EFFECT OF VELOCITY SLIP AND FRICTIONAL FORCE OF POROUS TRIANGULAR PLATES LUBRICATED WITH COUPLE STRESS FLUIDS

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Abstract: The theoretical investigation made in this paper is to study the effect of slip velocity on the performance characteristics of porous squeeze film lubrication between triangular plates with couple stress fluid. A most general modified Reynolds equation is derived using stokes constitutive equations for couple stress fluid. The fluid in the film region and in the porous region has been modeled as a couple stress fluid. The analysis takes into account velocity slip at the porous interface using Beavers-Joseph criterion. The modified form of Reynolds equation is solved and closed form expressions are obtained for pressure, load capacity, friction on the triangular plates and coefficient of friction to examine possible effects of the system.

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1. Introduction

Lubrication is the most important components of tribology as it takes care of friction and wear. The primary objective of lubrication is to reduce the severity of friction, wear and shear stresses in solid surface contacts. Lubrication are mainly used to reduce electrical and mechanical power consumption (by reducing friction) protect bearings and other components from corrosion and rust etc. Basically Lubrication are of types like Hydrodynamic, Hydrostatic lubrication and Boundary lubrication etc, depending on the functions.

In the recent years the study of squeeze films of Non-Newtonian fluids have gained interest among the researchers. Bujurke and Jayaraman (1982) studied the influence of couple stress in squeeze films. Naduvinanami, Syeda Tasneem and Hiremanth have discussed the squeeze film characteristics between the porous rectangular plates lubricated with couple stress fluid (2003).

Ramanaiah and Sarkar (1978) studied the performance of squeeze film in the thrust bearings Ramanaiah (1979) gave an excellent account on the squeeze film between finite plates of various shapes lubricated by fluids with the couple stress. The effect of bearings deformation on the characteristics of squeeze film between circular and rectangular plates and that of the slider bearings has been analyzed by Ramanaiah and Sundarammal Kesavan (1982). Gupta and Sharma (1988) have investigated the couple stress effects in a hydrostatic thrust bearings. Sudha, Sundarammal and Ramamurthy (2005, 2006, and 2008) have analysed squeeze film characteristics of rectangular, elliptic and porous triangular plates lubricated with couple stress. Beavers and Joseph in [1] proposed an alternate boundary condition which admits that a non-zero tangential velocity i.e. a slip velocity at the preamble wall influences the squeezing action. Sparrow et.al. in [3] studied its effect on porous walled squeeze films. Using the slip boundary condition, Wu studied the effects of slip velocity for porous annular disc (see [3]) and between porous rectangular plates (see [4]). Murti in [5] considered the problem of squeeze film between two circular disks. Prakash and Viji in [6] considered the squeeze film between porous plates of different shapes to include the effect of velocity slip. These investigation shows that the effect of velocity slip further reduces the load capacity and the squeeze film approach time. The study of velocity slip was also considered for various bearing.
2. Formulation of the Problem

The squeezing flow of couple stress fluid between two triangular plates is considered. The upper plate is approaching the lower plate with constant velocity. Under the usual assumptions of hydrodynamic lubrication of thin films, the continuity and momentum equations derived by stokes for the couple stress fluid takes the form

\[\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \frac{\partial p}{\partial x} = 0,\]  \hspace{1cm} (1)

\[\mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \frac{\partial p}{\partial y} = 0,\]  \hspace{1cm} (2)

\[\frac{\partial p}{\partial z} = 0,\]  \hspace{1cm} (3)

\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.\]  \hspace{1cm} (4)

Integrating equation (1) & (2) with respect to the boundary condition \(u = 0\) at \(z = 0, h\), we get

\[u = \frac{\partial p}{2\mu} \left\{ (z^2 - zh) + 2l^2 \left[ 1 - \frac{\cosh \left( \frac{2z-h}{2l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right\}, \]  \hspace{1cm} (5)

\[v = \frac{\partial p}{2\mu} \left\{ (z^2 - zh) + 2l^2 \left[ 1 - \frac{\cosh \left( \frac{2z-h}{2l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right\}. \]  \hspace{1cm} (6)

The flow of the couple stress fluid in the porous matrix is governed by the modified form of Darcy’s law which account for the polar effect is

\[u^* = -\frac{k}{\mu(1 - \beta)} \frac{\partial p^*}{\partial x},\]  \hspace{1cm} (7)

\[v^* = -\frac{k}{\mu(1 - \beta)} \frac{\partial p^*}{\partial y},\]  \hspace{1cm} (8)

\[w^* = -\frac{k}{\mu(1 - \beta)} \frac{\partial p^*}{\partial z}.\]  \hspace{1cm} (9)

