FOURTH GEOMETRIC ARITHMETIC INDEX OF NA_m^n NANOTUBE

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Abstract: Let G be a molecular graph. The length of a shortest path between two vertices of G is called distance. In a connected graph G , the *eccentricity* e_v of vertex v is the distance between v and a vertex farthest from v in G . In this paper, we consider NA_m^n nanotube and compute the Fourth Geometric Arithmetic Index.

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1. Introduction

Graph theory has provided chemist with a variety of useful tools, such as topological indices. Molecules and molecular compounds are often modeled

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by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. *Cheminformatics* is new subject which is a combination of chemistry, mathematics and information science. It studies Quantitative structure-activity (QSAR) and structure-property (QSPR) relationships that are used to predict the biological activities and properties of chemical compounds.

A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix. A topological index is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topological indices, eccentricity based topological indices, degree based topological indices and counting related polynomials and indices of graphs. For more study and definition of the topological indices and polynomials see [5], [9], [7], [8], [14], [18], ..., [23].

In more precise way, a topological index $Top(G)$ of a graph, is a number with the property that for every graph H isomorphic to G , $Top(H) = Top(G)$ [4].

Let G be a molecular graph with vertex set $V(G)$ and edge set $E(G)$. The vertices of G denotes atoms and an edge between two vertices denotes the chemical bond between these vertices. If no vertices in $u - v$ walk are repeated then it is called $u - v$ path in graph G . The length of a path is the number of edges in it. The distance $d(u, v)$ from vertex u to vertex v is the length of a shortest $u - v$ path in a graph G where $u, v \in G$. In a connected graph G , the eccentricity e_v of vertex v is the distance between v and a vertex farthest from v in G .

The concept of topological indices came from Wiener while he was working on boiling point of paraffin, named this index as *path number*. Later on, the path number was renamed as *Wiener index* defined as half sum of the distances between all the pairs of vertices in a graph [24].

Let G be a graph. Then the Wiener index of G is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u, v) \quad (1)$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is $u - v$ geodesic. Among the topological indices, the degree based topological indices are very important and they have a prominent role in chemistry. For a connected graph G the Geometric arithmetic index is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v} \quad (2)$$

where d_u denotes the degree of the vertex u and d_v denotes the degree of the vertex v of G [3]. In 2010, *M. Ghorbani* and *A. Khaki* defined the eccentricity based geometric arithmetic index named as *eccentric version of Geometric arithmetic index* [10]. Now the eccentric version of Geometric arithmetic index is denoted by $GA_4(G)$ and defined as

$$GA_4(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\epsilon_u \epsilon_v}}{\epsilon_u + \epsilon_v} \quad (3)$$

where ϵ_u denotes the eccentricity of the vertex u and ϵ_v denotes the eccentricity of the vertex v of G [6], [13], [15]. The aim of this paper is to compute the eccentricity version of Geometric arithmetic index of NA_m^n nanotube.

2. NA_m^n Nanotube

We consider the $m \times n$ quadrilateral section P_m^n with $m \geq 2$ hexagons on the top and bottom sides and $n \geq 2$ hexagons on the lateral sides cut from the regular hexagonal lattice L as shown in Figure 1. If we identify two lateral sides of P_m^n such that we identify the vertices u_0^j and u_m^j , for $j = 0, 1, 2, \dots, n$ then we obtain the nanotube NA_m^n [3].

In this paper, we consider NA_m^n nanotube with $n = m$ and we compute the eccentricity version of geometric arithmetic index. For the eccentricity version of geometric arithmetic index of NA_m^n nanotube we have three cases of n when $n \equiv 0(mod 2)$, $n \equiv 1(mod 4)$ and $n \equiv 3(mod 4)$.

