International Journal of Pure and Applied Mathematics
Volume 112 No. 1 2017, 177-188
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v112i1.14

ψ-CONTRACTIVE TYPE FUZZY MAPPING AND ITS APPLICATIONS

J. Jeyachristy Priskillal[1 §], P. Thangavelu[2]

[1]Department of Mathematics
Karunya University
Coimbatore, Tamil Nadu, 641114, INDIA

[2]Ramanujam Centre for Mathematical Sciences
Thiruppuvanam, Tamil Nadu, 630611, INDIA

Abstract: This manuscript contains a fixed fuzzy point theorems using ψ-contractive fuzzy mapping in a complete metric space and gives applications to fuzzy differential equations.

AMS Subject Classification: 47H10, 34A07, 37C25, 54E40, 54E50
Key Words: metric space, fuzzy mapping, fixed fuzzy point, fuzzy differential equations

1. Introduction

Vital tool in theory of metric spaces is the Banach fixed point theorem. Many Mathematicians has studied the concept of fixed point theorems and its applications. As an overview of Banach contraction theorem, ψ-contractive type mapping was introduced by Berinde [2]. Fuzzy mapping was introduced by Heilpern [6]. Further, contractive type fuzzy mapping was introduced by Byung Soo Lee and Sung Jin Cho [3]. Continuing this, many authors had considered the fixed fuzzy point theorem for fuzzy mappings [1], [4], [5], [11]. In 2003, Lakshmikantham and Mohapatra [10] applied the Banach contraction theorem

Received: November 29, 2016
Revised: January 12, 2017
Published: January 26, 2017

§Correspondence author

to fuzzy differential equations. Progressing this, Hemant Kumar Nashine [7] et al proved a fixed fuzzy point theorem and gave application of fuzzy differential equations [8], [9]. This manuscript contains a fixed fuzzy point theorems using ψ-contractive fuzzy mapping in a complete metric space and gives applications to fuzzy differential equations.

2. Preliminaries

According to Berinde [2] the function ψ is defined as follows:

Let Ψ be the family of nondecreasing functions $\ni \sum_{n=1}^{+\infty} \psi^n(s) < +\infty, \forall s > 0$, where ψ^n is the n-th iterate of ψ.

Lemma 2.1. [2] If $\psi \in \Psi$, then the following are satisfied.

1. $\psi(s) < s, \forall s > 0$;
2. $\psi(0) = 0$;
3. ψ is right continuous at $s = 0$;

Definition 2.2. [2] In a metric space (X, d), we say that a mapping $F : X \to X$ is an ψ-contraction if $\exists$ a function $\psi \in \Psi \ni d(Fu, Fv) \leq \psi(d(u, v))$,for all $u, v \in X$.

Definition 2.3. [6] Let A be a fuzzy set in X and $\alpha \in [0, 1]$. Then the α-level set A_α of A is defined as $A_\alpha = \{u : A(u) \geq \alpha\}$.

Definition 2.4. [6] Let A be a fuzzy set in a metric linear space (X, d) and $\alpha \in [0, 1]$. A is an approximate quantity iff A_α is compact and convex in X and $\sup_{u \in X} Au = 1$.

Definition 2.5. [6] Let (X, d) be a metric space and $W(X)$ be a collection of approximation quantities. The family $W_\alpha(X) = \{A \in I^X : A_\alpha$ is nonempty, compact and convex$\}$. Let $A, B \in W(X)$. Then we shall define a distance between two approximate quantities. Define,

$$p_\alpha(A, B) = \inf_{u \in A_\alpha, v \in B_\alpha} d(u, v),$$
$$D_\alpha(A, B) = H(A_\alpha, B_\alpha),$$
$$D(A, B) = \sup_\alpha D_\alpha(A, B).$$

p_α is called a α-space, D_α is called a α-distance, D is called the distance and H is called the Hausdorff distance.

Definition 2.6. [6] Let $\alpha \in (0,1]$. The fuzzy point u_α of X is the fuzzy set of X defined by $u_\alpha(u) = \alpha$ and $u_\alpha(w) = 0$ if $w \neq u$.

