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STRONG RAINBOW EDGE-COLOURING OF VARIANTS OF CUBIC HALIN GRAPHS

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Abstract: A non trivial connected graph $G = (V, E)$ is strongly rainbow connected if every two vertices u and v of G are connected by at least one shortest $u - v$ path in which no two edges have same colours. The strong rainbow connection number of G, denoted by $src(G)$ is the minimum number of colours that makes G strongly rainbow connected. In this paper we explore the strong rainbow connection number of variants of cubic Halin graphs.

AMS Subject Classification: 05C15, 05C40
Key Words: diameter, strong rainbow edge colouring, cubic Halin graph

1. Introduction

The theory of rainbow colouring was put forward by Chartrand in 2008 [5]. The rainbow connection number of an arbitrary graph is NP-Hard [4]. An edge colouring c of a graph G is a function from its edge set to the set of natural numbers. A $u-v$ path P in an edge coloured graph G is called a rainbow path if no two edges of it get the same colour. An edge coloured graph is strongly rainbow connected if every two vertices is connected by at least one shortest rainbow path. Such a colouring is called a strong rainbow edge colouring. The strong rainbow connection number $src(G)$ is the minimum number of colours

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needed for strong rainbow edge colouring of a graph G.

This paper focuses on the strong rainbow connection number of variants of cubic Halin graphs. A Halin graph $G = T \cup C$ is a plane graph that consists of a plane embedding of a tree T and a cycle C connecting the leaves (vertices of degree 1) of the tree such that C is the boundary of the exterior face and the degree of each interior vertex of T is at least three [5]. The tree T and the cycle C are in general referred to as characteristic tree and adjoint cycle of G respectively [5]. A caterpillar is a tree where all vertices of degree ≥ 3 lie on a path, called the spine of the caterpillar. A cubic Halin graph is a Halin graph whose characteristic tree T is a caterpillar. In this paper, we study cubic Halin graphs whose characteristic tree T is a caterpillar and the degree of each vertex of T is 3.

2. Preliminaries

Graphs studied in this paper are simple, finite and undirected. The distance between two vertices u and V in G, denoted by $d(u, v)$ is the length of a shortest path between them in G. The eccentricity $e(v)$ of a vertex v in a connected graph G is $ecc(v) = max_{u \in V(G)} d(u, v)$. The maximum eccentricity of all vertices in a graph G is called the diameter of G and is denoted by $diam(G)$. Two edges $e = uv$ and $f = xy$ are antipodal in G if $d(v, x) = diam(G)$. If a graph has an edge colouring c, then $c(G)$ denotes the set of colours appeared in G. The chromatic index $\chi(G)$, of a graph G is the fewest number of colours necessary to colour each edge of G such that no two edges incident on the same vertex have the same colour.

3. Main Results

Let G be a cubic Halin graph. The characteristic tree T of G is a caterpillar of order $2n+2$, $n \geq 1$. A pendent edge (leaf edge) is an edge of the caterpillar that is incident at a vertex of the outer cycle. An edge of G is called a cycle edge if it lies on the outer cycle. We denote the starting vertex(head) of the spine by u_0, the intermediate vertices along the spine P_n of the caterpillar by $v_1, v_2, ..., v_n$ and the end vertex(tail) of the spine by u_{n+1}. Other pendent vertices adjacent with v_i is denoted by u_i, $2 \leq i \leq n$. Note that the vertices $u_0, u_1, ..., u_n, u_{n+1}$ lie on the adjoint cycle C_{n+2}. Moreover, if the vertices $u_0, u_1, ..., u_n, u_{n+1}$ in C_{n+2} are in order and the pendent edges of the caterpillar are in the same direction (either up or down), then G is called a necklace graph with n pendent edges and is denoted by Ne_n. The edges of Ne_n can be labeled as follows: The cycle edges are denoted by $\{e_i | e_i = u_i u_{i+1}, 0 \leq i \leq n\} \cup \{e_{n+1} = u_{n+1}u_0\}$. The edges along the spine of the caterpillar are denoted by $\{s_i | s_i = v_i v_{i+1}, 1 \leq i \leq n-1\}$. Since v_1 is adjacent to u_0 and v_n is adjacent to u_{n+1} along the spine, denote the spine edges $u_0 v_1$ by s_0 and $v_n u_{n+1}$ by s_n. The pendent edges are denoted by $\{p_i | p_i = v_i u_i, 1 \leq i \leq n\}$. The edge chromatic number $\chi(G)$ of G is 3. The diameter of G is 1 for $n = 1$. For $n \geq 2$, $diam(G) = \left\lfloor \frac{n+3}{2} \right\rfloor$. Figure 1 is the necklace graph of dimension 4.