3. Results for the Eccentricity Version of Geometric Arithmetic Index of NA_m^n Nanotube

Theorem 3.1. *For every $n \equiv 0(mod 2)$, consider the graph of $G \cong NA_m^n$ nanotube. Then the eccentricity version of geometric arithmetic index is equal to*

$$GA_4(G) = \sum_{i=1}^{\frac{n}{2}} \sum_{2n-\frac{n}{2}}^{2n-1} \frac{(12i-6)\sqrt{p^2+p}}{2p+1} + \sum_{p=n} \frac{6n\sqrt{4p^2+2p}}{4p+1} + \sum_{p=n} \frac{4p+2}{2p+1}$$

Figure 1: NA_m^n nanotube.

$$\begin{aligned}
 & + \sum_{i=\frac{n}{2}+3} \sum_{p=2n+1}^{3n-\frac{n}{2}-1} \left(\frac{n}{2}-1\right)(6i-16) \times \frac{2\sqrt{p^2+p}}{2p+1} \\
 & + \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} \sum_{p=3n-\frac{n}{2}}^{3n-1} \frac{(12n-2i)\sqrt{p^2+p}}{2p+1}.
 \end{aligned}$$

Proof. Let G be the graph of NA_m^n nanotube. Then formula for the eccentricity version of geometric arithmetic index is equal to

$$GA_4(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\epsilon_u \epsilon_v}}{\epsilon_u + \epsilon_v}$$

By using the values from Table 1 we get

$$\begin{aligned}
 GA_4(G) &= \sum_{i=1}^{\frac{n}{2}} (6i-3) \times \sum_{2n-\frac{n}{2}}^{2n-1} \frac{2\sqrt{(p)(p+1)}}{p+p+1} + 3n \times \sum_{p=n} \frac{2\sqrt{(2p)(2p+1)}}{2p+2p+1} \\
 &+ 2 \times \sum_{p=n} \frac{2\sqrt{(2p+1)(2p+1)}}{2p+1+2p+1} + \sum_{i=\frac{n}{2}+3} \left(\frac{n}{2}-1\right) \times (6i-16) \times \sum_{p=2n+1}^{3n-\frac{n}{2}-1} \frac{2\sqrt{(p)(p+1)}}{p+p+1}
 \end{aligned}$$

$$+ \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} (6n-i) \times \sum_{p=3n-\frac{n}{2}}^{3n-1} \frac{2\sqrt{(p)(p+1)}}{p+p+1}.$$

After an easy simplification, we receive

$$\begin{aligned} GA_4(G) &= \sum_{i=1}^{\frac{n}{2}} \sum_{2n-\frac{n}{2}}^{2n-1} \frac{(12i-6)\sqrt{p^2+p}}{2p+1} + \sum_{p=n} \frac{6n\sqrt{4p^2+2p}}{4p+1} + \sum_{p=n} \frac{4p+2}{2p+1} \\ &\quad + \sum_{i=\frac{n}{2}+3}^{3n-\frac{n}{2}-1} \sum_{p=2n+1} (\frac{n}{2}-1)(6i-16) \times \frac{2\sqrt{p^2+p}}{2p+1} \\ &\quad + \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} \sum_{p=3n-\frac{n}{2}}^{3n-1} \frac{(12n-2i)\sqrt{p^2+p}}{2p+1}. \quad \square \end{aligned}$$

Theorem 3.2. For every $n \equiv 1(mod 4)$, consider the graph of $G \cong NA_m^n$ nanotube. Then the eccentricity version of geometric arithmetic index is equal to

$$\begin{aligned} GA_4(G) &= \sum_{i=(\frac{n-1}{2}) \times 2} i + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1(mod 2)} \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0(mod 2)} \frac{8i\sqrt{p^2+p}}{2p+1} \\ &\quad + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 0(mod 2)} \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1(mod 2)} \frac{(16i-4)\sqrt{p^2+p}}{2p+1} \\ &\quad + \sum_{i=2n+2}^{\frac{5n-1}{2}, p \equiv 0(mod 2)} \sum_{p=2n} (\frac{n+3}{4}) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\ &\quad + \sum_{i=4n}^{\frac{5n-3}{2}, p \equiv 1(mod 2)} \sum_{p=2n+1} (\frac{n-1}{4}) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\ &\quad + \sum_{i=\frac{n-1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1(mod 2)} \frac{32i\sqrt{p^2+p}}{2p+1} + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1(mod 2)} \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0(mod 2)} \frac{8i\sqrt{p^2+p}}{2p+1} + 1. \end{aligned}$$

