ON PRE GENERALIZED B-CLOSED SET
IN TOPOLOGICAL SPACES

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Abstract: In this article, we introduce a new class of sets called pre generalized $b$-closed sets in topological spaces (briefly $pgb$-closed set). Also we discuss some of their properties and investigate the relations between the associated topology.

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1. Introduction

type of generalized closed sets in topological space called \( b \) closed sets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A. A. Omari and M. S. M. Noorani [2] made an analytical study and result the concepts of generalized \( b \)-closed sets in topological spaces.


The aim of this paper is to continue the study of pre generalized \( b \)-closed sets. The notion of pre generalized \( b \)-closed sets and its different characterizations are given in this paper. It has been proved that the class of pre generalized \( b \)-closed set lies between the class of \( b \)-closed sets and \( rg \)-closed sets. Throughout this paper \((X, \tau)\) and \((Y, \sigma)\) represents the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let \( A \subseteq X \), the closure of \( A \) and interior of \( A \) will be denoted by \( cl(A) \) and \( int(A) \) respectively, union of all \( b \)-open sets \( X \) contained in \( A \) is called \( b \)-interior of \( A \) and it is denoted by \( bint(A) \), the intersection of all \( b \)-closed sets of \( X \) containing \( A \) is called \( b \)-closure of \( A \) and it is denoted by \( bcl(A) \).

2. Preliminaries

In this section, we referred some of the closed set definitions which was already defined by various authors.

**Definition 2.1.** [15] Let a subset \( A \) of a topological space \((X, \tau)\), is called a pre-open set if \( A \subseteq int(cl(A)) \).

**Definition 2.2.** [11] Let a subset \( A \) of a topological space \((X, \tau)\), is called a semi-open set if \( A \subseteq cl(int(A)) \).

**Definition 2.3.** [16] Let a subset \( A \) of a topological space \((X, \tau)\), is called a \( \alpha \) -open set if \( A \subseteq int(cl(int(A))) \).

**Definition 2.4.** [4] Let a subset \( A \) of a topological space \((X, \tau)\), is called a \( b \)-open set if \( A \subseteq cl(int(A)) \cup int(cl(A)) \).
Definition 2.5. [10] Let a subset $A$ of a topological space $(X, \tau)$, is called a generalized closed set (briefly $g$-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.

Definition 2.6. [12] Let a subset $A$ of a topological space $(X, \tau)$, is called a generalized $\alpha$ closed set (briefly $g\alpha$ - closed) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$ open in $X$.

Definition 2.7. [2] Let a subset $A$ of a topological space $(X, \tau)$, is called a generalized $b$- closed set (briefly $gb$- closed) if $b\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.

Definition 2.8. [13] Let a subset $A$ of a topological space $(X, \tau)$, is called a generalized $\alpha^*$-closed set (briefly $g\alpha^*$-closed) if $\alpha\text{cl}(A) \subseteq \text{int}U$ whenever $A \subseteq U$ and $U$ is $\alpha$ open in $X$.

Definition 2.9. [18] Let a subset $A$ of a topological space $(X, \tau)$, is called a pre-generalized closed set (briefly $pg$- closed) if $p\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open in $X$.

Definition 2.10. [6] Let a subset $A$ of a topological space $(X, \tau)$, is called a semi generalized closed set (briefly $sg$- closed) if $s\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $X$.

Definition 2.11. [19] Let a subset $A$ of a topological space $(X, \tau)$, is called a generalized $\alpha b$- closed set (briefly $g\alpha b$- closed) if $b\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$ open in $X$.

Definition 2.12. [14] Let a subset $A$ of a topological space $(X, \tau)$, is called a regular generalized $b$- closed set (briefly $rgb$- closed) if $b\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.

3. Pre Generalized $b$-Closed Sets

In this section, we introduce pre generalized $b$-closed set and investigate some of their properties.

Definition 3.1. Let a subset $A$ of a topological space $(X, \tau)$, is called a pre generalized $b$- closed set (briefly $pgb$- closed) if $b\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre open in $X$. 
**Theorem 3.2.** Every closed set is $pgb$-closed.

*Proof.* Let $A$ be any closed set in $X$ such that $A \subseteq U$, where $U$ is pre open. Since $bcl(A) \subseteq cl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set in $X$.

The converse of above theorem need not be true as seen from the following example.

