NEW INTERVAL VALUED INTUITIONISTIC FUZZY SET OF CUBE ROOT TYPE

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Abstract: Intuitionistic fuzzy set (IFS) is very useful in providing a flexible model to elaborate uncertainty and vagueness involved in decision making.

In this paper an interval valued intuitionistic fuzzy set of cube root type (IVIFSCRT) is proposed. We have proved some important notions and basic algebraic properties of IVIFSCRT.

Key Words: intuitionistic fuzzy sets, interval valued intuitionistic fuzzy set of cube root type

1. Introduction

Fuzzy sets (FS) introduced by Zadeh in 1965, has showed meaningful applications in many fields of study. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the

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membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by Atanassov, 1983, 1986, as intuitionistic fuzzy set (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degree respectively). The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin (see Atanassov, 1994, 1999, R. Biswas, 2001) gave an intuitionistic fuzzy sets approach in medical diagnosis using three steps such as; determination of symptoms, formulation of medical knowledge based on intuitionistic fuzzy relations, and determination of diagnosis on the basis of composition of intuitionistic fuzzy relations.

Following the definition IFS (see Atanassov and Gargov, 1986) introduced interval valued- intuitionistic fuzzy sets (IVIFS) which is generalisation of the IFS. The fundamental characteristic of IVIFS is that the value of its membership function and non-membership function are interval at $[0,1]$ rather than exact number. The IVIFSs has been studied and applied in a variety of fields as pattern recognition, medical diagnosis, decision making, data mining, conflict analysis, algebra and so on.

In this paper the main objective is to introduce a interval value of intuitionistic fuzzy set of cube root type and also define some operations and their properties

2. Preliminaries

In this section, we give some definitions of various types of IFS

Definition 2.1. (see Atanassov, 1986) Let X be a non empty set. An IFS A in X is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where the functions $\mu_A(x) : X \rightarrow [0,1]$ and $\nu_A(x) : X \rightarrow [0,1]$ denote the membership and non-membership function of A respectively, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \text{ for each } x \in X.$$

Definition 2.2. (see Atanassov and Gargov 1989) Interval Value Intuitionistic fuzzy sets (IVIFS) A in X , is defined as an object of the form

$$A = \{ \langle x, M_A(x), N_A(x) \rangle : x \in X \},$$

where the functions $M_A(x) : X \rightarrow [I]$ and $N_A(x) : X \rightarrow [I]$ denotes the degree of membership and non-membership function of A respectively, where:

$$M_A(x) = [M_{AL}(x), M_{AU}(x)],$$

$$N_A(x) = [N_{AL}(x), N_{AU}(x)],$$

$$0 \leq M_{AU}(x) + N_{AL}(x) \leq 1 \text{ for each } x \in X.$$

Definition 2.3. (see Rizwan and Nabeel) An Intuitionistic fuzzy set of cube root type(IFSCRT) A in X, is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denotes the degree of membership and non-membership function of A, respectively $0 \leq \mu_A(x)^{1/3} + \nu_A(x)^{1/3} \leq 1$ for each $x \in X$.

Definition 2.4. Let $[I]$ be the set of all closed subintervals of the interval $[0, 1]$ and

$$M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$$

and

$$N_A(x) = [N_{AL}(x), N_{AU}(x)] \in [I]$$

then $N_A(x) \leq M_A(x)$ if and only if $N_{AL}(x) \leq M_{AL}(x)$ and $N_{AU}(x) \leq M_{AU}(x)$.

Definition 2.5. Let $[I]$ be the set of all closed subintervals of the interval $[0, 1]$ and

$$M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$$

and $f : [0, 1] \rightarrow [0, 1]$ then $f(M_A(x)) = [f(M_{AL}(x)), f(M_{AU}(x))]$.

3. Interval Value of Intuitionistic Fuzzy set of Cube Root Type

Definition 3.1. Let X be a non empty set. Interval Value of Intuitionistic fuzzy set of cube root type(IVIFSCRT) A in X, is defined as an object of the form

$$A = \{ \langle x, M_A(x), N_A(x) \rangle : x \in X \},$$

where the functions $M_A(x) : X \rightarrow [I]$ and $N_A(x) : X \rightarrow [I]$ denote the degree of membership and non-membership function of A, respectively and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$, $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, where

$$0 \leq M_{AU}(x)^{1/3} + N_{AU}(x)^{1/3} \leq 1$$

for each $x \in X$.

