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$\mathcal{T}$-CURVATURE ON LORENTZIAN α-SASAKIAN MANIFOLDS

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Abstract: We study Lorentzian α-Sasakian manifolds endowed with a $\mathcal{T}$-curvature tensor which satisfies the conditions $\mathcal{T}(\xi, X)\cdot R = 0$, $\mathcal{T}(\xi, X)\cdot S = 0$, $\mathcal{T}$-flat, ξ-$\mathcal{T}$-flat. We also applied the results to several curvatures which are the particular cases of $\mathcal{T}$-curvature tensor.

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Key Words: Lorentzian α-Sasakian manifold, $\mathcal{T}$-curvature tensor, $\mathcal{T}$-flat, ξ-$\mathcal{T}$-flat, η-Einstein manifold

1. Introduction

It is well known that, Weyl conformal curvature tensor [3] and conharmonic curvature tensor [5] are invariant under conformal and conharmonic transformations respectively. And, projective curvature tensor [3] vanishes if and only if semi-Riemannian manifold is locally projectively flat. The next important cur-

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vature tensor in semi-Riemannian point of view is concircular curvature tensor [13], which is also invariant under concircular transformation. Yano and Sawaki [14] generalized the conformal curvature tensor and concircular curvature tensor to quasi-conformal curvature tensor.

With these ideas in mind, in 2011, Tripathi and Gupta [10] introduced a new curvature tensor called $\mathcal{T}$-curvature tensor on the semi-Riemannian manifold and is given by

$$\begin{aligned} \mathcal{T}(X,Y)Z =& a_0R(X,Y)Z + a_1S(Y,Z)X + a_2S(X,Z)Y \\ &+ a_3S(X,Y)Z + a_4g(Y,Z)QX + a_5g(X,Z)QY \\ &+ a_6g(X,Y)QZ + a_7r[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{1}$$

where, $a_0, ..., a_7$ are some smooth functions on M; and R, S, Q and r are the curvature tensor, the Ricci tensor, the Ricci operator of type $(1,1)$ and the scalar curvature respectively.

With this above generalized form they have shown that quasi-conformal, conformal, conharmonic, concircular, pseudo-projective, projective, M-projective, W_i-curvature tensors $(i = 0, ..., 9)$, $W_j^{\square}$-curvature tensors $(j = 0, 1)$ are the particular cases of the $\mathcal{T}$-curvature tensor. Later, the authors Tripathi and Gupta [11] also studied $\mathcal{T}$ curvature tensor in k-contact and Sasakian manifolds. In 2012 Nagaraja [8] and Somashekhara studied the $\mathcal{T}$-curvature tensor in (k,μ) manifold. In 2013, Shah [9] studied the same curvature on LP-Sasakian manifold. Later in 2014, Ingalahalli and Bagewadi [4] studied the curvature on $N(k)$-contact manifold and arrived at some interesting results.

The present paper is organized as follows: In Section 2, some definitions and preliminary results are presented which will be needed thereafter. In Section 3 we study Lorentzian α-Sasakian manifold satisfying the condition $\mathcal{T}(\xi, X)\cdot R = 0$ and shown that as η-Einstein manifold. And in section 4 we study Lorentzian α-Sasakian manifold satisfying the condition $\mathcal{T}(\xi, X) \cdot S = 0$ and arrived at some interesting results. Section 5 and Section 6 contains discussions about $\mathcal{T}$-flat and ξ-$\mathcal{T}$-flat conditions respectively. We have also discussed the results of each sections to several curvatures which are particular cases of $\mathcal{T}$-curvature tensor.