**Example 3.3.** Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{a, c\}$ is $pgb$-closed set but not a closed set.

**Theorem 3.4.** Every $b$-closed set is $pgb$-closed.

*Proof.* Let $A$ be any $b$-closed set in $X$ such that $U$ be any pre open set containing $A$. Since $A$ is $b$ closed, $bcl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.5.** Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. The set $\{a, c\}$ is $pgb$-closed set but not a $b$-closed set.

**Theorem 3.6.** Every $g\alpha$-closed set is $pgb$-closed set.

*Proof.* Let $A$ be any $g\alpha$-closed set in $X$ and $U$ be any pre open set containing $A$. Since $A$ is $g\alpha$ closed, $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.7.** Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. The set $\{c\}$ is $pgb$-closed set but not a $g\alpha$-closed set.

**Theorem 3.8.** Every $g\alpha^*$-closed set is $pgb$-closed set.

*Proof.* Let $A$ be any $g\alpha^*$-closed set in $X$ and $U$ be any pre open set containing $A$. Since $A$ is $g\alpha^*$ closed, $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.9.** Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. The set $\{c\}$ is $pgb$-closed set but not a $g\alpha^*$-closed set.

**Theorem 3.9.** Every $g$-closed set is $pgb$-closed set.
Proof. Let $A$ be any $g$-closed set in $X$ and $U$ be any open set containing $A$. Since every open set is pre open set, $bcl(A) \subseteq cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example. □

Example 3.11. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. The set $\{c\}$ is $pgb$-closed set but not a $g$-closed set.

Theorem 3.12. Every $gab$-closed set is $pgb$-closed set.

Proof. Let $A$ be any $gab$-closed set in $X$ and $U$ be any $\alpha$-open set containing $A$. Since every $\alpha$-open set is pre open set, $bcl(A) \subseteq cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example. □

Example 3.13. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. The set $\{a\}$ is $pgb$-closed set but not a $gab$-closed set.


Proof. Let $A$ be any $rgb$-closed set in $X$ and $U$ be any regular open set containing $A$. Since every regular open sets are $\alpha$-open sets and every $\alpha$-sets are pre open set, $bcl(A) \subseteq U$ and $U$ is pre open. Hence $A$ is $pgb$-closed set.

The converse of above theorem need not be true as seen from the following example. □

Example 3.15. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{a\}$ is $pgb$-closed set but not a $rgb$-closed set.

Theorem 3.16. Every $pgb$-closed set is $pg$-closed set.

Proof. Let $A$ be any $pgb$-closed set such that $U$ is pre open set containing $A$. Since $A$ is $pgb$ closed, $bcl(A) \subseteq pcl(A) \subseteq U$. Hence $A$ is $pg$-closed set.

The converse of above theorem need not be true as seen from the following example. □

Example 3.17. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{a, b\}$ is $pg$-closed set but not a $pgb$-closed set.

Theorem 3.18. Every $sg$-closed set is $pgb$-closed set.

Proof. Let $A$ be any $sg$-closed set such that $U$ is pre open set containing $A$. Since $A$ is $sg$ closed, $bcl(A) \subseteq scl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$
is $pgb$-closed set. The converse of above theorem need not be true as seen from the following example.

Example 3.19. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, c\}\}$. The set $\{b\}$ is $pgb$-closed set but not a $sg$-closed set.

4. Characteristics of $pgb$-Closed Sets

In this section, we investigate about the characteristics of pre generalized $b$-closed set.

Theorem 4.1. If $A$ and $B$ are $pgb$-closed sets in $X$ then $A \cup B$ is $pgb$-closed set in $X$.

Proof. Let $A$ and $B$ are $pgb$-closed sets in $X$ and $U$ be any pre open set containing $A$ and $B$. Therefore $bcl(A) \subseteq U$, $bcl(B) \subseteq U$. Since $A \subseteq U$, $B \subseteq U$ then $A \cup B \subseteq U$. Hence $bcl(A \cup B) = bcl(A) \cup bcl(B) \subseteq U$. Therefore $A \cup B$ is $pgb$-closed set in $X$. □

Theorem 4.2. If a set $A$ is $pgb$-closed sets iff $bcl(A) - A$ contains no non empty pre closed set.

Proof. Necessary: Let $F$ be a pre closed set in $X$ such that $F \subseteq bcl(A) - A$. Then $A \subseteq X - F$. Since $A$ is $pgb$-closed set and $X - F$ is pre open then $bcl(A) \subseteq X - F$. (i.e.) $F \subseteq X - bcl(A)$. So $F \subseteq (X - bcl(A)) \cap (bcl(A) - A)$. Therefore $F = \phi$.