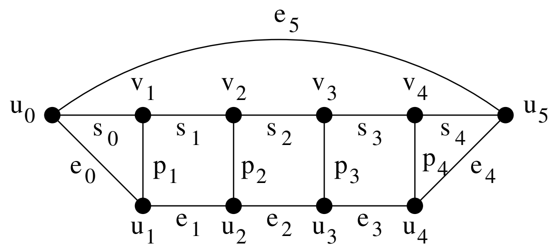

Figure 1: Ne_4

Theorem 1. *Let G be a necklace graph Ne_n. For $n \geq 3$, $src(Ne_n) = diam(Ne_n) + 1$ if n is even and $src(Ne_n) = diam(Ne_n)$ if n is odd.*

Proof. It is easy to check that $src(Ne_1) = 1$, $src(Ne_2) = 2$.
Case 1. n is even and $n \geq 3$.
The colouring algorithm is given as follows. $c(s_i) = i + 1$ for $0 \leq i \leq \frac{n}{2}$ and $c(s_i) = i - \frac{n}{2}$ for $\frac{n}{2} + 1 \leq i \leq n$. Also $c(e_i) = i + 1$ for $0 \leq i \leq \frac{n}{2}$ and $c(e_i) = i - \frac{n}{2}$ for $\frac{n}{2} + 1 \leq i \leq n+1$. $c(p_i) = \frac{n}{2} + 2$, $1 \leq i \leq n$. This gives a strong rainbow $\frac{n}{2} + 2$ edge colouring for Ne_n, n is even. Since $src(G) \geq diam(G)$ and $diam(G) = \frac{n}{2} + 1$ when n is even and $n \geq 3$, we find that $src(G) = \frac{n}{2} + 2$. Hence $src(Ne_n) = diam(Ne_n) + 1$.
Case 2. n is odd and $n \geq 3$.
The colouring algorithm is given below. $c(s_i) = i + 1$ for $0 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $c(s_i) = i - \left(\left\lceil \frac{n}{2} \right\rceil - 1 \right)$ for $\left\lceil \frac{n}{2} \right\rceil \leq i \leq n$, $c(e_i) = i + 1$ for $0 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $c(e_i) = i - \left(\left\lceil \frac{n}{2} \right\rceil - 1 \right)$ for $\left\lceil \frac{n}{2} \right\rceil \leq i \leq n + 1$. Also $c(p_i) = \left\lceil \frac{n}{2} \right\rceil + 1$, $1 \leq i \leq n$. This gives a strong rainbow $\left\lceil \frac{n}{2} \right\rceil + 1$-edge colouring for Ne_n, n is

odd. Since $diam(G) = \left\lceil \frac{n}{2} \right\rceil + 1$ when n is odd and $n \geq 3$, This bound is sharp and $src(G) = diam(G)$. □

Observation 2. *Antipodal edges in each of C_4 and C_{n+2} acquire the same colour in strong rainbow edge colouring.*

Observation 3. *If the vertices u_0 and u_{n+1} are removed from the necklace graph Ne_n, then the graph becomes a bipartite graph.*

Next, we study another class of cubic Halin graph in which the characteristic tree is constructed by drawing the pendent edges (leaf edges) up and down alternately starting from the leftmost to the rightmost on the spine. Here n represents the number of up and down pendent edges from the leftmost to the rightmost drawn vertically to the spine. Denote this class of graphs by G_{1*n}. The diameter of G_{1*n} is $\left\lceil \frac{n+3}{2} \right\rceil$. Label the cycle edges of G_{1*n} from the leftmost, in clockwise direction by $\{e_i | 0 \leq i \leq n+1\}$, that along the spine from the leftmost by $\{s_i | 0 \leq i \leq n\}$. Label the pendant edges above the spine from the leftmost to the rightmost by $\left\{p_i | 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil\right\}$ and that which is below the spine from rightmost to the leftmost by $\left\{p_i | \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n\right\}$. An illustration is given in Figure 2.

