

ROTATING BOUNDARY LAYER FLOW DUE TO
A PERMEABLE EXPONENTIALLY SHRINKING
SHEET IN NANOFUID

S.N.A. Salleh¹, N. Bachok² §, N.M. Arifin³

¹Department of Mathematics and Institute for
Mathematical Research
Universiti Putra Malaysia
43400 UPM Serdang, Selangor, MALAYSIA

Abstract: In this study, the effect of the suction on the boundary layer flow and heat transfer characteristics over an exponentially shrinking surface in a rotating nanofluid is contemplated for three types of nanoparticles namely, copper Cu, titania TiO₂ and alumina Al₂O₃. Similarity transformations have been applied to transform the partial differential equations into a system of ordinary differential equations, which are then solved numerically using a shooting method in Maple software. The effects of the rotation Ω , suction s and nanoparticle volume fraction parameters on the velocity field, temperature distribution, local skin friction coefficients and local Nusselt number are taken into account. Results obtain in this study are graphically presented and further discussion have been discussed in detail. The dual solutions are found to exist for a certain values of the governing parameters. It is revealed from the study that the presence of the rotation would increase the skin friction coefficients and heat transfer rate at the surface.

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Key Words: exponentially shrinking sheet, nanofluid, rotating flow, suction

1. Introduction

The deliberation of the rotating flow and heat transfer in the boundary layer

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§Correspondence author

flow over a stretching sheet have earned a huge number of response by many researcher. The term of heat transfer was recognizable in industrial and engineering proposes due to the heating and cooling processes. Some common applications in real world are aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in cooling bath, the material handling conveyors, the cooling and drying of paper and so forth. In order to get the good quality of the final product, heat transfer rate at the stretched surface is carefully observed during the manufacturing process. The fluid flow past a stretching sheet was first explored by [1], where an exact analytical solution to the Navier-Stokes equations was reported. Later, [2] extend the problem to a permeable surface subject to blowing and suction. Nowadays, the various aspects involving the boundary layer flow of a rotating fluid over a stretching surface have been considered by some authors such as [3], [4] and [5]. In summation, the problems relate to the magnetic field have been focused by [6], [7] and [8].

The above mentioned ideas, however, have not been discussed with the nanofluid yet. Therefore, [9] came out with the problem of the rotating flow in a nanofluid over a stretching surface. Nanofluid is a fluid with a suspended nanoparticles that show significant enhancement of their properties. [10] was the first who introduced the term nanofluid in the real world application in fluid dynamic. He concluded that the addition of a very small amount ($< 1\%$ volume fraction) of nanoparticles to conventional heat transfer fluid enhance the thermal conductivity of the fluid up to two times. The convectonal heat transfer fluids like water, ethylene-glycol and mineral oil have low thermal conductivity which inadequate to achieve the qualification of the cooling rate. The nanoscale particles are dissolved in a base fluid, which are later known as a nanofluid. These fluid are used as an alternative way to enhance the heat transfer of such fluids. The convective heat transfer is an important appearance in industrial and engineering processes due to its applications in automotive cooling applications, heat exchangers and electronic cooling. Recent years, many works involving nanofluid have been found in the literature. [11] presented the stagnation-point flow of a nanofluid over a stretching sheet. While, [12], [13] and [14] have considered a several aspects involving the nanofluid.

However, less consideration have been given to the problems of the rotating boundary layer flow and heat transfer induced by a shrinking surface. The flow of the viscous fluid was studied by [15] due to a shrinking sheet in the presence of suction. The flow towards a shrinking sheet is found to exist, whenever an adequate suction on a boundary layer is imposed, [16] or a stagnation-point flow is considered, [17]. The study reveals that the mass suction is required to maintain the flow over a shrinking sheet. Later [18], [19] and [20] have discussed

the effects of the MHD over a shrinking surface in a rotating fluid. Recently, [21] performed the rotating flow over an exponentially shrinking sheet with mass suction at the surface. The idea of an exponentially shrinking sheet was first considered by [22] in a viscous fluid and have been extend by [23], [24], [25], [26] and [27] in several aspects.

The desire of the present paper is to combine the idea of the rotating flow in a nanofluid by [9] over an exponentially shrinking sheet, [22] with suction effect at the surface. The governing partial differential equations are transformed into a set of ordinary differential equations using a similarity transformation in exponential form, before being solved numerically using a shooting method. The problem is fixed for Prandtl number $Pr = 6.2$ (water).