Sufficiency: Let us assume that $bcl(A) - A$ contains no non empty pre closed set. Let $A \subseteq U, U$ is pre open. Suppose that $bcl(A)$ is not contained in $U$, $bcl(A) \cap U^c$ is a non-empty pre closed set of $bcl(A) - A$ which is contradiction. Therefore $bcl(A) \subseteq U$. Hence $A$ is $pgb$-closed. □

Theorem 4.3. The intersection of any two subsets of $pgb$-closed sets in $X$ is $pgb$-closed set in $X$.

Proof. Let $A$ and $B$ are any two sub sets of $pgb$-closed sets. $A \subseteq U, U$ is any pre open and $B \subseteq U, U$ is pre open. Then $bcl(A) \subseteq U, bcl(B) \subseteq U$, therefore $bcl(A \cap B) \subseteq U, U$ is pre open in $X$. Since $A$ and $B$ are $pgb$-closed set. Hence $A \cap B$ is a $pgb$-closed set. □

Theorem 4.4. If $A$ is $pgb$-closed set in $X$ and $A \subseteq B \subseteq bcl(A)$, Then $B$ is $pgb$-closed set in $X$. 
Proof. Since $B \subseteq bcl(A)$, we have $bcl(B) \subseteq bcl(A)$ then $bcl(B) - B \subseteq bcl(A)$. By theorem 4.2., $bcl(A) - A$ contains no non empty pre closed set. Hence $bcl(B) - B$ contains no non empty pre closed set. Therefore $B$ is $pgb$-closed set in $X$. □

**Theorem 4.5.** If $A \subseteq Y \subseteq X$ and suppose that $A$ is $pgb$-closed set in $X$ then $A$ is $pgb$-closed set relative to $Y$.

Proof. Given that $A \subseteq Y \subseteq X$ and $A$ is $pgb$-closed set in $X$. To prove that $A$ is $pgb$-closed set relative to $Y$. Let us assume that $A \subseteq Y \cap U$, where $U$ is pre open in $X$. Since $A$ is $pgb$-closed set, $A \subseteq U$ implies $bcl(A) \subseteq U$. It follows that $Y \cap bcl(A) \subseteq Y \cap U$. That is $A$ is $pgb$-closed set relative to $Y$. □

**Theorem 4.6.** If $A$ is both pre open and $pgb$-closed set in $X$, then $A$ is pre closed set.

Proof. Since $A$ is pre open and $pgb$ closed in $X$, $bcl(A) \subseteq U$. But $A \subseteq bcl(A)$. Therefore $A = bcl(A)$. Hence $A$ is pre closed set. □

**Theorem 4.7.** For $x \in X$ then the set $X - \{x\}$ is a $pgb$-closed set or pre open.

Proof. Suppose that $X - \{x\}$ is not pre open, then $X$ is the only pre open set containing $X - \{x\}$. (i.e.) $bcl(X - \{x\}) \subseteq X$. Then $X - \{x\}$ is $pgb$-closed in $X$. □

## 5. Pre Generalized $b$-Open Sets and Pre Generalized $b$-Neighbourhoods

In this section, we introduce pre generalized $b$-open sets (briefly $pgb$-open) and pre generalized $b$-neighborhoods (briefly $pgb$-nbhd) in topological spaces by using the notions of $pgb$-open sets and study some of their properties.

**Definition 5.1.** A subset $A$ of a topological space $(X, \tau)$, is called pre generalized $b$-open set (briefly $pgb$-open set) if $A^c$ is $pgb$-closed in $X$. We denote the family of all $pgb$-open sets in $X$ by $pgb - O(X)$.

**Theorem 5.2.** If $A$ and $B$ are $pgb$-open sets in a space $X$. Then $A \cap B$ is also $pgb$-open set in $X$.

Proof. If $A$ and $B$ are $pgb$-open sets in a space $X$. Then $A^c$ and $B^c$ are $pgb$-closed sets in a space $X$. By Theorem 4.6. $A^c \cup B^c$ is also $pgb$-closed set
in $X$. (i.e.) $A^c \cup B^c = (A \cap B)^c$ is a pgb-closed set in $X$. Therefore $A \cap B$ is a pgb-open set in $X$. □

**Remark 5.3.** The union of two pgb-open sets in $X$ is generally not a pgb-open set in $X$.

**Example 5.4.** Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{b, c\}\}$. If $A = \{b\}, B = \{c\}$ are pgb-open sets in $X$, then $A \cup B = \{b, c\}$ is not pgb-open set in $X$.