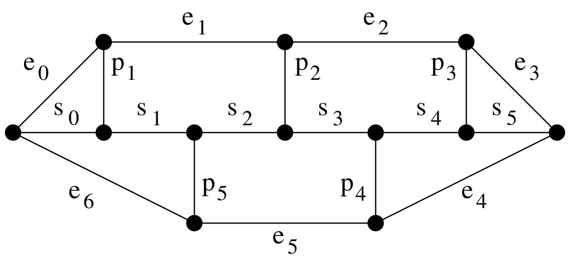

Figure 2: G_{1*5}

Theorem 4. *The strong rainbow connection number of G_{i*n} is n.*

Proof. We consider the following cases.
Case 1: n is even
Colouring Algorithm:
We shall first colour the cycles above the spine as follows:

1. The three edges of the leftmost cycle C_3 can be assigned with a colour c_1 (same colour). This implies that $c(e_0) = c(p_1) = c(s_0) = c_1$.

2. The cycle adjacent with C_3 is C_5. The cycles C_3 and C_5 have a common pendent edge p_1. Since C_3 is already coloured with a colour c_1, the remaining four edges of C_5 can be assigned with the colours c_2 and c_3 alternately. That is, the cycle C_5 constituted from the edges p_1, e_1, p_2, s_2, s_1 are assigned with the colours c_1, c_2, c_3, c_2, c_3 respectively.

3. Colour the cycle adjacent with C_5. Since one of the edges of the cycle C_5 is already coloured as discussed in step 2, the remaining four edges of C_5 can be assigned with the colours c_4 and c_5 alternately. Continue this process until all the cycles of length 5 have been assigned with colours. There are totally $1 + \frac{n-4}{2}$ cycles of length 5 and any two consecutive cycles have a common pendent edge. We find that totally $2\left(1 + \frac{n-4}{2}\right)$ colours are required for the $\left(1 + \frac{n-4}{2}\right)$ cycles C_5.

4. The rightmost cycle adjacent with C_5 is C_4. Since one of the edges (pendent edge) of C_4 is already assigned with a colour, assign a new colour to the cycle edge adjacent to the coloured edge. The two colours can be assigned to the remaining spine edges s_n, s_{n-1} alternately. That is, the edges $p_{\frac{n}{2}}, e_{\frac{n}{2}}, s_n, s_{n-1}$ receive the colours $c_{n-1}, c_n, c_{n-1}, c_n$ respectively.

5. All the cycles above the spine have been assigned with $1+2\left(1 + \frac{n-4}{2}\right)+ 1$ colours totally which is exactly n.

Now consider the cycles below the spine.

1. The leftmost cycle is C_4 and the rightmost cycle is C_3. The intermediate cycles are of length 5 and there are $\left(1 + \frac{n-4}{2}\right)$ intermediate cycles. We find that the colours of the spine edges can be applied to the cycles having the spine edges as its edges. Antipodal edges of the cycles acquire the same colours. That is, $c(s_i) = c(p_{n-\frac{i}{2}})$ for $0 \le i \le n-2$, $i \equiv 0 \pmod 2$ and $c(s_i) = c(e_{n+1-\lfloor \frac{i}{2} \rfloor})$ for $1 \le i \le n-1$, $i \equiv 1 \pmod 2$. The uncoloured edge is the cycle edge $e_{\frac{n}{2}+1}$ in the rightmost cycle C_3, which can be assigned with the colour c_1.

2. We find that colours assigned for the cycles above the spine have been used for the cycles below the spine.

Thus, totally $2 + 2\left(1 + \frac{n-4}{2}\right)$ colours are used to colour the edges of G_{1*n} if n is even. Thus $src(G_{r*n}) = n$. An illustration if given in Figure 3.

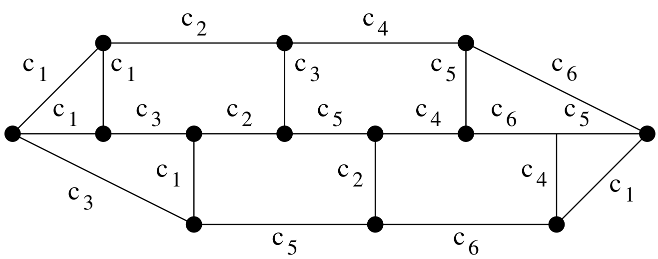

Figure 3: G_{1*6}

Case 2: n is odd
Study the cycles above the spine. The leftmost cycle and the rightmost cycle are of length 3 and there are $\frac{n-1}{2}$ intermediate cycles of length 5. We note that any two consecutive cycles have an edge in common. Colour the cycles using the following algorithm:

1. Take a look at the leftmost cycle C_3. Colour the edges of the cycle C_3 with a colour c_1.

2. C_5 is the cycle adjacent to C_3 and hence both have a pendent edge in common. The remaining four edges of C_5 can be assigned with the colours c_2 and c_3 alternately since C_3 is already coloured with a colour c_1. That is, the cycle C_5 constituted from the edges p_1, e_1, p_2, s_2, s_1 are assigned with the colours c_1, c_2, c_3, c_2, c_3 respectively.