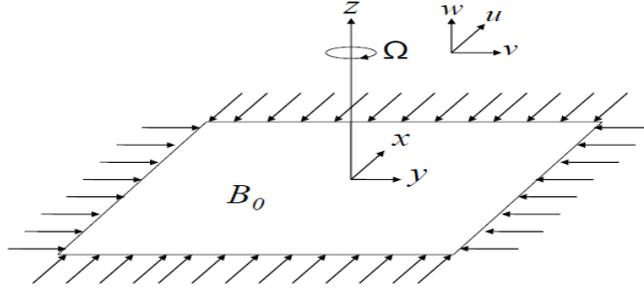


Figure 1: Physical model and coordinate system

2. Problem Formulation

Let us consider the steady three-dimensional rotating flow over an exponentially shrinking surface with mass suction in a water based nanofluid at $z = 0$. The physical model and coordinate system of the problem is shown in Figure 1. Three different types of nanoparticles namely copper Cu, titania TiO_2 and alumina Al_2O_3 are considered. We assume u , v , and w be the velocity components along the x , y , and z directions, respectively. Under these assumptions, the equations represent the rotating flow and heat transfer are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - 2\bar{\Omega}v = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + 2\bar{\Omega}v = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial z^2} \right) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial z^2} \right) \quad (4)$$

subject to the boundary conditions

$$u = -Ue^{x/L}, \quad v = 0, \quad w = -w_0e^{x/2L}, \quad T = T_w \quad \text{at } z = 0, \\ u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T \quad \text{as } z \rightarrow \infty \quad (5)$$

where w_0 is the constant mass flux with $w_0 < 0$ for suction and $w_0 > 0$ for injection, the $-$ sign represent a shrinking constant and $\bar{\Omega} = \Omega_0e^{x/L}$ is assumed to be the angular velocity of the rotating fluid. T is ambient temperature, T_w is wall temperature and T is the temperature of the fluid. The dynamic viscosity, density, thermal diffusivity and thermal conductivity of the nanofluid are given by μ_{nf} , ρ_{nf} , α_{nf} and k_{nf} , respectively and k_f is thermal conductivity of the base fluid. All these parameters related to the nanoparticle volume fraction given as follows

$$\rho_{nf} = \rho_f [1 - \varphi + \varphi \left(\frac{\rho_s}{\rho_f} \right)], \quad \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \\ \alpha_{nf} = \frac{k_{nf}}{(\rho C_\rho)_{nf}}, \quad (\rho C_\rho)_{nf} = (\rho C_\rho)_f [1 - \varphi + \varphi \left(\frac{(\rho C_\rho)_s}{(\rho C_\rho)_f} \right)], \\ \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \quad (6)$$

Here, $(\rho C_\rho)_s$, $(\rho C_\rho)_f$, and $(\rho C_\rho)_{nf}$ represents the volumetric heat capacities of solid nanoparticles, base fluid and nanofluid, respectively. Whereas, ρ_f and μ_f are respectively, the density and dynamic viscosity of base fluid and k_s is thermal conductivity of solid nanoparticles.

We are now introducing the following similarity transformations in exponential form:

$$u = Ue^{\frac{x}{L}} f(\eta), \quad v = Ue^{\frac{x}{L}} h(\eta), \quad w = - \left(\frac{U\nu_f}{2L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} [f(\eta) + \eta f'(\eta)], \\ \eta = \left(\frac{U}{2\nu_f L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} z, \quad \theta(\eta) = \frac{T - T_w}{T - T_w} \quad (7)$$

where primes refer as the differentiation with respect to η .

Using the above transformations, Eq. (1) is satisfied and Eqs. (2) - (4) become the following ordinary differential equations as shown below

$$\frac{1}{(1-\varphi)^{2.5}[1-\varphi+\varphi(\rho_s/\rho_f)]}f'' + ff' - 2f^2 + 4\Omega h = 0 \quad (8)$$

$$\frac{1}{(1-\varphi)^{2.5}[1-\varphi+\varphi(\rho_s/\rho_f)]}h'' + fh' - 2f'h - 4\Omega f = 0 \quad (9)$$

$$\frac{1}{Pr} \frac{k_{nf}/k_f}{[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f]} \theta'' + f\theta' = 0 \quad (10)$$

subjected to the boundary conditions (5) which become

$$f(0) = s, \quad f'(0) = -1, \quad h(0) = 0, \quad \theta(0) = 1$$

$$f(\eta) \rightarrow 0, \quad h(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (11)$$

$Pr = \nu/\alpha$ is the Prandtl number, $\Omega = \Omega_0 L/U$ is a nondimensional rotation parameter and s is the constant mass flux parameter with $s > 0$ for suction and $s < 0$ for injection.