Remark 3.2. It is obvious that for all real number $\alpha, \beta \in [0, 1]$

1. If $0 \leq \alpha + \beta \leq 1$
2. $0 \leq \alpha^{1/3} + \beta^{1/3} \leq 1$

Definition 3.3. Let X be a non empty set. Let A and B be two *IVIFSCRT* such that

$$A = \{\langle x, M_A(x), N_A(x) \rangle : x \in X\}$$

$$B = \{\langle x, M_B(x), N_B(x) \rangle : x \in X\}$$

$$M_A(x) = [M_{AL}(x), M_{AU}(x)], N_A(x) = [N_{AL}(x), N_{AU}(x)]$$

$$M_B(x) = [M_{BL}(x), M_{BU}(x)], N_B(x) = [N_{BL}(x), N_{BU}(x)]$$

define the following relations and operations on A and B :

$$(i) \ A \subset B \text{ if and only if } M_A(x) \leq M_B(x) \text{ and } N_A(x) \geq N_B(x), \forall x \in X$$

$$(ii) \ A = B \text{ if and only if } M_A(x) = M_B(x) \text{ and } N_A(x) = N_B(x), \forall x \in X$$

$$(iii) \ A \cup B = \{\langle x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], \\ [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))] \rangle : x \in X\}$$

$$(iv) \ A \cap B = \{\langle x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))], \\ [\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))] \rangle : x \in X\}$$

$$(v) \ A + B = \{\langle x, [M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3} - M_{AL}(x)^{1/3}M_{BL}(x)^{1/3}, \\ M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3} - M_{AU}(x)^{1/3}M_{BU}(x)^{1/3}], \\ [N_{AL}(x)^{1/3}N_{BL}(x)^{1/3}, N_{AU}(x)^{1/3}N_{BU}(x)^{1/3}] \rangle : x \in X\}.$$

Therefore

$$2A = \{\langle x, [1 - (1 - M_{AL}(x)^{1/3})^2, 1 - (1 - M_{AU}(x)^{1/3})^2], \\ [N_{AL}(x)^{2/3}N_{AU}(x)^{2/3}] \rangle : x \in X\},$$

$$nA = \{\langle x, [1 - (1 - M_{AL}(x)^{1/3})^n, 1 - (1 - M_{AU}(x)^{1/3})^n], \\ [N_{AL}(x)^{n/3} N_{AU}(x)^{n/3}] \rangle : x \in X\},$$

$$(vi) \ A.B = \{\langle x, [M_{AL}(x)^{1/3}, M_{BL}(x)^{1/3}, M_{AU}(x)^{1/3} M_{BU}(x)^{1/3}], \\ [N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3} - N_{AL}(x)^{1/3} N_{BL}(x)^{1/3}, \\ N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3} - N_{AU}(x)^{1/3} N_{BU}(x)^{1/3}] \rangle : x \in X\}.$$

Therefore

$$A^2 = \{\langle x, [M_{AL}(x)^{2/3}, M_{AU}(x)^{2/3}], [1 - (1 - N_{AL}(x)^{1/3})^2, \\ 1 - (1 - N_{AU}(x)^{1/3})^2] \rangle : x \in X\}$$

and

$$A^n = \{\langle x, [M_{AL}(x)^{n/3}, M_{AU}(x)^{n/3}], [1 - (1 - N_{AL}(x)^{1/3})^n, \\ 1 - (1 - N_{AU}(x)^{1/3})^n] \rangle : x \in X\}.$$

$$(vii) \ \overline{A} = \{\langle x, N_A(x), M_A(x) \rangle\}.$$

Proposition 3.4. Let $A, B, C \in IVIFSCRT$, then we have:

$$(i) \ \overline{\overline{A}} = A,$$

$$(ii) \ \overline{A \cup B} = \overline{A} \cap \overline{B},$$

$$(iii) \ \overline{A \cap B} = \overline{A} \cup \overline{B},$$

$$(iv) \ A \subset B, B \subset C \Rightarrow A \subset C.$$

Proof. Proof is obvious.