**Theorem 5.5.** If $\text{int}(B) \subseteq B \subseteq A$ and if $A$ is pgb-open in $X$, then $B$ is pgb-open in $X$.

**Proof.** Suppose that $\text{int}(B) \subseteq B \subseteq A$ and $A$ is pgb-open in $X$ then $A^c \subseteq B^c \subseteq \text{cl}(A^c)$. Since $A^c$ is pgb-closed in $X$, by theorem 5.2. B is pgb-open in $X$. □

**Definition 5.6.** Let $x$ be a point in a topological space $X$ and let $x \in X$. A subset $N$ of $X$ is said to be a pgb-nbhd of $x$ iff there exists a pgb-open set $G$ such that $x \in G \subset N$.

**Definition 5.7.** A subset $N$ of Space $X$ is called a pgb-nbhd of $A \subset X$ iff there exists a pgb-open set $G$ such that $A \subset G \subset N$.

**Theorem 5.8.** Every nbhd $N$ of $x \in X$ is a pgb-nbhd of $X$.

**Proof.** Let $N$ be a nbhd of point $x \in X$. To prove that $N$ is a pgb-nbhd of $x$. By definition of nbhd, there exists an open set $G$ such that $x \in G \subset N$. Hence $N$ is a pgb-nbhd of $x$. □

**Remark 5.9.** In general, a pgb-nbhd of $x \in X$ need not be a nbhd of $x$ in $X$ as seen from the following example.

**Example 5.10.** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{c\}, \{a, c\}\}$. Then $\text{pgb} - O(X) = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{b, c\}$ is pgb-nbhd of point $c$, since the pgb-open sets $\{c\}$ is such that $c \in \{c\} \subset \{b, c\}$. However, the set $\{b, c\}$ is not a nbhd of the point $c$, since no open set $G$ exists such that $c \in G \subset \{b, c\}$.

**Remark 5.11.** The pgb-nbhd $N$ of $x \in X$ need not be a pgb-open in $X$.

**Theorem 5.12.** If a subset $N$ of a space $X$ is pgb-open, then $N$ is pgb-nbhd of each of its points.
Proof. Suppose \( N \) is \( pgb \)-open. Let \( x \in N \), We claim that \( N \) is \( pgb \)-nbhd of \( x \). For \( N \) is a \( pgb \)-open set such that \( x \in N \subset N \). Since \( x \) is an arbitrary point of \( N \), it follows that \( N \) is a \( pgb \)-nbhd of each of its points. \( \square \)

**Remark 5.13.** In general, a \( pgb \)-nbhd of \( x \in X \) need not be a nbhd of \( x \) in \( X \) as seen from the following example.

**Example 5.14.** Let \( X = \{a, b, c\} \) with topology \( \tau = \{X, \phi, \{a, b\}\} \). Then \( pgb−O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\} \). The set \( \{b, c\} \) is \( pgb \)-nbhd of the point \( b \), since the \( pgb \)-open sets \( \{b\} \) is such that \( b \in \{b\} \subset \{b, c\} \). Also the set \( \{b, c\} \) is \( pgb \)-nbhd of each of its points. However, the set \( \{b, c\} \) is not a \( pgb \)-open set in \( X \).

**Theorem 5.15.** Let \( X \) be a topological space. If \( F \) is \( pgb \)-closed subset of \( X \) and \( x \in F^c \). Prove that there exists a \( pgb \)-nbhd \( N \) of \( x \) such that \( N \cap F = \phi \).

Proof. Let \( F \) be \( pgb \)-closed subset of \( X \) and \( x \in F^c \). Then \( F^c \) is \( pgb \)-open set of \( X \). So by theorem 5.12 \( F^c \) contains a \( pgb \)-nbhd of each of its points. Hence there exists a \( pgb \)-nbhd \( N \) of \( x \) such that \( N \subset F^c \). (i.e.) \( N \cap F = \phi \). \( \square \)

**Definition 5.16.** Let \( x \) be a point in a topological space \( X \). The set of all \( pgb \)-nbhd of \( x \) is called the \( pgb \)-nbhd system at \( x \), and is denoted by \( pgb−N(x) \).