The local skin friction coefficients in the x and y directions and the local Nusselt number can be expressed as

$$Cf_x = \frac{\tau_{xz}}{\rho_f (Ue^{x/L})^2}, \quad Cf_y = \frac{\tau_{yz}}{\rho_f (Ue^{x/L})^2}, \quad Nu_x = \frac{Lq_w}{k_f (T_w - T)}, \quad (12)$$

where the shear stress of x and y component τ_{xz} , τ_{yz} and heat flux q_w are given by

$$\tau_{xz} = \mu_{nf} \left(\frac{\partial u}{\partial z} \right)_{z=0}, \quad \tau_{yz} = \mu_{nf} \left(\frac{\partial v}{\partial z} \right)_{z=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial z} \right)_{z=0} \quad (13)$$

Using Eq. (6) - (7) and Eq. (13), Eq. (12) becomes

$$\begin{aligned} (2Re_x)^{1/2} Cf_x &= \frac{1}{(1-\varphi)^{2.5}} f'(0), \quad (2Re_x)^{1/2} Cf_y = \frac{1}{(1-\varphi)^{2.5}} h'(0), \\ (\sqrt{2}Re_x^{-1/2}) Nu_x &= \frac{-k_{nf}}{k_f} \theta'(0) \end{aligned} \quad (14)$$

where $Re_x = ULe^{x/L}/\nu_f$ is refer as the local Reynolds number.

Table 1: Thermophysical properties of the base fluid and nanoparticles

Physical properties	Fluid phase	Cu	Al ₂ O ₃	TiO ₂
C_p (J/kg K)	4179	385	765	686.2
ρ (kg/m ³)	997.1	8933	3970	4250
k (W/mk)	0.613	400	40	8.9538

3. Result and Discussion

Equations (8)–(10) associated with the conditions (11) were solved numerically via a shooting method in Maple software. The unknown values of $f(0)$, $h(0)$ and $-\theta(0)$ are set as an initial guesses in which all the profiles must satisfy the conditions (11) asymptotically with the different shapes. The physical parameters involved in this study are rotation parameter Ω , suction parameter s and nanoparticle volume fraction parameter φ . We take the Prandtl number as $Pr = 6.2$ (water) and nanoparticle volume fraction is considered from 0 to 0.2 ($0 \leq \varphi \leq 0.2$) according to [28] in which $\varphi = 0$ is refer as a regular fluid. Table 1 presents the thermophysical properties of the base fluid and nanoparticles used in the present study.

Figures 2 - 4 show the variations of the reduced skin friction of x and y component $f(0)$, $h(0)$ and local heat flux $-\theta(0)$ with suction s for several values of rotation Ω for Cu-water when $\varphi = 0.1$. The values of $f(0)$, $h(0)$ and $-\theta(0)$ are seem to increase as the Ω increases. The side effect of increasing the rotation in the fluid flow will reduce the momentum boundary layer thickness which leads to increase both the shear stress of x components, $f(0)$ and y components, $h(0)$ on the surface, respectively. When the values of Ω are small enough ($\Omega = 0.015$), the dual solutions are found to exist for $s \geq 2.2640$ and beyond this interval no solutions appear. s_c represents the critical values of s for which Eqs. (8) – (10) has no solutions.

Figures 5 - 7 displays the influence of the nanoparticle volume fraction on the variations of $f(0)$, $h(0)$ and $-\theta(0)$ with s for Cu-water when $\Omega = 0.015$. Figures 5 - 6 indicate that as the values of φ increase, the variations of $f(0)$ and $h(0)$ increase significantly. While in Fig. 7, the reverse effect occurs for the $-\theta(0)$. The values of s_c for different values of φ are shown in Table 2. When the values of φ increase, the values of s_c are found to decrease. However, increase the values of s will increase the values of s_c . This is due to the fact that, suction widens the range for which the solutions exist.

