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STRUCTURES OF GENERALIZED FUZZY SETS IN NON-ASSOCIATIVE RINGS

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Abstract: Since the introduction of the concept of fuzzy sets, the theoretical application of fuzzy sets has been restricted to associative algebraic structures (groups, semigroups, associative rings, semi-rings etc). In addition, the study of fuzzy sets, where the base set is a commutative structure, has attracted the attention of many researchers. On the other hand there are many sets which are naturally endowed with two compatible binary operations forming a non-associative ring and we may dig out examples which investigate a non-associative structure in the context of fuzzy sets. Intuitively one can apply the concept of fuzzy sets to non-commutative and non-associative structures.In this paper, we introduce the concept of (α, β)-fuzzy ideals in LA-rings (a non-associative structure). We discuss the important features of a non-associative regular LA-ring by using (α, β) -fuzzy bi-ideals, (α, β)-fuzzy generalized bi-ideals, (α, β)-fuzzy quasi-ideals, and (α, β)-fuzzy interior ideals. Ultimately, we identify upper and lower parts of these structures and characterize regular LA-rings using the identified properties of these structures.

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1. Introduction

After fuzzy sets were introduced by Zadeh [24], the concepts of classical mathematics began to be reconsidered. The theory of fuzzy semigroups was first developed by Kuroki [13]. Mordeson et al. (2002) deal with the application of the fuzzy approach to the concept of automata and formal languages. Later, in 2003, Mordeson et al. established theoretical results for fuzzy semigroups, and used these concepts in fuzzy coding, fuzzy finite-state machines, and fuzzy languages. In 2004, Murali [14] defined fuzzy points belonging to fuzzy subsets.

Bhakat et al. [2] introduced the concept of $(\in, \in \vee q)$-fuzzy subgroups, and also discussed (α, β)-fuzzy subgroups. The concept of $(\in, \in \vee q_k)$-fuzzy subgroups is a valuable generalization of Rosenfeld's fuzzy subgroups provided by Jun [10]. In 2006, Jun et al. [11] published a study on the (α, β)-fuzzy interior ideals of a semigroup. Davvaz (2006) investigated the concept of $(\in, \in \vee q_k)$-fuzzy sub-near rings and the ideal characteristics of a near ring. Kazanci et al. [12] studied $(\in, \in \vee q_k)$-fuzzy bi-ideal of semigroup. Shabir et al. [16] characterised semigroups based on $(\in, \in \vee q_k)$-fuzzy ideals. Yin et al. [23] introduced more general forms of .$(\in, \in \vee q)$-fuzzy filters and defined $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy filters, providing some interesting results relating to these notions. Gulistan et al. [**?**] developed the concept of the Hv-LA-semigroup, and in an another paper (2015) they discussed (α, β)-fuzzy KU-ideals of KU-algebras. A generalisation of fuzzy sets was provided by Atanassov [1] (1986), using the concept of intuitionistic fuzzy sets. Fuzzy rings and fuzzy ideals were discussed by Dib et al. [4]. Earlier, Dib et al. [5] examined the properties of fuzzy Cartesian products, fuzzy relations, and fuzzy functions. In addition, Volf [22] investigated the properties of fuzzy subfields.

Left Almost ring (LA-ring) is actually an off shoot of LA-semigroup and LA-group. It is a non-commutative and non-associative structure. Due to its peculiar characteristics, it has been emerging as useful non-associative class which intuitively would have reasonable contribution to enhance non-associative ring theory. By an LA-ring, we mean a non-empty set R with at least two elements such that $(R, +)$ is an LA-group, $(R, \cdot)$ is an LA-semigroup, both left and right distributive laws hold. The Left Almost ring (LA-ring) of finitely nonzero functions was introduced by Shah et al. (2010). Later, Shah et al. [17] investigated some characteristics of LA-rings based on their ideal. The concept of ideal theory provided a platform to investigate the application of fuzzy sets, intuitionistic fuzzy sets, and soft sets in LA-rings. Moreover, Shah et al. [18] defined intuitionistic fuzzy sets in LA-rings, and provided some useful results. Rehman et al. [15] did some computational work using Mace4, exploring some interesting characteristics of LA-rings. Shah et al.[19] discussed soft M-systems in a class of non-associative rings. Recently, Shah et al. [20] have applied the concept of soft sets to LA-rings, and have established some useful results for non-associative soft-set theory.