Proposition 3.5. Let $A, B, C \in IVIFSCRT$, then we have:

$$(i) \ A \cup B \in IVIFSCRT,$$

$$(ii) \ A \cap B \in IVIFSCRT,$$

$$(iii) \ A + B \in IVIFSCRT,$$

$$(iv) \ A.B \in IVIFSCRT.$$

Proof. (i) Let $\max(M_{AU}(x), M_{BU}(x)) = M_{AU}(x)$. Since

$$\min(N_{AU}(x), N_{BU}(x)) \leq N_{AU}(x),$$

we have

$$\begin{aligned}
 0 &\leq M_{(A \cup B)U}(x)^{1/3} + N_{(A \cup B)U}(x)^{1/3} \\
 &\leq (\max(M_{AU}(x), M_{BU}(x)))^{1/3} + (\min(N_{AU}(x), N_{BU}(x)))^{1/3} \\
 &\leq M_{AU}(x)^{1/3} + (\min(N_{AU}(x), N_{BU}(x)))^{1/3} \\
 0 &\leq M_{AU}(x)^{1/3} + N_{AU}(x)^{1/3} \leq 1.
 \end{aligned}$$

If $\max(M_{AU}(x), M_{BU}(x)) = M_{BU}(x), \min(N_{AU}(x), N_{BU}(x)) \leq N_{BU}(x)$, then

$$\begin{aligned}
 0 &\leq M_{(A \cup B)U}(x)^{1/3} + N_{(A \cup B)U}(x)^{1/3} \\
 &\leq (\max(M_{AU}(x), M_{BU}(x)))^{1/3} + (\min(N_{AU}(x), N_{BU}(x)))^{1/3} \\
 &\leq M_{BU}(x)^{1/3} + (\min(N_{AU}(x), N_{BU}(x)))^{1/3} \\
 0 &\leq M_{BU}(x)^{1/3} + N_{BU}(x)^{1/3} \leq 1.
 \end{aligned}$$

This completes the proof of (i). The proof of (ii) is similar to that of (i).

(iii) It is easy to see that

$$\begin{aligned}
 A + B = \{ \langle x, [M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3} - M_{AL}(x)^{1/3}M_{BL}(x)^{1/3}, \\
 M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3} - M_{AU}(x)^{1/3}M_{BU}(x)^{1/3}], \\
 [N_{AL}(x)^{1/3}N_{BL}(x)^{1/3}, N_{AU}(x)^{1/3}N_{BU}(x)^{1/3}] \rangle : x \in X \}.
 \end{aligned}$$

So

$$\begin{aligned}
 M_{(A+B)U}(x)^{1/3} + N_{(A+B)U}(x)^{1/3} &= (M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3} \\
 &\quad - M_{AU}(x)^{1/3}M_{BU}(x)^{1/3})^{1/3} + (N_{AU}(x)^{1/3}N_{BU}(x)^{1/3})^{1/3} \\
 &= [M_{AU}(x)^{1/3}(1 - M_{BU}(x)^{1/3}) + M_{BU}(x)^{1/3}]^{1/3} + [N_{AU}(x)^{1/3}N_{BU}(x)^{1/3}]^{1/3} \geq 0
 \end{aligned}$$

and

$$\begin{aligned}
 M_{(A+B)U}(x)^{1/3} + N_{(A+B)U}(x)^{1/3} &= (M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3} \\
 &\quad - M_{AU}(x)^{1/3}M_{BU}(x)^{1/3})^{1/3} + (N_{AU}(x)^{1/3}N_{BU}(x)^{1/3})^{1/3} \\
 &\leq [(1 - N_{AU}(x)^{1/3}) + (1 - N_{BU}(x)^{1/3}) - (1 - N_{AU}(x)^{1/3}) \\
 &\quad (1 - N_{BU}(x)^{1/3})]^{1/3} + [N_{AU}(x)^{1/3}N_{BU}(x)^{1/3}]^{1/3}
 \end{aligned}$$

$$\begin{aligned}
&= [1 - N_{AU}(x)^{1/3} N_{BU}(x)^{1/3}]^{1/3} + [N_{AU}(x)^{1/3} N_{BU}(x)^{1/3}]^{1/3} \\
&= [1 - u]^{1/3} + u^{1/3} \text{ where } u = N_{AU}(x)^{1/3} N_{BU}(x)^{1/3} \\
&= [1 - u]^{1/3} + u^{1/3} \text{ if and only if } N_{AU}(x) = 0, \text{ or } N_{BU}(x) = 0. \\
&\text{Hence } A + B \in IVIFSCRT.
\end{aligned}$$

(iv) Proof of (iv) is similar to the proof of (iii).