3. There are totally $\frac{n-1}{2}$ cycles of length 5 and any two consecutive cycles have a pendent edge in common. Each cycle requires exactly two colours as the common pendent edge is already assigned with a colour as given in step 2. Repeat this process until all the 5-cycles are coloured.

4. The two edges $e_{\lceil \frac{n}{2} \rceil}$, s_n of the rightmost cycle C_3 are assigned with the colour of the common pendent edge $p_{\lceil \frac{n}{2} \rceil}$ which is assigned in step 3. That is, $c(e_{\lceil \frac{n}{2} \rceil}) = c(s_n) = c(p_{\lceil \frac{n}{2} \rceil}) = c_n$.

Thus all the cycles above the spine have been assigned with n colours. Now consider the cycles below the spine. We give an algorithm which uses the same set of n colours for the cycles below the spine.

1. We find that colours of the spine edges can be applied to the cycles having the spine edges as its edges. Antipodal edges of the cycles acquire the same colours. This implies that $c(s_i) = c(p_{n-\frac{i}{2}})$ for $0 \le i \le n-4$, $i \equiv 0 \pmod 2$ and $c(s_i) = c(e_{n+1-\lfloor\frac{i}{2}\rfloor})$ for $1 \le i \le n-4$, $i \equiv 1 \pmod 2$. Moreover, $c(p_{\lceil\frac{n}{2}\rceil+1}) = c(p_{\lceil\frac{n}{2}\rceil+2}), c(s_{n-1}) = c(e_{\lceil\frac{n}{2}\rceil+1})$.

Now we study another class of cubic Halin graphs in which the characteristic tree is constructed by drawing $\left\lceil \frac{n}{2} \right\rceil$ pendent edges (leaf edges) above the spine and $\left\lfloor \frac{n}{2} \right\rfloor$ pendant edges below the spine from the leftmost to the rightmost on the spine. For convenience, denote this class of graphs by G_n^*. Label the cycle edges of G_n^* from the leftmost, in clockwise direction by $\{e_i|0 \le i \le n+1\}$ and that along the spine from the leftmost to the rightmost by $\{s_i|0 \le i \le n\}$. Label the pendant edges above the spine from the leftmost to the rightmost by $\left\{p_i|1 \le i \le \left\lceil \frac{n}{2} \right\rceil\right\}$ and that which is below the spine from leftmost to the rightmost by $\left\{p_i| \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n\right\}$. An illustration is given in Figure 4.

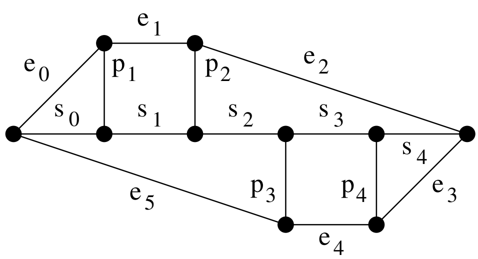

Figure 4: G_4^*

$\square$

Theorem 5. *For a graph* G_n^*, $diam(G_n^*) + 1 \le src(G_n^*) \le diam(G_n^*) + 2$.

Proof. We consider the following cases.
Case 1: n is even. For $n < 6$, the colouring algorithms do not follow a general pattern. The strong rainbow connection numbers of G_2^* and G_4^* are found to be 2 and 3 respectively. Figure 5 illustrates the strong rainbow edge colouring of G_4^*.

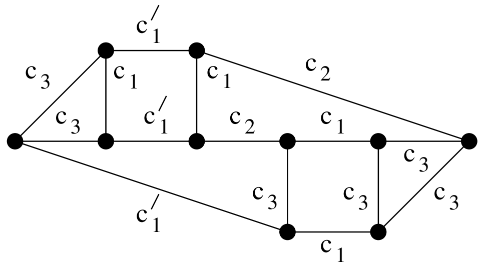

Figure 5: $src(G_4^*) = 4$

Now we discuss the case for $n \geq 6$. We concentrate on the cycles above the spine. The leftmost cycle is of length 3. The rightmost cycle is of length $\frac{n}{2}+3$. There are $\frac{n-2}{2}$ intermediate cycles of length 4. Also any two consecutive cycles have a common pendent edge. Below the spine, the leftmost cycle is of length $\frac{n}{2}+3$ and the rightmost cycle is of length 3. There are $\frac{n-2}{2}$ intermediate cycles of length 4. Now consider the rightmost cycle of length $\frac{n}{2}+3$ above the spine.