Figures 8 - 10 show the variations of $(2Re_x)^{1/2}Cf_x$, $(2Re_x)^{1/2}Cf_y$ and

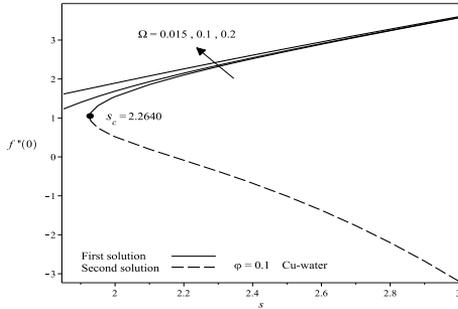


Figure 2: Variation of $f''(0)$ with s for different Ω for Cu-water when $\varphi = 0.1$.

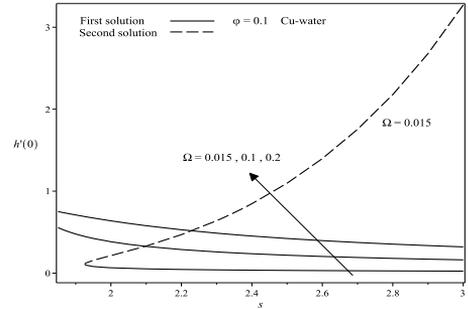


Figure 3: Variation of $h(0)$ with s for different Ω for Cu-water when $\varphi = 0.1$.

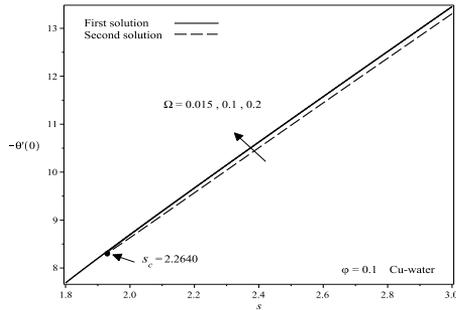


Figure 4: Variation of $-\theta'(0)$ with s for different Ω for Cu-water when $\varphi = 0.1$.

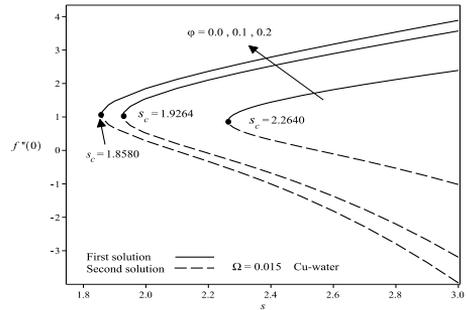


Figure 5: Variation of $f''(0)$ with s for different φ for Cu-water when $\Omega = 0.015$.

Table 2: Variations of s_c for different values of nanoparticle volume fraction φ for Cu-water when $\Omega = 0.015$

φ	Values of s_c
0.0	2.2640
0.1	1.9264
0.2	1.8580

$(\sqrt{2}Re_x^{-1/2})Nu_x$ with φ for different nanoparticles and for various values of Ω when $s = 2.5$. As the values of Ω and φ increase, the skin friction coefficients for both x and y component increases. The presence of the rotation in

the fluid flow will accelerate the fluid motion and causes the reduction of the momentum boundary layer thickness. This is due to the collision between the base fluid particles and nanoparticles. These kind of behaviors will increase the shear stress at the wall thus leads to increase the skin friction coefficient. However in Fig. 10, as the φ increase the local Nusselt number are found to decrease, while the opposite effect can be observed as the Ω increase. It is found from Fig. 10 that the lowest heat transfer rate is obtained for TiO_2 due to domination of conduction mode of heat transfer. This follow the fact that TiO_2 has the lowest value of thermal conductivity compared with Al_2O_3 and Cu. However, the thermal conductivity of Al_2O_3 is approximately one tenth of Cu. The comparison values of the thermal conductivity of nanoparticles are tabulated in Table 1.