2. Preliminaries

A $(2n+1)$-dimensional smooth manifold M is said to be Lorentzian α-Sasakian manifold if it admits a $(1,1)$-tensor field ϕ, a contravariant vector field ξ, global differentiable 1-form η and a Lorentzian metric g which satisfy [15]

$$\phi^2 = I + \eta \otimes \xi, \ \eta(\xi) = -1, \tag{2}$$
$$\phi\xi = 0, \ \eta \circ \phi = 0, \tag{3}$$
$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{4}$$
$$g(X,\xi) = \eta(X), \tag{5}$$

for all X,$Y \in TM$.
Also a Lorentzian α-Sasakian manifold M satisfies the following [15]:

$$\nabla_X \xi = \alpha\phi X, \tag{6}$$
$$(\nabla_X \eta)Y = \alpha g(X, \phi Y), \tag{7}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is constant.
Further, a Lorentzian α-Sasakian manifold M holds the following relations:

$$R(\xi, X)Y = \alpha^2(g(X,Y)\xi - \eta(Y)X), \tag{8}$$
$$R(X,Y)\xi = \alpha^2(\eta(Y)X - \eta(X)Y), \tag{9}$$
$$R(\xi, X)\xi = \alpha^2(\eta(X)\xi + X), \tag{10}$$
$$S(X,\xi) = 2n\alpha^2\eta(X), \tag{11}$$
$$Q\xi = 2n\alpha^2\xi, \tag{12}$$
$$S(\xi,\xi) = -2n\alpha^2, \tag{13}$$

for any vector fields X, Y, Z, where S is the Ricci curvature and Q is Ricci operator given by $S(X,Y) = g(QX,Y)$.

Definition 1. A Lorentzian α-Sasakian manifold M is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X,Y) = \lambda_1 g(X,Y) + \lambda_2 \eta(X)\eta(Y), \tag{14}$$

for any vector fields X, Y, where λ_1, λ_2 are functions on M. If $\lambda_1 = 0$ then M is a special η-Einstein manifold.

3. Lorentzian α-Sasakian Manifold Satisfying $\mathcal{T}(\xi, X) \cdot R = 0$

Theorem 2. *A $(2n+1)$ dimensional Lorentzian α-Sasakian manifold M satisfying the condition $\mathcal{T}(\xi, X) \cdot R = 0$ is an η-Einstein manifold, provided $a_1 \neq 0$.*

Proof. Suppose in a Lorentzian α-Sasakian manifold

$$\mathcal{T}(\xi, X) \cdot R = 0. \tag{15}$$

Which implies,

$$\begin{aligned} \mathcal{T}(\xi, X)R(Y, Z)U - R(\mathcal{T}(\xi, X)Y, Z)U \\ -R(Y, \mathcal{T}(\xi, X)Z)U - R(Y, Z)\mathcal{T}(\xi, X)U = 0. \end{aligned} \tag{16}$$

By taking inner product with ξ of the above equation, we have.

$$\begin{aligned} g(\mathcal{T}(\xi, X)R(Y, Z)U, \xi) - g(R(\mathcal{T}(\xi, X)Y, Z)U, \xi) \\ -g(R(Y, \mathcal{T}(\xi, X)Z)U, \xi) - g(R(Y, Z)\mathcal{T}(\xi, X)U, \xi) = 0. \end{aligned} \tag{17}$$

Putting $Y = U = \xi$ in (17) and using (1),(5),(8),(9),(10),(11), we obtain

$$\begin{aligned} \alpha^2\{ &- a_1 S(X, Z) - (a_0\alpha^2 + a_1 2n\alpha^2 + a_4 2n\alpha^2 + a_7 r)\eta(X)\eta(Z) \\ &- (a_0\alpha^2 + a_4 2n\alpha^2 + a_7 r)g(X, Z) - (a_0\alpha^2 + a_2 2n\alpha^2)g(X, Z) \\ &- (a_0\alpha^2 + a_2 2n\alpha^2)\eta(X)\eta(Z)\} = 0. \end{aligned} \tag{18}$$

After simplifying (18), we get

$$\begin{aligned} -a_1 S(X, Z) = (&\alpha^2(2a_0 + 2na_2 + 2na_4) + a_7 r)g(X, Z) \\ &+ (\alpha^2(2a_0 + 2na_1 + 2na_2 + 2na_4) + a_7 r)\eta(X)\eta(Z), \end{aligned} \tag{19}$$

