

**ON PRE GENERALIZED b -CLOSED
MAP IN TOPOLOGICAL SPACES**

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Abstract: In this paper, we introduce a new class of pre generalized b -closed map in topological spaces (briefly pgb -closed map) and study some of their properties as well as inter relationship with other closed maps.

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Key Words: pgb -closed set, b -closed map, gb -closed map, rgb -closed map and gp^* -closed map

1. Introduction

Different types of Closed and open mappings were studied by various researchers. In 1996, Andrijevic introduced new type of set called b -open set. A.A.Omari and M.S.M. Noorani [1] introduced and studied b -closed map.

The aim of this paper is to introduce pre generalized b -closed map and to continue the study of its relationship with various generalized closed maps. Through out this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

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2. Preliminaries

In this section, we referred some of the closed set definitions which was already defined by various authors.

Definition 2.1. [12] Let a subset A of a topological space (X, τ) , is called a pre-open set if $A \subseteq \text{int}(cl(A))$.

Definition 2.2. [7] Let a subset A of a topological space (X, τ) , is called a semi-open set if $A \subseteq cl(\text{int}(A))$.

Definition 2.3. [15] Let a subset A of a topological space (X, τ) , is called a α -open set if $A \subseteq \text{int}(cl(\text{int}(A)))$.

Definition 2.4. [3] Let a subset A of a topological space (X, τ) , is called a b -open set if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$.

Definition 2.5. [6] Let a subset A of a topological space (X, τ) , is called a generalized closed set (briefly g -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.6. [8] Let a subset A of a topological space (X, τ) , is called a generalized α closed set (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X .

Definition 2.7. [2] Let a subset A of a topological space (X, τ) , is called a generalized b -closed set (briefly gb -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.8. [9] Let a subset A of a topological space (X, τ) , is called a generalized α^* -closed set (briefly $g\alpha^*$ -closed) if $\alpha cl(A) \subseteq \text{int}U$ whenever $A \subseteq U$ and U is α open in X .

Definition 2.9. [17] Let a subset A of a topological space (X, τ) , is called a pre-generalized closed set (briefly pg -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in X .

Definition 2.10. [4] Let a subset A of a topological space (X, τ) , is called a semi generalized closed set (briefly sg -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .

Definition 2.11. [18] Let a subset A of a topological space (X, τ) , is called a generalized αb -closed set (briefly gab -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X .

Definition 2.12. [10] Let a subset A of a topological space (X, τ) , is called a regular generalized b - closed set (briefly rgb - closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

Definition 2.13. [13] Let a subset A of a topological space (X, τ) , is called pre generalized b - closed set (briefly pgb - closed set) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre open in X .

3. On Pre Generalized b -Closed Map

In this section, we introduce pre generalized b - closed map (pgb - closed map) in topological spaces by using the notions of pgb - closed sets and study some of their properties.

Definition 3.1. Let X and Y be two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \delta)$ is called pre generalized star b - closed (briefly, pgb - closed map) if the image of every closed set in X is pgb -closed in Y .

Theorem 3.2. *Every closed map is pgb - closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ is closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.3. Consider $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. The map is pgb - closed but not closed as the image of and $\{b\}$ in X is $\{c\}$ is not closed in Y .

Theorem 3.4. *Every b - closed map is pgb - closed set but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is b - closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.5. Consider $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. The map is pgb - closed but not b - closed as the image of and $\{a, b\}$ in X is $\{a, c\}$ is not b - closed in Y .

Theorem 3.6. *Every $g\alpha$ - closed map is pgb - closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $g\alpha$ -closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.7. Consider $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = b$. The map is pgb - closed but not $g\alpha$ - closed as the image of $\{b, c\}$ in X is $\{a, b\}$ is not $g\alpha$ -closed in Y .

Theorem 3.8. *Every $g\alpha^*$ - closed map is pgb - closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be $g\alpha^*$ - closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.9. Consider $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. The map is pgb - closed but not $g\alpha^*$ - closed as the image of $\{a\}$ in X is $\{b\}$ is not $g\alpha^*$ - closed in Y .

Theorem 3.10. *Every g - closed map is pgb - closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ g - closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.11. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. The map is pgb -closed but not g -closed as the image of $\{c\}$ in X is $\{b\}$ is not g -closed in Y .

Theorem 3.12. *Every gab - closed map is pgb - closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be gab - closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb - closed in Y . Then f is pgb - closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.13. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = b$. The map is pgb -closed but not $g\alpha b$ -closed as the image of and $\{b, c\}$ in X is $\{b, c\}$ is not $g\alpha b$ -closed in Y .