In this paper, we introduce the concept of (α, β)-fuzzy ideals in LA-rings. We discuss the important features of a non-associative regular LA-ring using (α, β)-fuzzy bi-ideals and (α, β)- fuzzy generalised bi-ideals.

2. Generalized Fuzzy Ideals

Let R be an LA-ring. A fuzzy subset f over R is a function from R into unit closed interval $[0,1]$, i.e. $f : R \longrightarrow [0,1]$. A fuzzy subset f over R of the form $f(y) = \begin{cases} t \in (0,1] & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$, is said to be a fuzzy point with support x and value t and is denoted by x_t. For a fuzzy point x_t and fuzzy set f over R we have

(i) $x_t \in f$ if $f(x) \geq t$.

(ii) $x_t q f$ if $f(x) + t > 1$.

(iii) $x_t \in \vee q f$ if $x_t \in f$ or $x_t q f$.

(iv) $x_t \in \wedge q f$ if $x_t \in f$ and $x_t q f$.

(v) $x_t q_k f$ if $f(x) + t + k > 1$, where $t, k \in (0,1]$.

(vi) $x_t \in_\gamma f$ if $f(x) \geq t > \gamma$ and $x_t q_\delta f$ if $f(x) + t > 2\delta$, where $t \in (\gamma, 1]$ and $\gamma, \delta \in (0,1]$.

Definition 1. An $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy subset f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy LA-subring of R, if $x_{t_1} \in f$, $y_{t_2} \in f$ (resp., $x_{t_1} \in f$, $y_{t_2} \in f$ and $x_{t_1} \in_\gamma f$, and $y_{t_2} \in_\gamma f$) $\Rightarrow$ $(x-y)_{\min\{t_1,t_2\}} \in \vee q f$(resp., $(x-y)_{\min\{t_1,t_2\}} \in \vee q_k f$ and $(x-y)_{\min\{t_1,t_2\}} \in_\gamma \vee q_\delta f$) and $(xy)_{\min\{t_1,t_2\}} \in \vee q f$(resp., $(xy)_{\min\{t_1,t_2\}} \in \vee q_k f$ and $(xy)_{\min\{t_1,t_2\}} \in_\gamma \vee q_\delta f$) where we required $k \in (0,1]$ for $(\in, \in \vee q_k)$-fuzzy subset and $t_1, t_2, \in (\gamma, 1]$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subset.

Corollary 2. *Every $(\in, \in \vee q)$-fuzzy LA-subring is an $(\in, \in \vee q_k)$-fuzzy LA-subring and every $(\in, \in \vee q_k)$-fuzzy LA-subring is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subring of R, but converse is not true.*

Example 1. $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ is an LA-ring defined by the following Cayley table.

$+$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	2	0	3	1	6	4	7	5
2	1	3	0	2	5	7	4	6
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	6	4	7	5	2	0	3	1
6	5	7	4	6	1	3	0	2
7	7	6	5	4	3	2	1	0

$\cdot$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	4	4	0	0	4	4	0
2	0	4	4	0	0	4	4	0
3	0	0	0	0	0	0	0	0
4	0	3	3	0	0	3	3	0
5	0	7	7	0	0	7	7	0
6	0	7	7	0	0	7	7	0
7	0	3	3	0	0	3	3	0

Let us define $f(0) = 0.81, f(1) = 0.8, f(2) = 0.75, f(3) = 0.7, f(4) = 0.65$, $f(5) = 0.6, f(6) = 0.55$, $f(7) = 0.55$ and let $t_1 = 0.54$ and $t_2 = 0.53$. Then by routine calculation it can be easily seen that f is an $(\in, \in \vee q)$ −fuzzy LA-subring of R, it is an $(\in \vee q_k)$-fuzzy LA-subring and it is also an $(\in_\gamma \vee q_\delta)$-fuzzy LA-subring of R, where k,$\gamma, \delta \in [0, 1]$ with condition that $\gamma < \delta$. If we define the fuzzy sets as

$$f(0) = 0.8, f(1) = 0.8, f(2) = 0.7, f(3) = 0.7,$$
$$f(4) = 0.6, f(5) = 0.6, f(6) = 0.5, f(7) = 0.49$$

and let $t_1 = 0.25$ and $t_2 = 0.31$ with $\gamma = 0.2$ and $\delta = 0.25$ then it is obvious that

1) f is an $(\in_{0.2}, \in_{0.2} \vee q_{0.25})$-fuzzy LA-subring of R.
2) f is not an $(\in, \in \vee q_{0.25})$-fuzzy LA-subring of R because $0_{0.25} \in f$ and $7_{0.31} \in f$ but $(0 - 7)_{0.25 \wedge 0.31} = 7_{0.25} \overline{\in \vee q_{0.25}} f$.
3) f is not an $(\in, \in \vee q)$-fuzzy LA-subring of R because $0_{0.25} \in f$ and $7_{0.31} \in f$ but $(0 - 7)_{0.25 \wedge 0.31} = 7_{0.25} \overline{\in \vee q} f$.

