)), d(z(t), z(t))\}]^\lambda. \end{aligned}$$

This implies that $d(S(t, z(t)), z(t)) = 1$ and hence $S(t, z(t)) = z(t)$. Since $z(t) = S(t, z(t)) \in S(t, X) \subset B(t, X)$, there exists $u(t) \in X$ such that $z(t) = B(t, u(t))$, $t \in \mathbb{R}$.

On putting $x(t) = z(t)$ and $y(t) = u(t)$ in (C_2) , we have

$$\begin{aligned} & d(S(t, z(t)), T(t, u(t))) \\ & \leq [\max\{d(A(t, z(t)), B(t, u(t))), d(A(t, z(t)), S(t, z(t))), \\ & \quad d(B(t, u(t)), T(t, u(t))), d(S(t, z(t)), B(t, u(t))), \\ & \quad d(A(t, z(t)), T(t, u(t)))\}]^\lambda \\ & = [\max\{1, 1, d(z(t), T(t, u(t))), 1, d(z(t), T(t, u(t)))\}]^\lambda \\ & = d^\lambda(z(t), T(t, u(t))). \end{aligned}$$

This implies that $d(z(t), T(t, u(t))) = 1$ and hence $T(t, u(t)) = z(t)$. Since B and T are compatible and $B(t, u(t)) = T(t, u(t)) = z(t)$, by Proposition 2.9, we have $d(B(t, T(t, u(t))), T(t, B(t, u(t)))) = 1$ and hence

$$B(t, z(t)) = B(t, T(t, u(t))) = T(t, B(t, u(t))) = T(t, z(t)).$$

Also, on putting $x(t) = z(t)$ and $y(t) = z(t)$ in (C_2) , we have

$$\begin{aligned} & d(z(t), B(t, z(t))) \\ & = d(S(t, z(t)), T(t, z(t))) \\ & \leq [\max\{d(A(t, z(t)), B(t, z(t))), d(A(t, z(t)), S(t, z(t))), \\ & \quad d(B(t, z(t)), T(t, z(t))), d(S(t, z(t)), B(t, z(t))), \\ & \quad d(A(t, z(t)), T(t, z(t)))\}]^\lambda \\ & = [\max\{d(z(t), B(t, z(t))), 1, 1, d(z(t), B(t, z(t))), d(z(t), B(t, z(t)))\}]^\lambda. \end{aligned}$$

This implies that $z(t) = B(t, z(t))$. Hence

$$A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t).$$

Therefore A, B, S and T have a common random fixed point $z(t)$ in X .

Similarly, we can also complete the proof when B is jointly continuous.

Next suppose that S is jointly continuous. Since A and S are compatible, by Proposition 2.10, it follows that

$$S(t, S(t, g_{2n}(t))) \rightarrow S(t, z(t)), \quad A(t, S(t, g_{2n}(t))) \rightarrow S(t, z(t))$$

as $n \rightarrow \infty$.

Now, we claim that $z(t) = S(t, z(t))$.

On putting $x(t) = S(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in (C_2) , we have

$$\begin{aligned} & d(S(t, S(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\ & \leq [\max\{d(A(t, S(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ & \quad d(A(t, S(t, g_{2n}(t))), S(t, S(t, g_{2n}(t))), \\ & \quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), \\ & \quad d(S(t, S(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ & \quad d(A(t, S(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^\lambda. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$\begin{aligned} & d(S(t, z(t)), z(t)) \\ & \leq [\max\{d(S(t, z(t)), z(t)), d(S(t, z(t)), S(t, z(t))), \\ & \quad d(z(t), z(t)), d(S(t, z(t)), z(t)), d(S(t, z(t)), z(t))\}]^\lambda \\ & = d^\lambda(S(t, z(t)), z(t)). \end{aligned}$$

This implies $z(t) = S(t, z(t))$. Since $z(t) = S(t, z(t)) \in S(t, X) \subset B(t, X)$, there exists $v(t) \in X$ such that $z(t) = B(t, v(t))$, $t \in \mathbb{R}$.

On putting $x(t) = S(t, g_{2n}(t))$ and $y(t) = v(t)$ in (C_2) , we have

$$\begin{aligned} & d(S(t, S(t, g_{2n}(t))), T(t, v(t))) \\ & \leq [\max\{d(A(t, S(t, g_{2n}(t))), B(t, v(t))), \\ & \quad d(A(t, S(t, g_{2n}(t))), S(t, S(t, g_{2n}(t))), d(B(t, v(t)), T(t, v(t))), \\ & \quad d(S(t, S(t, g_{2n}(t))), B(t, v(t))), d(A(t, S(t, g_{2n}(t))), T(t, v(t)))\}]^\lambda. \end{aligned}$$

Letting $n \rightarrow \infty$, we obtain

$$\begin{aligned} & d(S(t, z(t)), T(t, v(t))) \\ & \leq [\max\{1, 1, d(z(t), T(t, v(t))), 1, d(z(t), T(t, v(t)))\}]^\lambda \\ & = d^\lambda(z(t), T(t, v(t))). \end{aligned}$$