Theorem 3.14. *Every rgb -closed map is pgb -closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be rgb closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb -closed in Y . Then f is pgb -closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.15. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. The map is pgb -closed but not rgb -closed as the image of $\{a\}$ in X is $\{b\}$ is not rgb -closed in Y .

Theorem 3.16. *Every pgb -closed map is pgb -closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be pgb -closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence gb -closed in Y . Then f is gb -closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.17. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. The map is pg -closed but not pgb -closed as the image of $\{a, c\}$ in X is $\{a, b\}$ is not pgb -closed in Y .

Theorem 3.18. *Every sg -closed map is pgb -closed but not conversely.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be sg -closed map and V be an closed set in X then $f(V)$ is closed in Y . Hence pgb -closed in Y . Then f is pgb -closed. \square

The converse of above theorem need not be true as seen from the following example.

Example 3.19. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = a$. The map is pgb -closed but not sg -closed as the image of $\{c\}$ in X is $\{a\}$ is not sg -closed in Y .

Theorem 3.20. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous and pgb - closed set A is pgb - closed set of X then $f(A)$ is pgb closed in Y .*

Proof. Let $f(A) \subseteq U$ where U is regular open set in Y . Since f is continuous, $f^{-1}(U)$ is open set containing A . Hence $bcl(A) \subseteq f^{-1}(U)$ (as A is pgb - closed). Since f is pgb - closed $f(bcl(A)) \subseteq U$ is pgb closed set $\Rightarrow bcl(f(bcl(A))) \subseteq U$, Hence $bcl(A) \subseteq U$. So that $f(A)$ is pgb - closed set in Y . \square

Theorem 3.21. *If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous and closed set and A is pgb - closed then $f(A)$ is pgb - closed in Y .*

Proof. Let F be a closed set of A then F is pgb - closed set. By theorem 3.20 $f(A)$ is pgb - closed. Hence $f_A(F) = f(F)$ is pgb - closed set of Y . Here f_A is pgb - closed and also continuous. \square

Theorem 3.22. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is closed map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is pgb - closed map, then the composition $g \cdot f : (X, \tau) \rightarrow (Z, \eta)$ is pgb - closed map.*

Proof. Let F be any closed set in (X, τ) . Since f is closed map, $f(F)$ is closed set in (Y, σ) . Since g is pgb - closed map, $g(f(F))$ is pgb - closed set in (Z, η) . That is $g \cdot f(F) = g(f(F))$ is pgb closed. Hence $g \cdot f$ is pgb closed map. \square

Remark 3.23. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is pgb - closed map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is closed map, then the composition need not pgb - closed map.*

Theorem 3.24. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is pgb - closed if and only if for each subset S of (Y, σ) and each open set U containing $f^{-1}(S) \subset U$, there is a pgb - open set V of (Y, σ) such that $S \subset V$ and $f^{-1}(V) \subset U$.*

Proof. Suppose f is pgb - closed. Let $S \subset Y$ and U be an open set of (X, τ) such that $f^{-1}(S) \subset U$. Now $X - U$ is closed set in (X, τ) . Since f is pgb - closed, $f(X - U)$ is pgb - closed set in (Y, σ) . There fore $V = Y - f(X - U)$ is an pgb - open set in (Y, σ) . Now $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$. (ie) $f^{-1}(V) \subset U$.

Conversely,

Let F be a closed set of (X, τ) . Then $f^{-1}(f(F^c)) \subset F^c$ and F^c is an open in (X, τ) . By hypothesis, there exist a pgb - open set V in (Y, σ) such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c \Rightarrow F \subset f^{-1}(V)^c$. Hence $V^c \subset f(F) \subset$

$f(((f^{-1}(V))^c)^c \subset V^c \Rightarrow f(V) \subset V^c$. Since V^c is pgb - closed, $f(F)$ is pgb - closed. (ie) $f(F)$ is pgb - closed in Y . Therefore f is pgb - closed. \square

Theorem 3.25. *If $f : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$, then $f : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is pgb closed map.*

Proof. Let $U_1 \times U_2 \subset X_1 \times X_2$ where $U_i \in pgbcl(X_i)$, for $i = 1, 2$. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2) \in pgbcl(X_1 \times Y_2)$. Hence f is pgb - closed set. \square

Theorem 3.26. *Let $h : X \rightarrow X_1 \times X_2$ be pgb - closed map and Let $f_i : X \times X_i$ be define as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$, then $f_i : X \times X_i$ is pgb - closed map for $i = 1, 2$.*

Proof. Let $U_1 \times U_2 \in X_1 \times X_2$, then $f_1(U_1) = h_1(U_1 \times X_2) \in pgbcl(X)$, there fore f_1 is pgb - closed. Similarly we have f_2 is pgb - closed. Thus f_i is pgb - closed map for $i = 1, 2$. \square

Theorem 3.27. *For any bijection map $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

- (i) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is pgb - continuous.
- (ii) f is pgb - open map.
- (iii) f is pgb - closed map.