Definition 3. An $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy subset f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy left (resp: right) ideal of R, if it satisfies, $y_t \in f$(resp., $y_t \in f$, $y_t \in_\gamma f$) and $x \in R \Longrightarrow (xy)_t \in \vee q f$(resp., $(xy)_t \in \vee q_k f$, $(xy)_t \in_\gamma \vee q_\delta f$), for all $x, y \in R$ with $t \in [\gamma, 1)$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy set where we required $k, \delta \in (0, 1]$ for $(\in, \in \vee q_k)$-fuzzy subset.

The concept of $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy right ideals is defined in the similar way.

Definition 4. A fuzzy subset f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy ideal of R if it is both an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy left ideal and $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy right ideal of R.

Corollary 5. *Every $(\in, \in \vee q)$-fuzzy ideal is an $(\in, \in \vee q_k)$-fuzzy ideal and every $(\in, \in \vee q_k)$-fuzzy ideal is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideal of R, but the converse is not true.*

Definition 6. An $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy LA-subring f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy bi ideal of R, if for $x, y, z \in R$ and for all $t_3, t_4, k \in (0, 1]$, $x_{t_3} \in f, z_{t_4} \in f$(resp., $x_{t_3} \in f, z_{t_4} \in f$, $x_{t_3} \in_\gamma f, z_{t_4} \in_\gamma f$) $\Longrightarrow ((xy)z)_{\min\{t_3,t_4\}} \in \vee q f$(resp., $((xy)z)_{\min\{t_3,t_4\}} \in \vee q_k f$, $((xy)z)_{\min\{t_3,t_4\}} \in_\gamma \vee q_\delta f$) with $t_3, t_4 \in [\gamma, 1)$, $\delta \in (0, 1]$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy set and $k \in (0, 1]$ for $(\in, \in \vee q_k)$-fuzzy subset.

Corollary 7. *Every $(\in, \in \vee q)$-fuzzy bi-ideal is an $(\in, \in \vee q_k)$-fuzzy bi-ideal and every $(\in, \in \vee q_k)$-fuzzy bi-ideal is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-ideal of R, but the converse is not true.*

Definition 8. An $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy set f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy generalized bi-ideal of R, if for $x, y, z \in R$ and for all $t_3, t_4, k \in (0, 1]$, $x_{t_3} \in f, z_{t_4} \in f$(resp., $x_{t_3} \in f, z_{t_4} \in f$, $x_{t_3} \in_\gamma f, z_{t_4} \in_\gamma f$) $\Longrightarrow ((xy)z)_{\min\{t_3,t_4\}} \in \vee q f$(resp., $((xy)z)_{\min\{t_3,t_4\}} \in \vee q_k f$, $((xy)z)_{\min\{t_3,t_4\}} \in_\gamma \vee q_\delta f$) with $t_3, t_4 \in [\gamma, 1)$, $\delta \in (0, 1]$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy set and $k \in (0, 1]$ for $(\in, \in \vee q_k)$-fuzzy subset.

Corollary 9. *Every $(\in, \in \vee q)$-fuzzy generalized bi-ideal is an $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal and every $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy generalized bi-ideal of R, but the converse is not true.*

Definition 10. An $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy LA-subring f of an LA-ring R is called an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy interior ideal of R, if for $x, a, y \in R$ and for all $t, k \in (0, 1]$, $a_t \in f$, (resp., $a_t \in f$, $a_t \in_\gamma f$) $\Longrightarrow ((xa)y)_t \in \vee q f$(resp., $((xa)y)_t \in \vee q_k f$, $((xa)y)_t \in_\gamma \vee q_\delta f$) with $t \in [\gamma, 1)$, $\delta \in (0, 1]$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy set and $k \in (0, 1]$ for $(\in, \in \vee q_k)$-fuzzy subset.