**Theorem 5.17.** Let a \( pgb \)-nbhd \( N \) of \( x \) be a topological space and each \( x \in X \). Let \( pgb−N(X, \tau) \) be the collection of all \( pgb \)-nbhd of \( x \). Then we have the following results:

\[(i) \ \forall x \in X, pgb−N(x) \neq \phi. \]

\[(ii) N \in pgb−N(x) \Rightarrow x \in N. \]

\[(iii) N \in pgb−N(x), M \supset N \Rightarrow M \in pgb−N(x). \]

\[(iv) N \in pgb−N(x), M \in pgb−N(x) \Rightarrow N \cap M \in pgb−N(x). \]

\[(v) N \in pgb−N(x) \Rightarrow \text{there exists } M \in pgb−N(x) \text{ such that } M \subset N \text{ and } M \in pgb−N(y) \text{ for every } y \in M. \]

Proof. (i) Since \( X \) is \( pgb \)-open set, it is a \( pgb \)-nbhd of every \( x \in X \). Hence there exists at least one \( pgb \)-nbhd (namely-\( X \)) for each \( x \in X \). Therefore \( pgb−N(x) \neq \phi \) for every \( x \in X \):
(ii) If $N \in \text{pgb} - N(x)$, then $N$ is $\text{pgb-}\text{nbhd}$ of $x$. By definition of $\text{pgb-}\text{nbhd}$, $x \in N$.

(iii) Let $N \in \text{pgb} - N(x)$ and $M \supset N$. Then there is a $\text{pgb-open}$ set $G$ such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so $M$ is $\text{pgb-}\text{nbhd}$ of $x$. Hence $M \in \text{pgb} - N(x)$.

(iv) Let $N \in \text{pgb} - N(x)$, $M \in \text{pgb} - N(x)$. Then by definition of $\text{pgb-}\text{nbhd}$, there exists $\text{pgb-open}$ sets $G_1$ and $G_2$ such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$. Hence $x \in G_1 \cap G_2 \subset N \cap M$. Since $G_1 \cap G_2$ is a $\text{pgb-open}$ set,(being the intersection of two regular open sets), it follows from $x \in G_1 \cap G_2 \subset N \cap M$ that $N \cap M$ is a $\text{pgb-}\text{nbhd}$ of $x$. Hence $N \cap M \in \text{pgb} - N(x)$.

(v) Let $N \in \text{pgb} - N(x)$, Then there is a $\text{pgb-open}$ set $M$ such that $x \in M \subset N$. Since $M$ is $\text{pgb-open}$ set, it is $\text{pgb-}\text{nbhd}$ of each of its points. Therefore $M \in \text{pgb} - N(y)$ for every $y \in M$.

**Theorem 5.18.** Let $X$ be a nonempty set, and for each $x \in X$, let $\text{pgb} - N(x)$ be a nonempty collection of subsets of $X$ satisfying following conditions:

(i) $N \in \text{pgb} - N(x) \Rightarrow x \in N$.

(ii) $N \in \text{pgb} - N(x), M \in \text{pgb} - N(x) \Rightarrow N \cap M \in \text{pgb} - N(x)$.

Let $\tau$ consists of the empty set and all those non-empty subsets of $G$ of $X$ having the property that $x \in G$ implies that there exists an $N \in \text{pgb} - N(x)$ such that $x \in N \subset G$. Then $\tau$ is a topology for $X$.

**Proof.** (i) $\phi \in \tau$ By definition. We have to show that $x \in \tau$. Let $x$ be any arbitrary element of $X$. Since $\text{pgb} - N(x)$ is non-empty, there is an $N \in \text{pgb} - N(x)$ and so $x \in N$ by (i). Since $N$ is a subset of $X$, we have $x \in N \subset X$. Hence $x \in \tau$.

(ii) Let $G_1 \in \tau$ and $G_2 \in \tau$. If $x \in G_1 \cap G_2$ then $x \in G_1$ and $x \in G_2$. Since $G_1 \in \tau$ and $G_2 \in \tau$, there exists $N \in \text{pgb} - N(x)$ and $M \in \text{pgb} - N(x)$, such that $x \in N \subset G_1$ and $x \in M \subset G_2$. Then $x \in N \cap M \subset G_1 \cap G_2$. But $N \cap M \in \text{pgb} - N(x)$ by (2). Hence $G_1 \cap G_2 \in \tau$.

**6. Conclusions**

The classes of pre generalized b-closed sets defined using pre open sets form a topology. The $\text{pgb-closed}$ sets can be used to derive a new decomposition of continuity, closed maps and open maps, contra continuous function, almost
contra continuous function, closure and interior. This idea can be extended to fuzzy topological space and intuitionistic fuzzy topological spaces.

References