Proposition 3.6. *For every IVIFSCRTA, we have:*

- (i) $m \geq n \rightarrow A^m \subset A^n$,
- (ii) $m \geq n \rightarrow nA \subset mA$,
- (iii) $A^n = \overline{nA}$,

where m and n are both positive numbers.

Proof. (i)

$$\begin{aligned}
A^n &= \{ \langle x, [M_{AL}(x)^{n/3}, M_{AU}(x)^{n/3}], [1 - (1 - N_{AL}(x)^{1/3})^n, \\
&\quad 1 - (1 - N_{AU}(x)^{1/3})^n] \rangle : x \in X \}, \\
A^m &= \{ \langle x, [M_{AL}(x)^{m/3}, M_{AU}(x)^{m/3}], [1 - (1 - N_{AL}(x)^{1/3})^m, \\
&\quad 1 - (1 - N_{AU}(x)^{1/3})^m] \rangle : x \in X \}.
\end{aligned}$$

Since $m \geq n$, then $M_{AU}(x)^{n/3} \geq M_{AU}(x)^{m/3}$, $M_{AL}(x)^{n/3} \geq M_{AL}(x)^{m/3}$.
Hence $M_{A^n}(x) \geq M_{A^m}(x)$. Also since $N_{AL}(x) \leq 1$, $N_{AU}(x) \leq 1$, we have

$$\begin{aligned}
(1 - N_{AU}(x)^{1/3})^m &\leq (1 - N_{AU}(x)^{1/3})^n, \\
(1 - N_{AL}(x)^{1/3})^m &\leq (1 - N_{AL}(x)^{1/3})^n, \\
1 - (1 - N_{AU}(x)^{1/3})^n &\leq 1 - (1 - N_{AU}(x)^{1/3})^m, \\
1 - (1 - N_{AL}(x)^{1/3})^n &\leq 1 - (1 - N_{AL}(x)^{1/3})^m.
\end{aligned}$$

Therefore $N_{A^n}(x) \leq N_{A^m}(x)$.

This complete the proof. The proof of (iii) is clear.

Proposition 3.7. *Let $A, B \in IVIFSCRT$. Then:*

- (i) $A \subset B \rightarrow nA \subset nB$,
- (ii) $A \subset B \rightarrow A^n \subset B^n$,

$$(iii) (A \cup B)^n = A^n \cup B^n,$$

$$(iv) (A \cap B)^n = A^n \cap B^n,$$

$$(v) n(A \cup B) = nA \cup nB,$$

$$(v) n(A \cap B) = nA \cap nB.$$

Proof. (i) since $A \subset B$, then $M_A(x) \leq M_B(x)$ hence

$$\begin{aligned} M_A(x)^{1/3} \leq M_B(x)^{1/3} &\Rightarrow 1 - M_B(x)^{1/3} \leq 1 - M_A(x)^{1/3} \\ (1 - M_B(x)^{1/3})^n &\leq (1 - M_A(x)^{1/3})^n. \end{aligned}$$

Finally

$$\begin{aligned} 1 - (1 - M_A(x)^{1/3})^n &\leq (1 - M_B(x)^{1/3})^n \\ M_{nA}(x) &\leq M_{nB}(x). \end{aligned}$$

Also since $A \subset B$, then $N_B(x) \leq N_A(x)$, hence

$$N_B(x)^{n/3} \leq N_A(x)^{n/3} \Rightarrow N_{nB}(x) \leq N_{nA}(x)$$

The proof is complete.

$$(ii) A \subset B \Rightarrow \overline{B} \subset \overline{A} \Rightarrow n\overline{B} \subset n\overline{A},$$