Definition 2.7. [6] Let X be an arbitrary set, Y be any metric linear space and $I = [0,1]$. A fuzzy set of X is an element of I^X. T is a fuzzy mapping iff T is a mapping from the set X into a family $W(X) \subset I^X$, that is $T(u) \in W(X) \forall u \in X$.

Definition 2.8. [6] Let (X,d) be a metric space. Let $A, B \in W(X)$ and $u, v \in X$.

1. $u_\alpha \subset A$ if $p_\alpha(u, A) = 0$,
2. $p_\alpha(u, A) \leq d(u, v) + p_\alpha(v, A)$,
3. If $u_\alpha \subset A$, then $p_\alpha(u, A) \leq D_\alpha(A, B)$.

Definition 2.9. [6] A fuzzy point u_α in X is called a fixed fuzzy point of a fuzzy mapping T if $(u)_\alpha \subset Tu$, that is, $Tu(u) \geq \alpha$ or $u \in (Tu)_\alpha$. If $(u)_1 \subset Tu$, then u is a fixed point of fuzzy mapping T.

3. Main Results

Definition 3.1. In a metric space (X,d), the fuzzy mapping $T : X \to W_\alpha(X)$ is an ψ-contractive fuzzy mapping if $\exists$ a function $\psi \in \Psi \ni D_\alpha(T(u), T(v)) \leq \psi(d(u,v)), \forall u, v \in X$.

Theorem 3.2. *Let (X,d) be a complete metric linear space and $T : X \to W_\alpha(X)$ be an ψ-contractive fuzzy mapping. Then there exists a fixed fuzzy point.*

Proof. Let $u_0 \in X$ and $T : X \to W_\alpha(X)$ be a fuzzy mapping. Suppose $\exists u_1 \in (T(u_0))_\alpha$. Since $T(u_1))_\alpha$ is nonempty compact subset of X, then $\exists u_2 \in (T(u_1))_\alpha \ni$ by lemma 2.8(3) and by our hypothesis,

$$\begin{aligned} d(u_1, u_2) &= p_\alpha(u_1, T(u_1)) \\ &\leq D_\alpha(T(u_0), T(u_1)) \\ &\leq \psi(d(u_0, u_1)). \end{aligned}$$

Proceeding like this, we assemble a sequence $\{u_n\}$ in $X \ni u_n \in (T(u_{n-1}))_\alpha$, by lemma 2.8(3) and by our hypothesis,

$$\begin{aligned}
d(u_n, u_{n+1}) &= p_\alpha(u_n, T(u_n)) \\
&\leq D_\alpha(T(u_{n-1}), T(u_n)) \\
&\leq \psi(d(u_{n-1}, u_n)) \\
&= \psi(p_\alpha(u_{n-1}, T(u_{n-1}))) \\
&\leq \psi(D_\alpha(T(u_{n-2}), T(u_{n-1}))) \\
&\leq \psi(\psi(d(u_{n-2}, u_{n-1}))) \\
&\vdots \\
&\leq \psi^n(d(u_0, u_1)).
\end{aligned}$$

Continuing this process, we can get,

$$\begin{aligned}
d(u_n, u_{n+m}) &\leq d(u_n, u_{n+1}) + \ldots + d(u_{n+m-1}, u_{n+m}) \\
&\leq \psi^n(d(u_0, u_1)) + \ldots + \psi^{n+m-1}(d(u_0, u_1)) \\
&= \sum_{i=n}^{n+m-1} \psi^i(d(u_0, u_1))
\end{aligned}$$

Since $\sum_{n=1}^{\infty} \psi^n(s) < \infty$, $\{u_n\}$ is a Cauchy sequence in X.