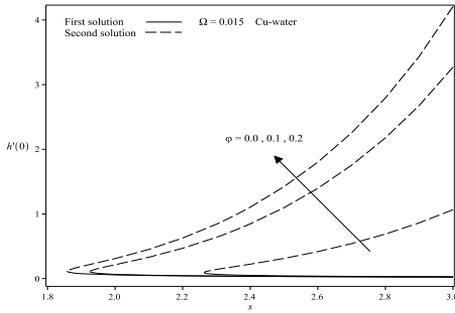


Figure 6: Variation of $h(0)$ with s for different φ for Cu-water when $\Omega = 0.015$.

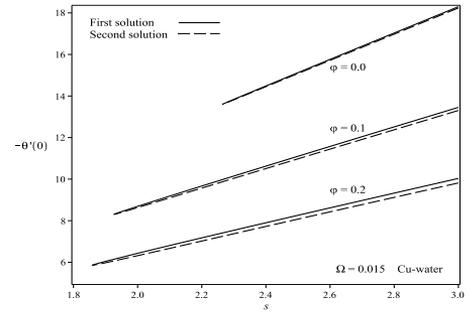


Figure 7: Variation of $-\theta(0)$ with s for different φ for Cu-water when $\Omega = 0.015$.

The sample of the profiles are shown in Figs. 11 - 16 for different values of Ω and s . From Figs. 11 - 12, it is noticed that the values of $f(\eta)$ and $h(\eta)$ increases as the Ω increase, while in Fig. 13 the opposite trend is observed for $\theta(\eta)$. In summation, Fig. 14 indicate that increase the values of s results in an increase in the values of $f(\eta)$ for the first solution and decrease for the second solution. Whereas the effect of the $h(\eta)$ are opposite and it is shown in Fig. 15. Fig. 16 show that the values of $\theta(\eta)$ decreases as the values of s increase and these will make the thermal boundary layer thickness reduced. Figures 11 - 16 show that all the profiles have satisfied the conditions (11) asymptotically and these behaviors will support the accuracy of the numerical results obtained in this study.

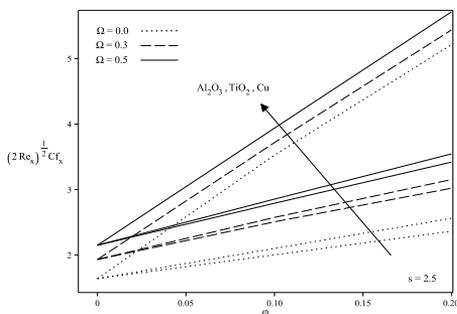


Figure 8: Variation of the skin friction coefficient of x -component with φ for different nanoparticles and Ω when $s = 2.5$.

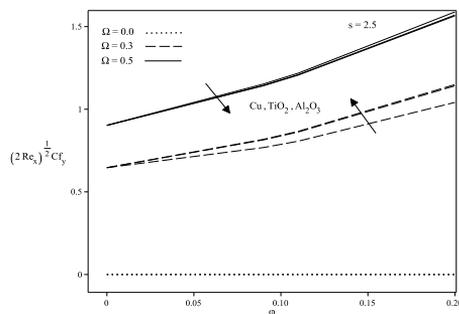


Figure 9: Variation of the skin friction coefficient of y -component with φ for different nanoparticles and Ω when $s = 2.5$.

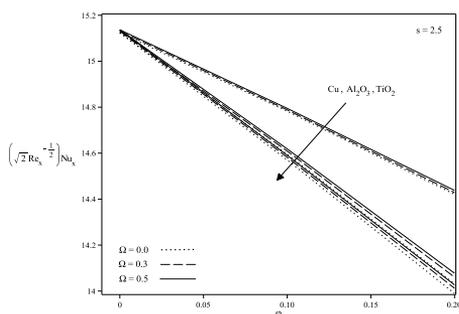


Figure 10: Variation of the local Nusselt number with φ for different nanoparticles and Ω when $s = 2.5$.

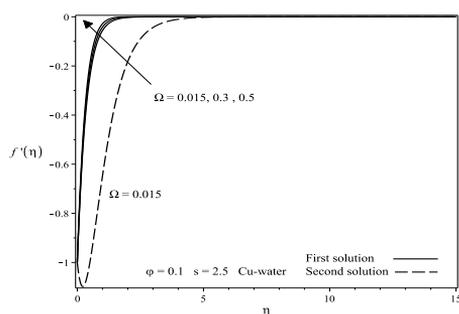


Figure 11: Velocity profiles $f'(\eta)$ for different Ω for Cu-water when $\varphi = 0.1$ and $s = 2.5$.