If $a_1 \neq 0$, then

$$S(X, Z) = \lambda_1 g(X, Z) + \lambda_2 \eta(X)\eta(Z), \tag{20}$$

where,

$$\lambda_1 = \frac{-(2a_0\alpha^2 + a_2 2n\alpha^2 + a_4 2n\alpha^2 + a_7 r)}{a_1} \tag{21}$$

and

$$\lambda_2 = \frac{-(2a_0\alpha^2 + a_1 2n\alpha^2 + a_2 2n\alpha^2 + a_4 2n\alpha^2 + a_7 r)}{a_1}.$$

Hence M is a η-Einstein manifold.
Further, from [10] and equation (20), we have also derived the result of the above theorem for particular cases of $\mathcal{T}$-curvature tensor as follows:

Curvature	**Values of λ_1 and λ_2**
Conformal curvature tensor $\mathcal{C}$	$\lambda_1 = 2(n-1)\alpha^2 + \frac{r}{2n}$ $\lambda_2 = 2(2n-1)\alpha^2 + \frac{r}{2n}$
Conharmonic curvature tensor $\mathcal{L}$	$\lambda_1 = 2\alpha^2$ $\lambda_2 = 2(n-1)\alpha^2$
Projective curvature tensor $\mathcal{P}$	$\lambda_1 = 3\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{M}$-Projective curvature tensor	$\lambda_1 = 8n\alpha^2$ $\lambda_2 = 6n\alpha^2$
$\mathcal{W}_0$-curvature tensor	$\lambda_1 = 4n\alpha^2$ $\lambda_2 = 2n\alpha^2$
$\mathcal{W}_0^{*}$-curvature tensor	$\lambda_1 = -4n\alpha^2$ $\lambda_2 - 6n\alpha^2$
$\mathcal{W}_1$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = -2n\alpha^2$
$\mathcal{W}_1^{*}$-curvature tensor	$\lambda_1 = 4n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_6$-curvature tensor	$\lambda_1 = 4n\alpha^2$ $\lambda_2 = 2n\alpha^2$
$\mathcal{W}_7$-curvature tensor	$\lambda_1 = 6n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_8$-curvature tensor	$\lambda_1 = 2\alpha^2$ $\lambda_2 = 6n\alpha^2$

For Quasi conformal curvature tensor $\mathcal{C}_{*}$:

$$\lambda_1 = 2A\alpha^2 - \frac{r}{2n+1}(\frac{A}{2n} + 2B)$$
$$\lambda_2 = \frac{-1}{B}[2A\alpha^2 + 2nB\alpha^2 - \frac{r}{2n+1}(\frac{A}{2n} + 2B)].$$

For Pseudo-projective curvature tensor $\mathcal{P}_{*}$:

$$\lambda_1 = 2A\alpha^2 + 2nB\alpha^2 - \frac{r}{2n+1}(\frac{A}{2n} + B)$$
$$\lambda_2 = 2A\alpha^2 - \frac{r}{2n+1}(\frac{A}{2n} + B).$$

Here A and B are some smooth functions on M.

In all the above cases M is a η-Einstein manifold. And since $a_1, a_7 = 0$ in $\mathcal{W}_2$, $\mathcal{W}_3$, $\mathcal{W}_4$, $\mathcal{W}_5$, $\mathcal{W}_9$-curvature tensors, the above result is not applicable for these curvature tensors. $\square$

4. Lorentzian α-Sasakian Manifold Satisfying $\mathcal{T}(\xi, X) \cdot S = 0$

Theorem 3. *A $(2n+1)$ dimensional Lorentzian α-Sasakian manifold M satisfying the condition $\mathcal{T}(\xi, Y) \cdot S = 0$ with*

$$a_0\alpha^2 + a_1 2n\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r \neq 0,$$

is an η-Einstein manifold.