Since (X, d) is a complete metric space, $\{u_n\}$ converges to some $u \in X$, that is, $(d(u_n, u)) \to 0$. Now by lemma 2.8(2,3) and ψ is continuous at the origin,

$$\begin{aligned}
p_\alpha(u, T(u)) &\leq d(u, u_n) + p_\alpha(u_n, T(u)) \\
&\leq d(u, u_n) + D_\alpha(T(u_{n-1}), T(u)) \\
&\leq d(u, u_n) + \psi(d(u_{n-1}, u)) \\
&\to 0 + 0 = 0.
\end{aligned}$$

Therefore, $p_\alpha(u, T(u)) = 0$ and by lemma 2.8(1),$u_\alpha \subset T(u)$. $\square$

Example 3.3. [12] Let $X = [0, 1], d : X \times X \to X$ be the Euclidean metric and $\alpha \in (0, \frac{1}{2})$. The fuzzy mapping $T : X \to I^X$ is defined by

$$T(0)(u) = \begin{cases} 1 & when \quad u = 0, \\ \alpha & when \quad u \in (0, \frac{1}{2}], \\ \frac{\alpha}{2} & when \quad u \in (\frac{1}{2}, 1], \end{cases} \quad T(1)(u) = \begin{cases} 1 & when \quad u = 0, \\ 2\alpha & when \quad u \in (0, \frac{1}{2}], \\ \frac{\alpha}{2} & when \quad u \in (\frac{1}{2}, 1], \end{cases}$$

and for $w \in (0,1)$,

$$T(w)(u) = \begin{cases} 1 & when \quad u = 0, \\ \alpha & when \quad u \in (0, \frac{1}{2}], \\ 0 & when \quad u \in (\frac{1}{2}, 1], \end{cases}$$

Then $T(0)_1 = T(w)_1 = T(1)_1 = \{0\}, T(0)_\alpha = T(w)_\alpha = T(1)_\alpha = [0, \frac{1}{2}]$ and $T(0)_{\frac{\alpha}{2}} = T(1)_{\frac{\alpha}{2}} = [0,1], T(w)_{\frac{\alpha}{2}} = [0, \frac{1}{2}]$. Now,

$$\begin{aligned} D_1(T(u),T(v)) &= H(T(u)_1, T(v)_1) = 0, \forall u, v \in X, \\ D_\alpha(T(u),T(v)) &= H(T(u)_\alpha, T(v)_\alpha) = 0, \forall u, v \in X, \\ D_{\frac{\alpha}{2}}(T(u),T(v)) &= H(T(u)_{\frac{\alpha}{2}}, T(v)_{\frac{\alpha}{2}}) = 0, \forall u, v \in \{0,1\} \quad \text{and} \forall u, v \in (0,1), \\ D_{\frac{\alpha}{2}}(T(u),T(v)) &= H(T(u)_{\frac{\alpha}{2}}, T(v)_{\frac{\alpha}{2}}) = \frac{1}{2}, \forall u \in \{0,1\} \quad \text{and} \forall v \in (0,1). \end{aligned}$$

Define $\psi : [0,\infty) \to [0,\infty)$ by $\psi(s) = \frac{s}{s+1}$. Clearly,$\sum_{n=1}^{\infty} \psi^n(s) < \infty$ where $s \in (0,\infty)$.
We obtain $D_\alpha(T(u),T(v)) = 0 \leq \psi(d(u,v)), \forall u, v \in X$.
Then 0 is the fixed fuzzy point. The theorem is justified.

Theorem 3.4. *Let $\alpha \in (0,1], (X,d)$ be a complete metric space, T_1 and T_2 be two fuzzy mappings from X onto $W_\alpha(X)$ and $\psi \in \Psi \ni D_\alpha(T_1(u), T_2(v)) \leq \psi(d(u,v)), \forall u, v \in X$. Then u_α is a common fixed fuzzy point of T_1 and T_2.*

Proof. Let $u_0 \in X$ and $T_1 : X \to W_\alpha(X)$ and $T_1 : X \to W_\alpha(X)$ be two fuzzy mappings. Since $(T_1(u_0))_\alpha$ is nonempty subset of X, then $\exists u_1 \in (T_1(u_0))_\alpha$,also since $(T_1(u_1))_\alpha$ is nonempty subset of X, then $\exists u_2 \in (T_2(u_1))_\alpha$ such that by lemma 2.8(3) and by our hypothesis,

$$\begin{aligned} d(u_1,u_2) &= p_\alpha(u_1, T_2(u_1)) \\ &\leq D_\alpha(T_1(u_0), T_2(u_1)) \\ &\leq \psi(d(u_0,u_1)). \end{aligned}$$