This implies that $d(z(t), T(t, v(t))) = 1$ and hence $T(t, v(t)) = z(t)$. Since B and T are compatible and $B(t, v(t)) = T(t, v(t)) = z(t)$, by Proposition 2.9, $d(B(t, T(t, v(t))), T(t, B(t, v(t)))) = 1$ and hence

$$B(t, z(t)) = B(t, T(t, v(t))) = T(t, B(t, v(t))) = T(t, z(t)).$$

On putting $x(t) = g_{2n}(t)$ and $y(t) = z(t)$ in (C_2) , we have

$$\begin{aligned}
 & d(S(t, g_{2n}(t)), T(t, z(t))) \\
 & \leq [\max\{d(A(t, g_{2n}(t)), B(t, z(t))), d(A(t, g_{2n}(t)), S(t, g_{2n}(t))), \\
 & \quad d(B(t, z(t)), T(t, z(t))), d(S(t, g_{2n}(t)), B(t, z(t))), \\
 & \quad d(A(t, g_{2n}(t)), T(t, z(t)))\}]^\lambda \\
 & = [\max\{d(z(t), T(t, z(t))), 1, 1, d(z(t), T(t, z(t))), d(z(t), T(t, z(t)))\}]^\lambda \\
 & = d^\lambda(z(t), T(t, z(t))).
 \end{aligned}$$

This implies that $d(z(t), T(t, z(t))) = 1$ and hence $T(t, z(t)) = z(t)$. Since $T(t, X) \subset A(t, X)$, there exists a point $w(t) \in X$ such that $z(t) = T(t, z(t)) = A(t, w(t))$.

On putting $x(t) = w(t)$ and $y(t) = z(t)$ in (C_2) , we have

$$\begin{aligned}
 & d(S(t, w(t)), z(t)) \\
 & = d(S(t, w(t)), T(t, z(t))) \\
 & \leq [\max\{d(A(t, w(t)), B(t, z(t))), d(A(t, w(t)), S(t, w(t))), \\
 & \quad d(B(t, z(t)), T(t, z(t))), d(S(t, w(t)), B(t, z(t))), \\
 & \quad d(A(t, w(t)), T(t, z(t)))\}]^\lambda \\
 & = [\max\{1, d(z(t), S(t, w(t))), 1, d(S(t, w(t)), z(t)), 1\}]^\lambda.
 \end{aligned}$$

This implies that $S(t, w(t)) = z(t)$. Since A and S are compatible and $S(t, w(t)) = A(t, w(t)) = z(t)$, we get $d(A(t, S(t, w(t))), S(t, A(t, w(t)))) = 1$ by Proposition 2.9 and hence

$$A(t, z(t)) = A(t, S(t, w(t))) = S(t, A(t, w(t))) = S(t, z(t)).$$

Hence

$$A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t).$$

Therefore A, B, S and T have a common random fixed point $z(t)$ in X .

Similarly, we can complete the proof when T is continuous.

Finally, suppose that $z(t)$ and $v(t)$ ($z(t) \neq v(t)$) are two common random

fixed points of A, S, B and T . From (C_2) , we have

$$\begin{aligned} & d(z(t), v(t)) \\ &= d(S(t, z(t)), T(t, v(t))) \\ &\leq [\max\{d(A(t, z(t)), B(t, v(t))), d(d(A(t, z(t)), S(t, z(t))), \\ &\quad d(B(t, v(t)), T(t, v(t))), d(S(t, z(t)), B(t, v(t))), \\ &\quad d(d(A(t, z(t)), T(t, v(t))))\}]^\lambda \\ &= [\max\{d(z(t), v(t)), 1, 1, d(z(t), v(t)), d(z(t), v(t))\}]^\lambda. \end{aligned}$$

This implies that $z(t) = v(t)$. Therefore, A, B, S and T have a unique common fixed point in X . This completes the proof. $\square$

From Theorem 3.1, Remarks 2.4 and 2.6, we follow the following corollary.

Corollary 3.2. *Let (X, d) be a complete multiplicative metric space and A, B, S and $T : \mathbb{R} \times X \rightarrow X$ be mappings satisfying the conditions (C_1) – (C_3) .*

Assume that the pairs A, S and B, T are commuting. Then A, B, S and T have a unique common random fixed point.

From Theorem 3.1 and Remark 2.6, we follow the following corollary.

Corollary 3.3. *Let (X, d) be a complete multiplicative metric space and A, B, S and $T : \mathbb{R} \times X \rightarrow X$ be mappings satisfying the conditions (C_1) – (C_3) .*

Assume that the pairs A, S and B, T are weakly commuting. Then A, B, S and T have a unique common random fixed point.

Remark 3.4. 1. In Theorem 3.1, if we take $\mathbb{R}$ to be a singleton set, then the result can reduce into the result of Kang et al. [4]. Also Theorem 3.1 generalize the result of Kumar et al. [6, Theorem 3.1].

2. In Corollary 3.2, if we take $\mathbb{R}$ to be a singleton set, then the result can reduce into the result of Gu et al. [2].

3. In Corollary 3.3, if we take $\mathbb{R}$ to be a singleton set, then the result can reduce into the result of He et al. [3].

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