Proof. Consider a Lorentzian α-Sasakian manifold satisfying

$$\mathcal{T}(\xi, X) \cdot S = 0. \tag{22}$$

Which implies,

$$S(\mathcal{T}(\xi, X)Y, Z) + S(Y, \mathcal{T}(\xi, X)Z) = 0. \tag{23}$$

Taking inner product of (23) with ξ, we get

$$g(S(\mathcal{T}(\xi, X)Y, Z), \xi) + g(S(Y, \mathcal{T}(\xi, X)Z), \xi) = 0. \tag{24}$$

And taking $Z = \xi$ in (24) and using (1),(5),(8),(9),(10),(11), we obtain

$$\begin{aligned}
&- a_1 2n\alpha^2 S(X, Y) - 2n\alpha^2(a_0\alpha^2 + a_4 2n\alpha^2 + a_7 r)g(X, Y) \\
&+ 2n\alpha^2(a_0\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r)\eta(X)\eta(Y) \\
&+ 4n^2\alpha^4(a_3 + a_6 + a_1)\eta(X)\eta(Y) \\
&+ 2n\alpha^2(a_0\alpha^2 + a_4 2n\alpha^2 + a_7 r)\eta(X)\eta(Y) \\
&- (a_0\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r)S(X, Y) \\
&+ 2n\alpha^2(a_3 2n\alpha^2 + a_6 2n\alpha^2)\eta(X)\eta(Y) = 0.
\end{aligned} \tag{25}$$

Simplifying (25), we have

$$S(X, Y) = \lambda_1 g(X, Y) + \lambda_2 \eta(X)\eta(Y), \tag{26}$$

where,

$$\lambda_1 = \frac{-2n\alpha^2(a_0\alpha^2 + a_4 2n\alpha^2 + a_7 r)}{a_0\alpha^2 + a_1 2n\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r}$$

and

$$\lambda_2 = \frac{2n\alpha^2(2a_0\alpha^2 + 2n\alpha^2[a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6])}{a_0\alpha^2 + a_1 2n\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r}.$$

The particular cases of Theorem 3 for different curvature tensors is as follows:

Curvature	**Values of λ_1 and λ_2**
Conformal curvature tensor $\mathcal{C}$	$\lambda_1 = 2n\alpha^2[\frac{-2n(2n-1)\alpha^2+4n^2\alpha^2-r}{2n(2n-1)\alpha^2+4n^2\alpha^2-r}]$ $\lambda_2 = \frac{8n^2\alpha^4(2n-1)}{2n(2n-1)\alpha^2+4n^2\alpha^2-r}$
Conharmonic curvature tensor $\mathcal{L}$	$\lambda_1 = \frac{2n\alpha^2}{4n-1}$ $\lambda_2 = \frac{4n(2n-1)\alpha^2}{4n-1}$
Concircular curvature tensor $\mathcal{V}$	$\lambda_1 = \frac{-4n^2(2n+1)\alpha^2+2nr}{r}$ $\lambda_2 = \frac{8n^2(2n+1)\alpha^2}{r}$
Projective curvature tensor $\mathcal{P}$	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{M}$-Projective curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 8n\alpha^2$
$\mathcal{W}_0$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_0^{*}$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_1$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_1^{*}$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_2$-curvature tensor	$\lambda_1 = 0$ $\lambda_2 = 2n\alpha^2$

Curvature	**Values of λ_1 and λ_2**
$\mathcal{W}_4$-curvature tensor	$\lambda_1 = -n\alpha^2$ $\lambda_2 = n\alpha^2$
$\mathcal{W}_5$-curvature tensor	$\lambda_1 = -2n\alpha^2$ $\lambda_2 = 4n\alpha^2$
$\mathcal{W}_9$-curvature tensor	$\lambda_1 = 0$ $\lambda_2 = 6n\alpha^2$