By induction we construct a sequence $\{u_n\}$ in $X \ni u_{2n+1} \in (T_1(u_{2n}))_\alpha$ and

$u_{2n+2} \in (T_2(u_{2n+1}))_\alpha$, by lemma 2.8(3) and by our hypothesis,

$$\begin{aligned} d(u_{2n+1}, u_{2n+2}) &= p_\alpha(u_{2n+1}, T_2(u_{2n+1})) \\ &\leq D_\alpha(T_2(u_{2n}), T_2(x_{2n+1})) \\ &\leq \psi(d(u_{2n}, u_{2n+1})) \\ &= \psi(p_\alpha(u_{2n}, T_1(u_{2n}))) \\ &\leq \psi(D_\alpha(T_2(u_{2n-1}), T_1(u_{2n}))) \\ &\leq \psi(\psi(d(u_{2n-1}, u_{2n}))) \\ &\vdots \\ &\leq \psi^{2n+1}(d(u_0, u_1)). \end{aligned}$$

That is,

$$d(u_{n+1}, u_{n+2}) \leq \psi^{n+1}(d(u_0, u_1)).$$

Similarly we can prove,

$$d(u_n, u_{n+1}) \leq \psi^n(d(u_0, u_1)).$$

Continuing this process, we can get,

$$\begin{aligned} d(u_n, u_{n+m}) &\leq d(u_n, u_{n+1}) + \ldots + d(u_{n+m-1}, u_{n+m}) \\ &\leq \psi^n(d(u_0, u_1)) + \ldots + \psi^{n+m-1}(d(u_0, u_1)) \\ &= \sum_{i=n}^{n+m-1} \psi^i(d(u_0, u_1)) \end{aligned}$$

Since $\sum_{n=1}^{\infty} \psi^n(t) < \infty, \{u_n\}$ is a Cauchy sequence in X.

Since (X, d) is a complete metric space, $\{u_n\}$ converges to some $x \in X$, that is, $(d(u_n, u)) \to 0$.

Suppose n is odd, by lemma 2.8(2,3) and ψ is continuous at the origin,

$$\begin{aligned} p_\alpha(u, T_1(u)) &\leq d(u, u_n) + p_\alpha(u_n, T_1(u)) \\ &\leq d(u, u_n) + D_\alpha(T_1(u_{n-1}), T_1(u)) \\ &\leq d(u, u_n) + \psi(d(u_{n-1}, u)) \\ &\to 0 + 0 = 0. \end{aligned}$$

Therefore, $p_\alpha(u, T_1(u)) = 0$ and by lemma 2.8(1),$u_\alpha \subset T_1(u)$.

Similarly, Suppose n is even, we can prove $u_\alpha \subset T_2(u)$.

Therefore, u_α is a common fixed fuzzy point of T_1 and T_2. □

Example 3.5. [7] Let $X = \{0, 1, 2\}, d : X \times X \to X$ be the Euclidean metric and $\alpha \in (0, \frac{1}{3})$. Define, two fuzzy mappings $T_1 : X \to W_\alpha(X)$ and $T_2 : X \to W_\alpha(X)$ as follows:

$$(T_1 0)(u) = (T_2 1)(u) = \begin{cases} \alpha & if \quad u = 0, \\ \frac{\alpha}{2} & if \quad u = 1, \\ 0 & if \quad u = 2, \end{cases}$$

$$(T_1 1)(u) = (T_2 0)(u) = \begin{cases} \alpha & if \quad u = 0, \\ 0 & if \quad u = 1, \\ \frac{\alpha}{3} & if \quad u = 2, \end{cases}$$

$$(T_1 2)(u) = (T_2 2)(u) = \begin{cases} \frac{\alpha}{3} & if \quad u = 0, \\ \alpha & if \quad u = 1, \\ \frac{\alpha}{4} & if \quad u = 2. \end{cases}$$

Note that $(T_1 0)_\alpha = (T_1 1)_\alpha = (T_2 0)_\alpha = (T_2 1)_\alpha = \{0\}, (T_1 2)_\alpha = (T_2 2)_\alpha = \{1\}$.