Proof. Let G be the graph of NA_m^n nanotube. Then formula for the eccentricity version of geometric arithmetic index is equal to

$$GA_4(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\epsilon_u \epsilon_v}}{\epsilon_u + \epsilon_v}$$

By the use of the values in Table 2 we obtain

$$\begin{aligned}
 GA_4(G) = & 1 \times \sum_{p=\frac{3n+1}{2}} \frac{2\sqrt{(p)(p)}}{p+p} + \sum_{i=(\frac{n-1}{2}) \times 2} i \times \sum_{p=\frac{3n+1}{2}}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p)}}{p+p} \\
 & + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1 \pmod{2}} 4i \times \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 0 \pmod{2}} (8i-2) \times \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=2n+2} (\frac{n+3}{4}) \times i \times \sum_{p=2n}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=4n} (\frac{n-1}{4}) \times i \times \sum_{p=2n+1}^{\frac{5n-3}{2}, p \equiv 1 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=\frac{n-1}{4}}^1 16i \times \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1 \pmod{2}} 4i \times \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1}.
 \end{aligned}$$

After an easy simplification, we get

$$\begin{aligned}
 GA_4(G) = & \sum_{i=(\frac{n-1}{2}) \times 2} i + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1 \pmod{2}} \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0 \pmod{2}} \frac{8i\sqrt{p^2+p}}{2p+1} \\
 & + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 0 \pmod{2}} \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1 \pmod{2}} \frac{(16i-4)\sqrt{p^2+p}}{2p+1} \\
 & + \sum_{i=2n+2}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \sum_{p=2n} (\frac{n+3}{4}) \times i \times \frac{2\sqrt{p^2+p}}{2p+1}
 \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=4n} \sum_{p=2n+1}^{\frac{5n-3}{2}, p \equiv 1 \pmod{2}} \left(\frac{n-1}{4}\right) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n-1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1 \pmod{2}} \frac{32i\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1 \pmod{2}} \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0 \pmod{2}} \frac{8i\sqrt{p^2+p}}{2p+1} + 1. \quad \square
\end{aligned}$$

Theorem 3.3. For every $n \equiv 3 \pmod{4}$, consider the graph of $G \cong NA_m^n$ nanotube. Then the eccentricity version of geometric arithmetic index is equal to

$$\begin{aligned}
GA_4(G) = & \sum_{i=(\frac{n+1}{2}) \times 2} i + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} \frac{(32i-20)\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} \frac{16i\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=4n} \sum_{p=2n+1}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} \left(\frac{n+1}{4}\right) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=2n+2, n \neq 3} \sum_{p=2n+2, n \neq 3}^{\frac{5n-3}{2}, p \equiv 0 \pmod{2}} \left(\frac{n-3}{4}\right) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n-3}{4}, n \neq 3}^1 \sum_{p=\frac{5n+3}{2}, n \neq 3}^{3n-2, p \equiv 0 \pmod{2}} \frac{32i\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n+1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-1, p \equiv 0 \pmod{2}} \frac{(16i-8)\sqrt{p^2+p}}{2p+1}.
\end{aligned}$$

Proof. Let G be the graph of NA_m^n nanotube. Then formula for the eccentricity version of geometric arithmetic index is equal to

$$GA_4(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\epsilon_u \epsilon_v}}{\epsilon_u + \epsilon_v}.$$

By using the values from Table 3 we have

$$\begin{aligned}
 GA_4(G) = & \sum_{i=(\frac{n+1}{2}) \times 2} i \times \sum_{p=\frac{3n+3}{2}}^{\frac{5n+1}{2}, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p)}}{p+p} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} (16i-10) \times \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} 8i \times \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=4n} (\frac{n+1}{4}) \times i \times \sum_{p=2n+1}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=2n+2, n \neq 3} (\frac{n-3}{4}) \times i \times \sum_{p=2n+2, n \neq 3}^{\frac{5n-3}{2}, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=\frac{n-3}{4}, n \neq 3}^1 16i \times \sum_{p=\frac{5n+3}{2}, n \neq 3}^{3n-2, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1} \\
 & + \sum_{i=\frac{n+1}{4}}^1 (8i-4) \times \sum_{p=\frac{5n+1}{2}}^{3n-1, p \equiv 0 \pmod{2}} \frac{2\sqrt{(p)(p+1)}}{p+p+1}.
 \end{aligned}$$