The boundary conditions for the velocity components at \(z = 0\) are

\[\frac{\alpha}{\sqrt{k}} (u - u^*) = \frac{\partial u}{\partial y}.\]  \hspace{1cm} (10)
The pressure \( p^* \) in the porous medium satisfies the Laplace equation
\[
\nabla^2 p^* = 0.
\] (14)

The equations (10) and (11) are Beavers Joseph slip boundary conditions for the tangential velocity slip at the porous interface. Here \( \alpha \) is the slip coefficient which is a dimensionless quantity depending on the material parameters and characterizes the structure of the permeable material within the boundary region.

Solving, we receive
\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu \frac{dh}{dt}}{f(h, c_1, c_2, l) + 12 \left( \frac{\delta k}{1 - \beta} \right)},
\] (15)

where
\[
f(h, c_1, c_2, l) = h^3(1 + c_2) - 6h^2lc_1 \tanh \left( \frac{h}{2l} \right)
- 12l^2 \left[ h - 2l \tanh \left( \frac{h}{2l} \right) \right],
\] (16)
\[
c_1 = \frac{s}{s + h},
\] (17)
\[
c_2 = \frac{3}{(1 - \beta)} \left[ \frac{2s^2\alpha^2}{(h^2 + sh)} + \frac{s(1 - \beta)}{(s + h)} \right].
\] (18)

Here \( s = \frac{\sigma}{h_0} \), \( \sigma = \frac{\sqrt{k}}{\alpha} \), \( l = \sqrt{\frac{\eta}{\mu}} \).

The pressure distribution is obtained by solving equation (14) with the boundary condition \( p = 0 \) on the boundary of the plates.

\[
p = -12\mu \frac{dh}{dt} \left[ f(h, c_1, c_2, l) + 12 \left( \frac{\delta k}{1 - \beta} \right) \right]^{-1}
\times \left( \frac{1}{2\sqrt{3}a} \left[ (x^2 - 3y^2) \left( \frac{\sqrt{3}}{2} a - x \right) \right] \right). \] (20)
The load is given by

\[
    w = -\mu A^2 R \frac{d\bar{h}}{dt} \left[ f(h, c_1, c_2, l) + 12 \left( \frac{\delta k}{1 - \beta} \right) \right]^{-1},
\]

where \( A \) is the area of the upper plate and

\[
    R = \frac{12}{A^2} \int \int_A \left( \frac{1}{2\sqrt{3}a} \left( x^2 - 3y^2 \right) \left( \frac{\sqrt{3}}{2} a - x \right) \right) dx dy,
\]

and

\[
    f(h, c_1, c_2, l) = h^3(1 + c_2) - 6h^2lc_1 \tanh \left( \frac{h}{2l} \right) - 12l^2 \left[ h - 2l \tanh \left( \frac{h}{2l} \right) \right],
\]

\[
    \frac{f(h, c_1, c_2, l)}{h^3} = (1 + c_2) - 6 \left( \frac{l}{h} c_1 \right) \tanh \left( \frac{h}{2l} \right) - \frac{12l^2}{h^2} \left[ 1 - \frac{2l}{h} \tanh \left( \frac{h}{2l} \right) \right],
\]

\[
    \frac{f(h, c_1, c_2, l)}{h^3} = (1 + c_2) - 6 \left( \frac{\overline{l}}{h} c_1 \right) \tanh \left( \frac{h}{2l} \right) - \frac{12l^2}{h^2} \left[ 1 - \frac{2\overline{l}}{h} \tanh \left( \frac{h}{2l} \right) \right],
\]

where \( \overline{l} = \frac{l}{l_0} \) and \( \overline{h} = \frac{h}{h_0} = F(\overline{h}, c_1, c_2, \overline{l}) \).

The non dimensional is given by

\[
    \overline{w} = -\frac{w h_0^2 \overline{t}}{\mu A^2}.
\]

The load is given by

\[
    \overline{w} = R \int_{\overline{h}}^{1} \left[ \frac{d\overline{h}}{\overline{h}^3} \left[ F(\overline{h}, c_1, c_2, \overline{l}) \right] + \frac{12\psi}{1 - \beta} \right].
\]

The shear stress along the surface is

\[
    \tau = \frac{\partial \bar{p}}{2} \left\{ (2z - h) - 2l \left[ \frac{\sinh \left( \frac{2z - h}{2l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right\}.
\]