Now,

$$D_\alpha(T_1(u), T_2(v)) = H(T_1(u)_\alpha, T_2(v)_\alpha) = 0, \forall u = v \quad \text{and} \quad \forall, u, v \in \{0, 1\},$$
$$D_\alpha(T_1(u), T_2(v)) = H(T_1(u)_\alpha, T_2(v)_\alpha) = 1, \forall u, v \in \{0, 2\} \quad \text{and} \quad \forall, u, v \in \{1, 2\}.$$

Define $\psi : [0, \infty) \to [0, \infty)$ by $\psi(t) = \frac{t}{2}$. Clearly,$\sum_{n=1}^{\infty} \psi^n(t) = \sum_{n=1}^{\infty} (\frac{1}{2})^n t < \infty$ where $t \in (0, \infty)$.

For all $u, v \in X, D_\alpha(T_1(u), T_2(v)) \leq \psi(d(u, v))$.

The hypotheses in the above theorem are satisfied. Then 0 is the fixed fuzzy point. The theorem is justified.

Remark 3.6. If $T_1 = T_2$ in Theorem 3.4, we can get Theorem 3.2.

4. Applications to Fuzzy Differential Equations

From the book of Lakshmikantham et al. [10], we have the follwing problem and some lemmas to apply our main results.

Let E^n be the space of all fuzzy subsets x of R^n where $x : R^n \to I = [0, 1]$.

Consider the boundary value problem

$$\begin{cases} x''(t) = f(t, x(t), x'(t)), t \in J = [a, b], \\ x(t_1) = x_1, x(t_2) = x_2, t_1, t_2 \in J, \end{cases} \tag{1}$$

where $f : J \times E^n \times E^n \to E^n$ is a continuous function. This problem is equivalent to the integral equation

$$x(t) = \int_{t_1}^{t_2} G(t,s)[f(s, x(s), x^{'}(s))]ds + \beta(t),$$

where Green's function G is given by

$$G(t,s) = \begin{cases} \frac{(t_2-t)(s-t_1)}{t_2-t_1} & t_1 \leq s \leq t \leq t_2, \\ \frac{(t_2-s)(t-t_1)}{t_2-t_1} & t_1 \leq t \leq s \leq t_2. \end{cases}$$

and $\beta(t)$ satisfies $\beta^{''} = 0, \beta(t_1) = x_1, \beta(t_2) = x_2$. Let us recall some properties of $G(t,s)$, namely,

$$\int_{t_1}^{t_2} |G(t,s)|\, ds \leq \frac{(t_2 - t_1)^2}{8},$$

and

$$\int_{t_1}^{t_2} |G_t(t,s)|\, ds \leq \frac{t_2 - t_1}{2}.$$

Now, we shall prove the existence of the result for the above boundary value problem by using our theorem 3.2 and 3.4.

Theorem 4.1. *Let $f : J \times E^n \times E^n \to E^n$. Suppose $\exists 0 < \gamma < 1, 0 < \delta < 1$ with $\gamma \leq \delta \ni \forall x, y \in C^1(J, E^n)$,*

$$\left| f(t, x, x^{'}) - f(t, y, y^{'}) \right| \leq \gamma \left| x - y \right| + \delta \left| x^{'} - y^{'} \right|,$$

Then the integral equation

$$x(t) = \int_{t_1}^{t_2} G(t,s)[f(s, x(s), x^{'}(s))]ds + \beta(t), t \in J$$

has a solution in $C^1[[t_1, t_2], E^n]$.