After an easy simplification, we get

$$\begin{aligned}
 GA_4(G) = & \sum_{i=(\frac{n+1}{2}) \times 2} i + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} \frac{(32i-20)\sqrt{p^2+p}}{2p+1} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} \frac{16i\sqrt{p^2+p}}{2p+1} \\
 & + \sum_{i=4n}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} (\frac{n+1}{4}) \times i \times \frac{2\sqrt{p^2+p}}{2p+1}
 \end{aligned}$$

(ϵ_u, ϵ_v)	Range	Frequency	Range
$(p, p+1)$	$2n - \frac{n}{2} \leq p \leq 2n - 1$	$3 + 6(i - 1)$	$1 \leq i \leq \frac{n}{2}$
$(2p, 2p+1)$	$p = n \equiv 0(mod 2)$	$3n$	$n \equiv 0(mod 2)$
$(2p+1, 2p+1)$	$p = n \equiv 0(mod 2)$	2	$i = \frac{n}{2} + 2$
$(p, p+1)$	$2n + 1 \leq p \leq 3n - \frac{n}{2} - 1$	$(\frac{n}{2} - 1) \times [2 + 6(i - 3)]$	$i = \frac{n}{2} + 3$
$(p, p+1)$	$3n - \frac{n}{2} \leq p \leq 3n - 1$	$6n - i$	$i = 3n + 6j$, where $0 \leq j \leq \frac{n}{2} - 1$.

Table 1: The edge partition of NA_m^n nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here $n \equiv 0(mod 2)$.

(ϵ_u, ϵ_v)	Range	Frequency	Range
(p, p)	$p = \frac{3n+1}{2}$	1	
(p, p)	$\frac{3n+1}{2} < p \leq \frac{5n-1}{2}, p \equiv 0(mod 2)$	i	$i = (\frac{n-1}{2}) \times 2$
$(p, p+1)$	$\frac{3n+1}{2} \leq p \leq 2n - 2, p \equiv 0(mod 2)$	$4i$	$1 \leq i \leq \frac{n-3}{2}, i \equiv 1(mod 2)$
$(p, p+1)$	$\frac{3n+3}{2} \leq p \leq 2n - 1, p \equiv 0(mod 2)$	$8i - 2$	$2 \leq i \leq \frac{n-1}{2}, i \equiv 0(mod 2)$
$(p, p+1)$	$2n \leq p \leq \frac{5n-1}{2}, p \equiv 0(mod 2)$	$i = 2n + 2$	$(\frac{n+3}{4}) \times i$
$(p, p+1)$	$2n + 1 \leq p \leq \frac{5n-3}{2}, p \equiv 1(mod 2)$	$i = 4n$	$(\frac{n-1}{4}) \times i$
$(p, p+1)$	$\frac{5n+1}{2} \leq p \leq 3n - 2, p \equiv 1(mod 2)$	$16i$	$\frac{n-1}{4} \leq i \leq 1$
$(p, p+1)$	$\frac{5n+3}{2} \leq p \leq 3n - 1, p \equiv 0(mod 2)$	$4i$	$\frac{n-3}{2} \leq i \leq 1, i \equiv 1(mod 2)$

Table 2: The edge partition of NA_m^n nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here $n \equiv 1(mod 4)$.

$$\begin{aligned}
& + \sum_{i=2n+2, n \neq 3} \sum_{p=\frac{5n-3}{2}, p \equiv 0(mod 2)} \left(\frac{n-3}{4}\right) \times i \times \frac{2\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n-3}{4}, n \neq 3}^1 \sum_{p=\frac{5n+3}{2}, n \neq 3}^{3n-2, p \equiv 0(mod 2)} \frac{32i\sqrt{p^2+p}}{2p+1} \\
& + \sum_{i=\frac{n+1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-1, p \equiv 0(mod 2)} \frac{(16i-8)\sqrt{p^2+p}}{2p+1}. \quad \square
\end{aligned}$$

4. Conclusion

In this paper, we discuss the Geometric arithmetic index and Eccentricity version of Geometric arithmetic index. We consider the molecular graph of NA_m^n