4. Conclusion

The problem involving the rotating flow over an exponentially shrinking sheet immersed in nanofluid with suction has been considered in this study. The set of ordinary differential equations have been transformed using the similarity transformation, before being solved numerically using a shooting method in Maple software. The results obtain in this study can be summarized as follow:

- increase the rotation parameter would increase the skin friction coefficients in both x and y component as well as local Nusselt number.

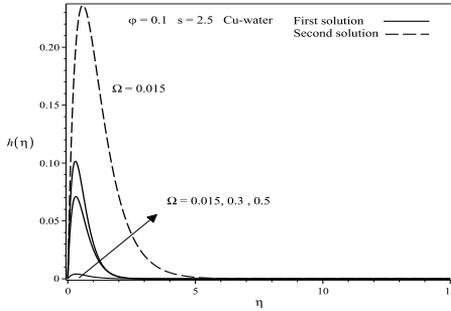


Figure 12: Velocity profiles $h(\eta)$ for different Ω for Cu-water when $\varphi = 0.1$ and $s = 2.5$.

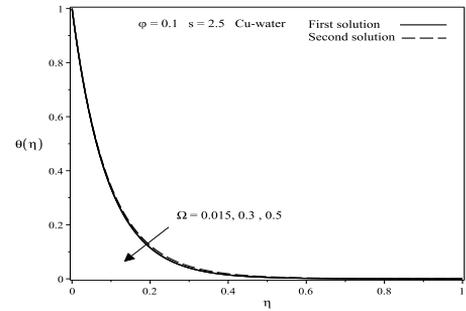


Figure 13: Temperature profiles $\theta(\eta)$ for different Ω for Cu-water when $\varphi = 0.1$ and $s = 2.5$.

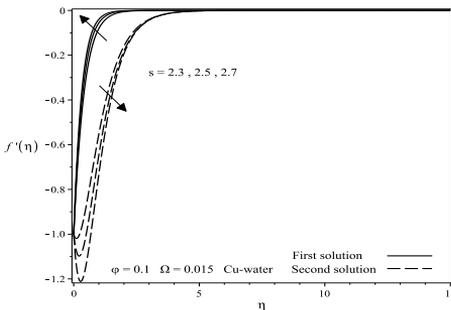


Figure 14: Velocity profiles $f'(\eta)$ for different s for Cu-water when $\varphi = 0.1$ and $\Omega = 0.015$.

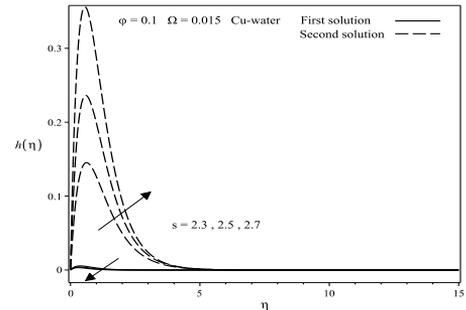


Figure 15: Velocity profiles $h(\eta)$ for different s for Cu-water when $\varphi = 0.1$ and $\Omega = 0.015$.

- increase the nanoparticle volume fraction results in an increase the skin friction coefficients in both x and y component, while the opposite trend is observed for the local Nusselt number.

- if the mass suction exceeds a certain value, i.e. $s \geq s_c$, the steady rotating flow due to an exponentially shrinking surface is possible and dual solution for velocity fields and temperature distribution were found. Besides, velocity increases with s for the first solution and decreases for second solution of x components. However, reverse effects occur for y components. The temperature distribution are seem to be decreased for both solutions with an increment of suction.

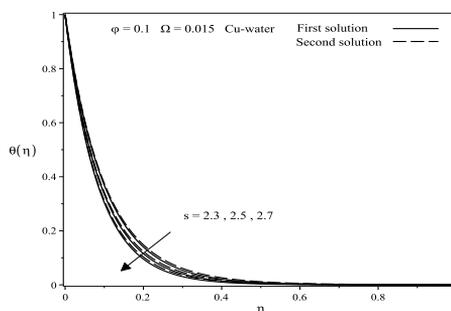


Figure 16: Velocity profiles $h(\eta)$ for different s for Cu-water when $\varphi = 0.1$ and $\Omega = 0.015$.

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