Proof. Consider $C = C^1[[t_1, t_2], E^n]$ with the metric

$$D(x,y) = \max_{t_1 \leq t \leq t_2} [\gamma \left| x(t) - y(t) \right| + \delta \left| x^{'}(t) - y^{'}(t) \right|].$$

The space (C, D) is a complete metric space. Define the operator $T : C \to C$ by

$$Tx(t) = \int_{t_1}^{t_2} G(t,s)[f(s, x(s), x^{'}(s))]ds + \beta(t).$$

By the properties of $G(t,s)$ and by our hypothesis,

$$\begin{aligned}
|Tx(t)-Ty(t)| &\leq \int_{t_1}^{t_2} |G(t,s)| \left| f(s,x(s),x^{'}(s)) - f(s,y(s),y^{'}(s)) \right| ds \\
&\leq D(x,y) \int_{t_1}^{t_2} |G(t,s)|\, ds \\
&\leq \frac{(t_2-t_1)^2}{8} D(x,y) \\
&\leq \frac{D(x,y)}{8}.
\end{aligned}$$

and

$$\begin{aligned}
\left|(Tx)^{'}(t)-(Ty)^{'}(t)\right| &\leq \int_{t_1}^{t_2} |G_t(t,s)| \left| f(s,x(s),x^{'}(s)) - f(s,y(s),y^{'}(s)) \right| ds \\
&\leq D(x,y) \int_{t_1}^{t_2} |G_t(t,s)|\, ds \\
&\leq \frac{(t_2-t_1)}{2} D(x,y) \\
&\leq \frac{D(x,y)}{2}.
\end{aligned}$$

Now, we have

$$\begin{aligned}
D[Tx,Ty] &\leq \gamma \frac{D(x,y)}{8} + \delta \frac{D(x,y)}{2} \\
&\leq (\frac{5}{8}\delta) D(x,y) \\
&= \psi(D(x,y)).
\end{aligned}$$

and $\sum_{n=1}^{\infty} \psi^n(s) = \sum_{n=1}^{\infty} (\frac{5}{8}\delta)^n s < \infty, \psi(0) = 0$ and ψ is continuous at the origin.

We obtain $D(Tx,Ty) \leq \psi(D(x,y))$.

Therefore, Theorem 3.2 applies to T which has a fixed point $x^* \in C$, that is x^* is a solution of the boundary value problem. $\square$

Theorem 4.2. *Let $f_1, f_2 : J \times E^n \times E^n \to E^n$. Suppose there exists $0 < \gamma < 1, 0 < \delta < 1$ with $\gamma \leq \delta$ such that for all $u, v \in C^1(J, E^n)$,*

$$\left| f_1(t,u,u^{'}) - f_2(t,v,v^{'}) \right| \leq \gamma \left| u - v \right| + \delta \left| u^{'} - v^{'} \right|,$$

Then the integral equation

$$u(t) = \int_{t_1}^{t_2} G(t,s)[f_i(s,u(s),u^{'}(s))]ds + \beta(t), t \in J, i \in \{1,2\}$$

has a common solution in $C^1[[t_1,t_2],E^n]$.

Proof. Consider $C = [[t_1,t_2],E^n]$ with the metric

$$D(u,v) = \max_{t_1 \le t \le t_2} [\gamma \left|u(t) - v(t)\right| + \delta \left|u^{'}(t) - v^{'}(t)\right|].$$

The space (C, D) is a complete metric space. Define the operator $F_i : C \to C$ by

$$F_i(u)(t) = \int_{t_1}^{t_2} G(t,s)[f_i(s,u(s),u^{'}(s))]ds + \quad \beta(t), \quad t \in J, i \in \{1,2\},$$

where $f_1, f_2 \in C(J \times E^n \times E^n, E^n), u \in C^1(J, E^n)$ and $\beta \in C(J, E^n)$. By the properties of $G(t,s)$ and by our hypothesis,

$$\begin{aligned}
\left|F_1(u)(t) - F_2(v)(t)\right| &\le \int_{t_1}^{t_2} \left|G(t,s)\right| \left|f_1(s,u(s),u^{'}(s)) - f_2(s,v(s),v^{'}(s))\right| ds \\
&\le D(u,v) \int_{t_1}^{t_2} \left|G(t,s)\right| ds \\
&\le \frac{(t_2-t_1)^2}{8} D(u,v) \\
&\le \frac{D(u,v)}{8}.
\end{aligned}$$

and

$$\begin{aligned}
\left|(F_1(u))^{'}(t) - (F_2(v))^{'}(t)\right| &\le \int_{t_1}^{t_2} \left|G_t(t,s)\right| \left|f_1(s,u(s),u^{'}(s)) - f_2(s,v(s),v^{'}(s))\right| ds \\
&\le D(u,v) \int_{t_1}^{t_2} \left|G_t(t,s)\right| ds \\
&\le \frac{(t_2-t_1)}{2} D(u,v) \\
&\le \frac{D(u,v)}{2}.
\end{aligned}$$