(ϵ_u, ϵ_v)	Range	Frequency	Range
(p, p)	$\frac{3n+3}{2} \leq p \leq \frac{5n+1}{2}$	i	$i = (\frac{n+1}{2}) \times 2$
$(p, p+1)$	$\frac{3n+1}{2} \leq p \leq 2n-1, p \equiv 1(mod 2)$	$16i-10$	$1 \leq i \leq \frac{n+1}{4}$
$(p, p+1)$	$\frac{3n+3}{2} \leq p \leq 2n, p \equiv 0(mod 2)$	$8i$	$1 \leq i \leq \frac{n+1}{4}$
$(p, p+1)$	$2n+1 \leq p \leq \frac{5n-1}{2}, p \equiv 1(mod 2)$	$i = 4n$	$(\frac{n+1}{4}) \times i$
$(p, p+1)$	$2n+2 \leq p \leq \frac{5n-3}{2}, p \equiv 0(mod 2), n \neq 3$	$i = 2n+2, n \neq 3$	$(\frac{n-3}{4}) \times i, n \neq 3$
$(p, p+1)$	$\frac{5n+3}{2} \leq p \leq 3n-2, p \equiv 1(mod 2), n \neq 3$	$16i$	$\frac{n-3}{4} \leq i \leq 1, n \neq 3$
$(p, p+1)$	$\frac{5n+1}{2} \leq p \leq 3n-1, p \equiv 0(mod 2)$	$8i-4$	$\frac{n+1}{4} \leq i \leq 1$

Table 3: The edge partition of NA_m^n nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here $n \equiv 3(mod 4)$.

nanotube and we compute the eccentricity version of Geometric arithmetic index.

References

- [1] A.R. Ashrafi, M. Ghorbani, M. Jalali, Eccentric connectivity polynomial of an infinite family of Fullerenes, *Optoelectronics and Advanced Materials-Rapid Communications*, **3** No. 8 (2009), 823-826 .
- [2] A.R. Ashrafi, M. Saheli, The eccentric connectivity index of a new class of nanostar dendrimers, *Optoelectronics and Advanced Materials-Rapid Communications*, **4** No. 6 (2010), 898-899.
- [3] M. Bac, J. Horvathova, M. Mokrisova, A. Semanicova, A. Suhanyiova, On topological indices of carbon nanotube network, *Canadian Journal of Chemistry*, **93** No. 10 (2015), 1157-1160, doi: 10.1139/cjc-2015-0175.
- [4] M. Deza, P. W. Fowler, A. Rassat, K. M. Rogers, Fullerenes as tiling of surfaces, *Journal of Chemical Information and Modeling*, **40** (2000), 550-558.
- [5] M.R. Farahani, M.R.R. Kanna, R.P. Kumar, Sh. Wang, The vertex Szeged index of Titanian Carbon Nanotubes $TiO_2(m, n)$, *International Journal of Pharmaceutical Sciences and Research*, **7** No. 9 (2017), 3734-3741, doi: 10.13040/IJPSR.0975-8232.7(9).3734-41.
- [6] W. Gao, M.R. Farahani, M.K. Jamil, The eccentricity version of Atom-Bond connectivity index of Linear Polycene Parallelogram Benzenoid $ABC_5(P(n, n))$, *Acta Chimica Slovenica*, **63** (2016), In press.
- [7] W. Gao, M. Imran, A.Q. Baig, H. Ali, M.R. Farahani. On Topological Indices of Certain Interconnection Networks, *Journal of Applied Mathematics and Computing*, (2016), In press, doi: 10.1016/j.amc.2014.07.064.
- [8] W. Gao, A.Q. Baig, H. Ali, W. Sajjad, M.R. Farahani. Margin Based Ontology Sparse Vector Learning Algorithm and Applied in Biology Science, *Saudi Journal of Biological Sciences*, (2016), In press, doi: 10.1016/j.sjbs.2016.09.001.
- [9] Y. Gao, M.R. Farahani, W. Gao. A Neighborhood Union Condition for Fractional (k, n, m) -Critical Deleted Graphs, *Transactions on Combinatorics*, (2016), In press. TOC203551468697400.