Proof. (i) \Rightarrow (ii) Let U be an open set of (X, τ) . By assumption, $(f^{-1})^{-1}(U) = f(U)$ is pgb - open in (Y, σ) and so f is pgb - open.

(ii) \Rightarrow (iii) Let F be a closed set of (X, τ) . Then F^c is open set in (X, τ) . By assumption $f(F^c)$ is pgb - open in (Y, σ) . Therefore $f(F^c) = f(F)^c$ is pgb - open in (Y, σ) . That is $f(F)$ is pgb - closed in (Y, σ) . Hence f is pgb - closed.

(iii) \Rightarrow (i) Let F be a closed set of (X, τ) . By assumption, $f(F)$ is pgb - closed in (Y, σ) . But $f(F) = (f^{-1})^{-1}(F) \Rightarrow (f^{-1})$ is continuous. \square

4. On Pre Generalized b -Open Map

In this section, we introduce pre generalized b - open map (briefly pgb - open) in topological spaces by using the notions of pgb - open sets and study some of their properties.

Definition 4.1. Let X and Y be two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \delta)$ is called pre generalized b - open (briefly, pgb - open) if the image of every open set in X is pgb - open in Y .

Theorem 4.2. Every open map is pgb - open but not conversely.

Proof. Let $f : (X, \tau) \rightarrow (Y, \delta)$ is open map and V be an open set in X then $f(V)$ is open in Y . Hence pgb - open in Y . Then f is pgb - open. \square

The converse of above theorem need not be true as seen from the following example.

Example 4.3. Consider $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$. The map is pgb - open but not open as the image of and $\{a, b\}$ in X is $\{a, b\}$ is not open in Y .

Theorem 4.4. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is pgb - closed set if and only if for each subset S of Y and for each open set U containing $f^{-1}(S) \subset U$ there is a pgb - open set V of Y such that $S \subset U$ and $f^{-1}(V) \subset U$.

Proof. Suppose f is pgb - closed set. Let $S \subset Y$ and U be an open set of (X, τ) such that $f^{-1}(S) \subset U$. Now $X - U$ is closed set in (X, τ) . Since f is pgb closed, $f(X - U)$ is pgb closed set in (Y, σ) . Then $V = Y - f(X - U)$ is pgb open set in (Y, σ) . There fore $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - V) = U$. (ie) $f^{-1}(V) \subset U$.

Conversely,

Let F be a closed set of (X, τ) . Then $f^{-1}(f(F^c)) \subset F^c$ and F^c is an open in (X, τ) . By hypothesis, there exists a pgb open set V in (Y, σ) such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c \Rightarrow F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f(((f^{-1}(V))^c)^c) \subset V^c \Rightarrow f(V) \subset V^c$. Since V^c pgb - closed, $f(F)$ is pgb - closed. (ie) $f(F)$ is pgb - closed in (Y, σ) and there fore f is pgb - closed. \square

Theorem 4.5. For any bijection map $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

(i) $f^{-1} : (X, \tau) \rightarrow (Y, \sigma)$ is pgb - continuous.

(ii) f is pgb open map.

(iii) f is pgb - closed map.

Proof. (i) \Rightarrow (ii) Let U be an open set of (X, τ) . By assumption $(f^{-1})^{-1}(U) = f(U)$ is pgb - open in (Y, σ) . Therefore f is pgb - open map.

(ii) \Rightarrow (iii) Let F be closed set of (X, τ) , Then F^c is open set in (X, τ) . By assumption, $f(F^c)$ is pgb - open in (Y, σ) . Therefore $f(F)$ is pgb - closed in (Y, σ) . Hence f is pgb - closed.

(iii) \Rightarrow (i) Let F be a closed set of (X, τ) , By assumption $f(F)$ is pgb - closed in (Y, σ) . But $f(F) = (f^{-1})^{-1}(F)$. Hence $f^{-1} : (X, \tau) \rightarrow (Y, \sigma)$ is pgb -continuous. \square

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