Now, we have

$$\begin{aligned} D[F_1u, F_2v] &\leq \gamma \frac{D(u,v)}{8} + \delta \frac{D(u,v)}{2} \\ &\leq (\frac{5}{8}\delta) D(u,v) \\ &= \psi(D(u,v)). \end{aligned}$$

and $\sum_{n=1}^{\infty} \psi^n(s) = \sum_{n=1}^{\infty} (\frac{5}{8}\delta)^n s < \infty, \psi(0) = 0$ and ψ is continuous at the origin.

We obtain $D(F_1u, F_2v) \leq \psi(D(u,v))$.

Therefore, Theorem 3.4 applies to F_1 and F_2 have a common fixed point $u^* \in C$, that is u^* is a solution of the boundary value problem. □

References

[1] M. Abbas, B. Damjanivic, and R. Lazovic, Fuzzy common fixed point theorems for generalized contractive mappings, *Applied Mathematics Letter*, **23**, No. 11 (2010), 1326-1330, **doi:** 10.1016/j.aml.2010.06.023.

[2] V. Berinde, *Iterative Approximation of Fixed Points*, Lecture Notes in Mathematics, Romania, 2007, **doi:** 10.1007/978-3-540-72234-2.

[3] Byung Soo Lee and Sung Jin Cho, A fixed point theorem for contractive-type fuzzy mappings, *Fuzzy Sets and Systems*, **61** (1994), 309-312, **doi:** 10.1016/0165-0114(94)90173-2.

[4] L. Ciric, M. Abbas, B. Damjanovic, R. Saadati, Common fuzzy fixed point theorems in ordered metric spaces, *Mathematical and Computer Modelling*, **53** (2011), 1737-1741, **doi:** 10.1016/j.mcm.2010.12.050.

[5] V.D. Estruch, A. Vidal, A note on fixed fuzzy points for fuzzy mappings, *Rend. Ist. Mat. Univ. Trieste.*, **32** (2001), 39-45.

[6] S. Heilpern, Fuzzy mappings and fixed point theorem, *Journal of Mathematical Analalysis and Applications*, **83** (1981), 566-569, **doi:** 10.1016/0022-247X(81)90141-4.

[7] Hemant Kumar Nashine, Calogero Vetro, Wiyada Kumam and Poom Kumam, Fixed point theorems for fuzzy mappings and applications to ordinary fuzzy differential equations, *Advances in Difference Equations*, **232**, No. 1 (2014), 1-14, **doi:** 10.1186/1687-1847-2014-232.

[8] Jong Yeoul Park, Hyo Keun Han, Fuzzy differential equations, *Fuzzy Sets and Systems*, **110** (2000), 69-77, **doi:** 10.1016/S0165-0114(98)00150-X.

[9] Jong Yeoul Park, S.Y. Lee, H.M. Kee, The existence of solution for fuzzy differential equations with infinite delays, *Indian Journal of Pure and Applied Mathematics*,**31**, No. 10 (2000), 137-151.

[10] V. Lakshmikantham, R. Mohapatra, *Theory of Fuzzy Differential Equations and Inclusions*, Taylor and Francis, London, 2003, **doi:** 10.1201/9780203011386.

[11] R.A. Rashwan, M.A. Ahmed, Common fixed point theorems for fuzzy mappings, *Arch. Math.*, **38** (2002), 219-226.

[12] D. Turkoglu, B.E. Rhoades, A fixed fuzzy point for fuzzy mapping in complete metric spaces, *Mathematical Communications*, **10** (2005), 115-121.