Subcase 1.1: The cycle of length $\frac{n}{2}+3$ is even. Above the spine, this rightmost cycle is constituted from the edges $p_{\frac{n}{2}}, e_{\frac{n}{2}}, s_n, s_{n-1}, ..., s_{\frac{n}{2}}$. The edges $p_{\frac{n}{2}}, e_{\frac{n}{2}}, s_n, s_{n-1}, ..., s_{\frac{n}{2}}$ are assigned with the colours $c_1, c_2, ..., c_{\frac{\frac{n}{2}+3}{2}}, c_1, c_2, ..., c_{\frac{\frac{n}{2}+3}{2}}$ respectively. Below the spine, the leftmost cycle is constituted from the edges $s_{\frac{n}{2}}, p_{\frac{n}{2}+1}, e_{n+1}, s_0, s_1, ...s_{\frac{n}{2}-1}$. We note that the edge $s_{\frac{n}{2}}$ is the common spine edge to both the cycles. The remaining edges $p_{\frac{n}{2}+1}, e_{n+1}, s_0, s_1, ...s_{\frac{n}{2}-1}$ are assigned with $\left(\frac{\frac{n}{2}+3}{2}\right)$ distinct colours. That is, the edges $s_{\frac{n}{2}}, p_{\frac{n}{2}+1}, e_{n+1}, s_0, s_1,$ $...s_{\frac{n}{2}-1}$ are assigned with the colours $c_{\left(\frac{\frac{n}{2}+3}{2}\right)}, c'_1, c'_2, ..., c_{\left(\frac{\frac{n}{2}+3}{2}\right)}, c'_1, c'_2, ..., c'_{\left(\frac{\frac{n}{2}+3}{2}\right)-1}$ respectively. The colour of the pendent edge $p_{\frac{n}{2}}$ of the rightmost cycle above the spine can be assigned to the remaining pendent edges $p_1, p_2, ..., p_{\frac{n}{2}-1}$ above the spine. Similarly, $c(p_{\frac{n}{2}+1}) = c(p_{\frac{n}{2}+2}) = ... = c(p_n)$. The remaining uncoloured cycle edges acquire the colour of the spine edges as they are antipodal. Thus, we assign $\left(\frac{\frac{n}{2}+3}{2}\right) + \left(\frac{\frac{n}{2}+3}{2}\right)$ distinct colours to the edges of G_n^*. After simplification we find that the minimum number of colours required for strong rainbow edge colouring is $\frac{n}{2}+3$. But $diam(G_n^*) = \frac{n}{2}+2$. Hence $src(G_n^*) = diam(G_n^*)+1 = \frac{n}{2}+3$.

Subcase 1.2: The cycle of length $\frac{n}{2}+3$ is odd. Above the spine, this rightmost cycle is constituted from the edges $p_{\frac{n}{2}}, e_{\frac{n}{2}}, s_n, s_{n-1}, ..., s_{\frac{n}{2}}$. The edges are assigned with the colours $c_1, c_2, ..., c_{\left\lceil \frac{\frac{n}{2}+3}{2} \right\rceil}$, $c_1, c_2, ..., c_{\left\lceil \frac{\frac{n}{2}+3}{2} \right\rceil -1}$ respectively. Below the spine, the leftmost cycle is constituted from the edges $s_{\frac{n}{2}}$, $p_{\frac{n}{2}+1}$, e_{n+1}, s_0, s_1, ... $s_{\frac{n}{2}-1}$. We note that the edge $s_{\frac{n}{2}}$ is the common spine edge to both the cycles. The remaining $p_{\frac{n}{2}+1}, e_{n+1}, s_0, s_1, ...s_{\frac{n}{2}-1}$ edges are assigned with $\left\lfloor \frac{\frac{n}{2}+3}{2} \right\rfloor$ distinct colours. Thus the edges $s_{\frac{n}{2}}, p_{\frac{n}{2}+1}, e_{n+1}, s_0, s_1, ...s_{\frac{n}{2}-1}$ are assigned with the colours $c'_1, c'_2, ..., c'_{\left\lfloor \frac{\frac{n}{2}+3}{2} \right\rfloor}, c_{\left\lfloor \frac{\frac{n}{2}+3}{2} \right\rfloor}, c'_1, c'_2, ..., c'_{\left\lfloor \frac{\frac{n}{2}+3}{2} \right\rfloor}$, respectively. The colour of the pendent edge $p_{\frac{n}{2}}$ of the rightmost cycle above the spine can be assigned to the remaining leaf edges $p_1, p_2, ..., p_{\frac{n}{2}-1}$ above the spine. Similarly $c(p_{\frac{n}{2}+1}) = c(p_{\frac{n}{2}+2}) = ... = c(p_n)$. The remaining uncoloured cycle edges acquire the colour of the spine edges as they are antipodal. Thus, we assign $\left\lceil \frac{\frac{n}{2}+3}{2} \right\rceil + \left\lfloor \frac{\frac{n}{2}+3}{2} \right\rfloor$ distinct colours to the edges of G_n^*. After simplification we find that which is equal to $\frac{n}{2}+3$. Hence $src(G_n^*) = diam(G_n^*)+1 = \frac{n}{2}+3$ for $n \geq 6$.