$$\overline{nA} \subset \overline{nB}, \quad A^n \subset B^n.$$

$$\begin{aligned} (iii) (A \cup B)^n &= \{ \langle x, [(max(M_{AL}(x), M_{BL}(x)))^{n/3}, (max(M_{AU}(x), \\ &\quad M_{BU}(x)))^{n/3}], [1 - (1 - min(N_{AL}(x), N_{BL}(x))^{1/3})^n, \\ &\quad 1 - (1 - min(N_{AU}(x), N_{BU}(x))^{1/3})^n] \rangle : x \in X \} \\ &= \{ \langle x, [max(M_{AL}(x)^{n/3}, M_{BL}(x)^{n/3}), max(M_{AU}(x)^{n/3}, M_{BU}(x)^{n/3})], \\ &\quad [1 - (1 - min(N_{AL}(x)^{1/3}, N_{BL}(x))^{1/3})^n, \\ &\quad 1 - (1 - min(N_{AU}(x)^{1/3}, N_{BU}(x)^{1/3})^n] \rangle : x \in X \} \\ &= \{ \langle x, [max(M_{AL}(x)^{n/3}, M_{BL}(x)^{n/3}), max(M_{AU}(x)^{n/3}, M_{BU}(x)^{n/3})], \\ &\quad [1 - (max(1 - N_{AL}(x)^{1/3}, 1 - N_{BL}(x))^{1/3})^n, \\ &\quad 1 - (max(1 - N_{AU}(x)^{1/3}, N_{BU}(x)^{1/3})^n] \rangle : x \in X \} \\ &= \{ \langle x, [max(M_{AL}(x)^{n/3}, M_{BL}(x)^{n/3}), max(M_{AU}(x)^{n/3}, M_{BU}(x)^{n/3})], \\ &\quad [min(1 - (1 - N_{AL}(x)^{1/3})^n, 1 - (1 - N_{BL}(x)^{1/3})^n] \rangle : x \in X \} \end{aligned}$$

$$\begin{aligned} & \min(1 - (1 - N_{AU}(x)^{1/3})^n, 1 - (1 - N_{BU}(x)^{1/3})^n) : x \in X \} \\ & = A^n \cup B^n. \end{aligned}$$

The proof of (iv) is similar to that of (iii).

$$\begin{aligned} (v) \quad n(A \cup B) &= \{ \langle x, [1 - (1 - \max(M_{AL}(x), M_{BL}(x))^{1/3})^n, \\ & \quad 1 - (1 - \max(M_{AU}(x), M_{BU}(x))^{1/3})^n], \\ & \quad [\min(N_{AL}(x), N_{BL}(x))^{n/3}, \min(N_{AU}(x), N_{BU}(x))^{n/3}] \rangle : x \in X \} \\ &= \{ \langle x, [1 - (1 - \max(M_{AL}(x)^{1/3}, M_{BL}(x)^{1/3})^n, \\ & \quad (1 - (1 - \max(M_{AU}(x)^{1/3}, M_{BU}(x)^{1/3}))^n], \\ & \quad [\min(N_{AL}(x), N_{BL}(x))^{n/3}, \min(N_{AU}(x), N_{BU}(x))^{n/3}] \rangle : x \in X \} \\ &= \{ \langle x, [1 - \min((1 - M_{AL}(x)^{1/3})^n, (1 - M_{BL}(x)^{1/3})^n), \\ & \quad 1 - \min((1 - M_{AU}(x)^{1/3})^n, (1 - M_{BU}(x)^{1/3})^n)], \\ & \quad [\min(N_{AL}(x), N_{BL}(x))^{n/3}, \min(N_{AU}(x), N_{BU}(x))^{n/3}] \rangle : x \in X \} \\ &= \{ \langle x, [\max(1 - (1 - M_{AL}(x)^{1/3})^n, 1 - (1 - M_{BL}(x)^{1/3})^n), \\ & \quad \max(1 - (1 - M_{AU}(x)^{1/3})^n, 1 - (1 - M_{BU}(x)^{1/3})^n)], \\ & \quad [\min(N_{AL}(x), N_{BL}(x))^{n/3}, \min(N_{AU}(x), N_{BU}(x))^{n/3}] \rangle : x \in X \} \\ &= nA \cup nB. \end{aligned}$$

The proof of (vi) is similar to that (v).

4. Conclusion

We have introduced Interval value of intuitionistic fuzzy set of cube root type (IVIFSCRT) as an extension to the intuitionistic fuzzy set. The basic algebraic properties of IVIFSCRT are also presented. Some operators on IVIFSCRT are defined and their relationship have been proved. IVIFSCRT is more comprehensive and practical than IVIFS in coping with fuzziness and uncertainty (A List of open problem as follows:

(i) define IFSCRT, norms, distance, metrics, metric spaces, etc. For the IFSCRT and study their properties;

(ii) It is still open to check whether there exist an IFSCRT in case of operators already defined on an IFS.

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