- [10] M. Ghorbani, A. Khaki, A note on fourth version of geometric arithmetic index, *Opto-electronics and Advanced Materials-Rapid Communications*, **4** No. 10 (2010), 2212-2215.
- [11] M. Ghorbani, M.A. Hosseinzadeh, A new version of Zagreb indices, *Filomat*, **26** No. 1 (2012), 93-100, doi: 10.2298/FIL1201093G.
- [12] S. Gupta, M. Singh, A. K. Madan, Application of graph theory: Relationship of eccentric connectivity index and Wiener's index with anti-inflammatory activity, *Journal of Mathematical Analysis and Applications*, **266** (2002), 259-268, doi:10.1006/jmaa.2000.7243.
- [13] S. Hayat, M. Imran, Computation of certain topological indices of nanotubes, *Journal of Computational and Theoretical Nanoscience*, **12** (2015), 70-76, doi: 10.1166/jctn.2015.3699.
- [14] Y. Huo, J.B. Liu, A.Q. Baig, W. Sajjad, M. Rezaei, Z. Foruzanfar, M.R. Farahani. Connective Eccentric Index of NA_m^n Nanotube, *Journal of Computational and Theoretical Nanoscience*, (2016), In press.
- [15] M.K. Jamil, M.R. Farahani, M.R.R. Kanna, Fourth Geometric arithmetic index of Polycyclic Aromatic Hydrocarbons PAH_K , *The Pharmaceutical and Chemical Journal*, **3** No. 1 (2016), 94-99, doi: 10.21042/AMNS.2016.1.00023.
- [16] J.B. Liu, Ch. Wang, Sh. Wang, B. Wei, Zagreb Indices and Multiplicative Zagreb Indices of Eulerian graphs, *Bulletin of the Malaysian Mathematical Sciences Society*, (2016), In press.
- [17] Ch. Wang, Sh. Wang, B. Wei, Cacti with Extremal PI Index, *Transactions on Combinatorics*, **5** (2016), 1-8.
- [18] Sh. Wang, M.R. Farahani, M.R.R. Kanna, R.P. Kumar, Schultz polynomials and their topological indices of Jahangir graphs $J_{2,m}$, *Applied Mathematics*, **7** (2016), 1632-1637, doi: 10.4236/am.2016.714140 .
- [19] Sh. Wang, B. Wei, Multiplicative Zagreb indices of Cacti, *Discrete Mathematics, Algorithms and Applications*, **8** No. 3 (2016), 1650040, doi: 10.1142/S1793830916500403.
- [20] Sh. Wang, M.R. Farahani, M.R.R. Kanna, M.K. Jamil, R.P. Kumar. The Wiener Index and the Hosoya Polynomial of the Jahangir Graphs, *Applied and Computational Mathematics*, **5** (2016), 138-141, doi: 10.11648/j.acm.20160503.17.
- [21] Sh. Wang, M.R. Farahani, A.Q. Baig, W. Sajja. The sadhana polynomial and the sadhana index of polycyclic aromatic hydrocarbons PAH_K , *Journal of Chemical and Pharmaceutical Research*, **8** No. 6 (2016), 526-531.
- [22] Sh. Wang, B. Wei, Multiplicative Zagreb indices of k-trees, *Discrete Applied Mathematics*, **180** (2015), 168-175, doi: 10.1016/j.dam.2014.08.017.
- [23] Sh. Wang, B. Basavanagoud, S.M. Hosamani, M.R. Farahani. The Schultz polynomial, Modified Schultz Polynomial and Their Indices for the Jahangir Graphs $J_{4,m}$, *Pacific Journal of Applied Mathematics*, (2016), In press.
- [24] H. Wiener, Structural determination of paraffin boiling points, *Journal of the American Chemical Society*, **69** (1947), 17-20, doi: 10.1021/ja01193a005.
- [25] Y. Yuan, B. Zhou, N. Trinajstić, On geometric arithmetic index, *Journal of Mathematical Chemistry*, **47** No. 2 (2010), 833-841, doi: 10.1007/s10910-009-9603-8.
- [26] B. Zhou, Z. Du, On eccentric connectivity index, *MATCH Communications in Mathematical and in Computer Chemistry*, **63** (2010), 181-198.