Case 2: $n \geq 3$ is odd G_n^*. consists of $\left\lceil \frac{n}{2} \right\rceil$ pendant edges above the spine and $\left\lfloor \frac{n}{2} \right\rfloor$ pendant edges below the spine.

The proof is similar to case 1. In this case, the rightmost cycle above the spine is of length $\left\lceil \frac{n}{2} \right\rceil + 2$ and leftmost cycle below the spine is of length $\left\lceil \frac{n}{2} \right\rceil + 3$. For convenience, we denote the rightmost cycle above the spine and the leftmost cycle below the spine by $C^R_{\lceil \frac{n}{2} \rceil + 2}$ and $C^L_{\lceil \frac{n}{2} \rceil + 3}$ respectively. First we colour the cycles $C^R_{\lceil \frac{n}{2} \rceil + 2}$ and $C^L_{\lceil \frac{n}{2} \rceil + 3}$, which exhaust all the spine edges. Then the colour of the spine edges can be assigned to their corresponding antipodal cycle edges.

Subcase 2.1: The cycle $C^R_{\lceil \frac{n}{2} \rceil + 2}$ is even and the cycle $C^L_{\lceil \frac{n}{2} \rceil + 3}$ is odd. The cycle $C^R_{\lceil \frac{n}{2} \rceil + 2}$ requires $\frac{\left\lceil \frac{n}{2} \right\rceil + 2}{2}$ colours and the cycle $C^L_{\lceil \frac{n}{2} \rceil + 3}$ requires $\left\lfloor \frac{\left\lceil \frac{n}{2} \right\rceil + 3}{2} \right\rfloor$

distinct colours for strong rainbow edge colouring. Hence totally $\left\lceil \frac{n}{2} \right\rceil + 3$ colours are required for strong edge colouring of G_n^*. But $diam(G_n^*) = \lceil \frac{n}{2} \rceil + 2$. Thus $src(G_n^*) = diam(G_n^*) + 1$. This completes the proof.

Subcase 2.2: The cycle $C^R_{\lceil \frac{n}{2} \rceil + 3}$ is odd and the cycle $C^L_{\lceil \frac{n}{2} \rceil + 2}$ is even. The cycle $C^R_{\lceil \frac{n}{2} \rceil + 2}$ requires $\left\lceil \frac{\left\lceil \frac{n}{2} \right\rceil + 2}{2} \right\rceil$ colours and the cycle $C^L_{\lceil \frac{n}{2} \rceil + 3}$ requires $\left(\frac{\left\lceil \frac{n}{2} \right\rceil + 3}{2} \right)$ distinct colours for strong rainbow edge colouring. Hence totally $\left\lceil \frac{n}{2} \right\rceil + 3$ colours are required for strong edge colouring of G_n^*. But $diam(G_n^*) = \left\lceil \frac{n}{2} \right\rceil + 1$. Thus $src(G_n^*) = diam(G_n^*) + 2$. This completes the proof. $\square$

4. Conclusion

In this paper we characterized three non isomorphic cubic Halin graphs Ne_n, G_{1*n} and G_n^*. We found that the strong rainbow connection numbers of Ne_n and G_n^* are closer to the diameter of the graph. However, the strong rainbow connection number of G_{1*n} is n. In this process, we develop techniques that are useful in finding the strong rainbow connection numbers of cycle related graphs.

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