For Quasi conformal curvature tensor $\mathcal{C}_{*}$:

$$\lambda_1 = -2n\alpha^2\left[\frac{2n(2n+1)A\alpha^2 + 4n^2(2n+1)B\alpha^2 - r(A+4nB)}{2n(2n+1)A\alpha^2 - 4n^2(2n+1)B\alpha^2 - r(A+4nB)}\right]$$

$$\lambda_2 = 2n\alpha^2\left[\frac{4n(2n+1)A\alpha^2 - r(A+4nB)}{2n(2n+1)A\alpha^2 - 2n(2n+1)B\alpha^2 - r(A+4nB)}\right].$$

For Pseudo-projective curvature tensor $\mathcal{P}_{\square}$:

$$\lambda_1 = -2n\alpha^2 \left[\frac{2n(2n+1)A\alpha^2 - r(A+2nB)}{2n(2n+1)A\alpha^2 + r(A+2nB)} \right]$$
$$\lambda_2 = 2n\alpha^2 \left[\frac{4n(2n+1)A\alpha^2 - r(A+2nB)}{2n(2n+1)A\alpha^2 + r(A+2nB)} \right].$$

Here A and B are some smooth functions on M.

From the above table one can observe that M is special η-Einstein manifold for $\mathcal{W}_2$ and $\mathcal{W}_9$-curvature tensors and η-Einstein manifold for other curvature tensors. Since $a_0\alpha^2 + a_1 2n\alpha^2 + a_2 2n\alpha^2 + a_5 2n\alpha^2 - a_7 r = 0$, in $\mathcal{W}_3$, $\mathcal{W}_6$, $\mathcal{W}_7$, $\mathcal{W}_8$-curvature tensors, the above result is not applicable for these curvature tensors. □

5. Lorentzian α-Sasakian Manifold Satisfying $\mathcal{T}(X,Y)Z = 0$

Theorem 4. *A $(2n+1)$ dimensional Lorentzian α-Sasakian manifold M satisfying the condition $\mathcal{T}(X,Y)Z = 0$ with $a_3 \neq 0$ is an η-Einstein manifold.*

Proof. Suppose Lorentzian α-Sasakian manifold is $\mathcal{T}$-flat, i.e.,

$$\mathcal{T}(X,Y)Z = 0. \tag{27}$$

Then we have

$$\begin{aligned} 0 =& a_0 g(R(X,Y)Z,W) + a_1 S(Y,Z)g(X,W) \\ &+ a_2 S(X,Z)g(Y,W) + a_3 S(X,Y)g(Z,W) \\ &+ a_4 g(Y,Z)g(QX,W) + a_5 g(X,Z)g(QY,W) \\ &+ a_6 g(X,Y)g(QZ,W) + a_7 r g(Y,Z)g(X,W) \\ &- a_7 r g(X,Z)g(Y,W). \end{aligned} \tag{28}$$

Taking $Z = W = \xi$ in (28), we get

$$S(X,Y) = \lambda_1 g(X,Y) + \lambda_2 \eta(X)\eta(Y), \tag{29}$$

where,

$$\lambda_1 = -\frac{a_6 2n\alpha^2}{a_3},$$

$$\lambda_2 = \frac{(a_1 + a_2 + a_4 + a_5)2n\alpha^2}{a_3}.$$

Further, from [10] and equation (29), we have also derived the result of the above theorem for particular cases of $\mathcal{T}$-curvature tensor as follows:
Since $a_3 = 0$ in conformal, conharmonic, concircular, pseudo-projective, projective, $\mathcal{M}$-projective,$\mathcal{W}_0, \mathcal{W}_0^*, \mathcal{W}_1, \mathcal{W}_1^*, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4, \mathcal{W}_5, \mathcal{W}_6$,and $\mathcal{W}_7$- curvature tensors, the above result is not applicable for these curvature tensors.
So, curvatures which are applicable and corresponding results with the values of λ_1 and λ_2 are given below:

Curvature	**Values of λ_1 and λ_2**
$\mathcal{W}_8$-curvature tensor	$\lambda_1 = 0$, $\lambda_2 = -2n\alpha^2$
$\mathcal{W}_9$-curvature tensor	$\lambda_1 = 0$, $\lambda_2 = -2n\alpha^2$

From the above table we can observe that M is Special η-Einstein manifold in both curvature tensors. □

6. Lorentzian α-Sasakian Manifold Satisfying $\mathcal{T}(X, Y)\xi = 0$

Theorem 5. *A $(2n + 1)$ dimensional Lorentzian α-Sasakian manifold M satisfying the condition $\mathcal{T}(X, Y)\xi = 0$ with $a_3 \neq -2n\alpha^2 a_0$ is a special η-Einstein manifold with scalar curvature given by*

$$r = -\frac{2n\alpha^2(a_1 + a_4 + a_5)}{a_0 2n\alpha^2 + a_3}. \tag{30}$$

Proof. Suppose Lorentzian α-Sasakian manifold is ξ-$\mathcal{T}$-flat, i.e.,

$$\mathcal{T}(X, Y)\xi = 0. \tag{31}$$

Then we have

$$\begin{aligned}(a_3 + a_6 2n\alpha^2)S(X, Y)\xi + (a_0\alpha^2 + a_1 2n\alpha^2 + a_4 2n\alpha^2 + a_7 r)\eta(Y)X \\ + (-a_0\alpha^2 + a_5 2n\alpha^2 - a_7 r)\eta(X)Y = 0.\end{aligned} \tag{32}$$

By taking inner product of (32)with ξ, we have

$$S(X, Y) = \lambda_1 g(X, Y) + \lambda_2 \eta(X)\eta(Y), \tag{33}$$

where,

$$\lambda_1 = 0 \text{ and } \lambda_2 = \frac{2n\alpha^2(a_1 + a_4 + a_5)}{a_0 2n\alpha^2 + a_3}.$$

On contracting above equation one can get the scalar curvature r as

$$r = -\frac{2n\alpha^2(a_1 + a_4 + a_5)}{a_0 2n\alpha^2 + a_3}.$$

Further, from [10] the result of the Theorem 5 for different curvature tensors is as follows:

Curvature	**r**
Conformal curvature tensor $\mathcal{C}$	$\frac{1}{2n-1}$
Conharmonic curvature tensor $\mathcal{L}$	$\frac{1}{2n-1}$
Concircular curvature tensor $\mathcal{V}$	0
Projective curvature tensor $\mathcal{P}$	$\frac{1}{2n}$
$\mathcal{M}$-Projective curvature tensor	0
$\mathcal{W}_0$-curvature tensor	0
$\mathcal{W}_0^{*}$-curvature tensor	0
$\mathcal{W}_1$-curvature tensor	$\frac{-1}{2n}$
$\mathcal{W}_1^{*}$-curvature tensor	$\frac{1}{2n}$
$\mathcal{W}_3$-curvature tensor	$\frac{-1}{2n}$
$\mathcal{W}_4$-curvature tensor	$\frac{-1}{2n}$
$\mathcal{W}_5$-curvature tensor	$\frac{1}{2n}$
$\mathcal{W}_6$-curvature tensor	$\frac{1}{2n}$
$\mathcal{W}_7$-curvature tensor	0
$\mathcal{W}_8$-curvature tensor	$\frac{2n\alpha^2}{1+4n\alpha^2}$
$\mathcal{W}_9$-curvature tensor	$\frac{2n\alpha^2}{1+4n\alpha^2}$
$\mathcal{W}_2$-curvature tensor	0
Quasi conformal curvature tensor $\mathcal{C}_{*}$	$\frac{-B}{A}$
Pseudo-projective curvature tensor $\mathcal{P}_{*}$	$\frac{-B}{A}$

Here A and B are some smooth functions on M. □

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