At the upper plate \( z = h \) the shear stress is

\[
    \tau = \frac{\partial \bar{p}}{2} \left\{ h - 2l \tanh \left( \frac{h}{2l} \right) \right\}.
\]
At the lower plate \( z = 0 \) the shear stress is

\[ \tau = \frac{\partial p}{\partial x} \left\{ -h + 2l \tanh \left( \frac{h}{2l} \right) \right\}. \quad (26) \]

Integrating the shear stress over the surface gives the frictional force \( F \).

\[ F = \int \tau dx. \quad (27) \]

The coefficient of friction is obtained in the form

\[ \overline{f} = \frac{F}{w}. \quad (28) \]

3. Results and Discussions

Friction can be defined as a force resisting movement—the force exerted by either of two contacting bodies tending to oppose relative tangential displacement of the other. Coefficient of friction is independent of sliding speed. The behavior of bearing characteristics such as pressure distribution, load carrying capacity and friction force can be obtained by the numerical solution of the modified Reynolds equation.

In this section we have presented the graphical results of the effect of slip velocity and friction for various parameter.

![Figure 1: Variation \( l \) on \( h \) when \( \alpha = 0.05, \beta = 0.25, s = 0.5 \)]

Figure 1, The slip parameter shows a constant distribution till the values of the fluid film thickness \( h = 0 \), and it drops suddenly. It shows that it
deviates from increasing values to decreasing values which shows that there is an accurate work input or the energy transfer between the fluid film surface and the fluid. We have taken constant value of the parameter $\beta$ which describe movement and filtering of fluids through porous material as $\beta = 0.25$ which shows that there is a constant transport of fluid in porous media which leads to high work capacity.

Figure 2: Variation $s$ on $h$ when $\alpha = 0.01, \beta = 0.50, l = 0.5$

Figure 2 shows that there is a uniform distribution as the value of slip velocity increases from minimum to maximum, the gap between the coordinates is constant which states that the pressure distribution is uniform which leads to high work load capacity. It shows the bearing characteristics in which it can resist high support in critical situation when it is lubricated with a couple stress fluid.

Figure 3 shows as we increase the values of couple stress parameter, the work load increases uniformly. In other words the bearings can hold the support and it increases the life span of the machine.

Figure 4 shows as the film thickness increases the gap between the coordinates increases which states that it can resist the obstacles which is coming during the working. The work load increases drastically with the couple stress as a lubricant.

Figure 5. It can be seen that as the slip parameter increases frictional force also increases.
Figure 3: Variation \( l \) on \( \bar{h} \) when \( \alpha = 0.20, \beta = 0.4, s = 1, \psi = 0.001 \)

4. Conclusion

On the basis of the microcontinuum theory, this paper predict the effect of the shear stress and friction of porous triangular plates lubricated with couple stress fluid considering the effects of slip velocity. The value of friction was found to increase with increase in slip velocity. The modified Reynolds equations was derived by using the stokes constitutive equations to account for the couple stress effects due to the lubricant. The film pressure was solved and applied to predict the load carrying capacity and friction force of the system. As the value of couple stress parameter approaches zero, the bearing characteristics approaches the Newtonian- lubricant case. According to the result evaluated, the influence of slip velocity effects on the triangular plates characteristics is physically significant.

References


Figure 4: Variation $s$ on $h$ when $\alpha = 0.2, \beta = 0.75, l = 0.5, \psi = 0.001$

Figure 5: Variation $\tau$ on $p$ when $s = 1.25, l = 0.2, h = 1.5$


**Appendix: Nomenclature**

- $A$ – Area of the plate
- $h$ – Fluid film thickness
- $\overline{h}$ – Non-dimensional film height $=\frac{h}{h_0}$
- $h_0$ – Initial film thickness
- $l$ – Couple stress parameter
- $\overline{l}$ – Non-dimensional couple stress parameter
- $p$ – Pressure in film region
- $s$ – Slip parameter
- $u, v, w$ – Components of fluid velocity in the film region
- $u^*, v^*, w^*$ – Components of fluid velocity in the porous region
- $w$ – Work load
- $\overline{w}$ – Dimensionless load
- $x, y, z$ – Cartesian co-ordinates
- $\alpha$ – Slip constant
- $\beta$ – Percolation parameter $=\frac{\eta}{\mu k}$
- $\mu$ – Viscosity
- $\eta$ – New material constant
- $\Psi$ – Permeability parameter $=\frac{k \delta}{\eta}$
- $\tau$ – Shear stress
- $F$ – Friction
- $\overline{f}$ – Co-efficient of friction
- $\delta$ – Thickness of the porous layer