Corollary 11. *Every $(\in, \in \vee q)$-fuzzy interior ideal is an $(\in, \in \vee q_k)$-fuzzy interior ideal and every $(\in, \in \vee q_k)$-fuzzy interior ideal is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy interior ideal of R, but the converse is not true.*

Theorem 12. *If f is a nonzero $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy LA-subring (resp.,fuzzy ideal, fuzzy bi-ideal, fuzzy generalized bi-ideal, fuzzy interior ideal) of R, then the set $f_0 = \{x \in R \mid f(x) > 0\}$ is an LA-subring(resp., ideal, bi-ideal, generalized bi-ideal, interior ideal) of R.*

Proof. If f is a nonzero $(\in, \in \vee q)$-fuzzy LA-subring and $x, y \in f_0$, then $f(x) > 0$ and $f(y) > 0$. Let $f(xy) = 0$ and $f(x-y) = 0$. Then $x_{f(x)} \in f$ and $y_{f(y)} \in f$ but $f(xy) = 0 < \min\{f(x), f(y)\}$, $f(x-y) = 0 < \min\{f(x), f(y)\}$ and $f(xy) + \min\{f(x), f(y)\} \leq 0 + 1 = 1$, $f(x-y) + \min\{f(x), f(y)\} \leq 0 + 1 = 1$. So

$$(xy)_{\min\{f(x),f(y)\}} \overline{\in \vee q} f$$

and

$$(x-y)_{\min\{f(x),f(y)\}} \overline{\in \vee q} f$$

a contradiction. Hence $f(xy) > 0$, $f(x-y) > 0$, that is $xy \in f_0$ and $x-y \in f_0$. Also $x_1 q f$ and $y_1 q f$ but $(xy)_1 \overline{\in \vee q} f$ and $(x-y)_1 \overline{\in \vee q} f$. Hence $f(xy) > 0$, $f(x-y) > 0$, that is, $xy \in f_0$ and $x - y \in f_0$. Thus f_0 is an LA-subring of R. Similarly the case for $(\in, \in \vee q_k)$ and $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy sets can easily be seen. □

Theorem 13. *If f is a fuzzy subset of an LA-ring R, then 1) f is an $(\in, \in \vee q)$-fuzzy LA-subring of R if and only if $f(xy) \geq \min\{f(x), f(y), 0.5\}$ and $f(x-y) \geq \min\{f(x), f(y), 0.5\}$. 2) f is an $(\in, \in \vee q_k)$-fuzzy LA-subring of R if and only if $f(xy) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$ and*

$$f(x-y) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$$

. 3) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subring of R if and only if $\max\{f(xy), \gamma\} \geq \min\{f(x), f(y), \delta\}$ and $\max\{f(x-y), \gamma\} \geq \min\{f(x), f(y), \delta\}$.

Proof. 1) Let f be an $(\in, \in \vee q)$-fuzzy LA-subring of R. Assume that there exist $x, y \in R$ such that $f(xy) < \min\{f(x), f(y), 0.5\}$ and $f(x-y) < \min\{f(x), f(y), 0.5\}$. Choose $t \in (0, 1]$ such that $f(xy) < t \leq \min\{f(x), f(y), 0.5\}$ and $f(x-y) < t \leq \min\{f(x), f(y), 0.5\}$.Then $x_t \in f$ and $y_t \in f$ but $f(xy) < t$ with $f(x-y) < t$ and $f(xy)+t+0.5 = 1$ with $f(x-y)+t+0.5 = 1$, so $(xy)_t \overline{\in \vee q} f$ and $(x-y)_t \overline{\in \vee q} f$, which is a contradiction. Hence $f(xy) \geq \min\{f(x), f(y), 0.5\}$ and $f(x-y) \geq \min\{f(x), f(y), 0.5\}$. Conversely, assume that $f(xy) \geq \min\{f(x), f(y), 0.5\}$ and $f(x-y) \geq \min\{f(x), f(y), 0.5\}$. Let $x_t \in f$ and $y_r \in f$ for $t, r \in (0, 1]$. Then $f(x) \geq t$ and $f(y) \geq r$. Now $f(xy) \geq$

$\min\{f(x), f(y), 0.5\} \geq \min\{t, r, 0.5\}$ and $f(x-y) \geq \min\{f(x), f(y), 0.5\} \geq \min\{t, r, 0.5\}$. We consider two cases: Case (i): If $t \wedge r > 0.5$, then $f(xy) + t \wedge r + 0.5 > 0.5 + 0.5 = 1$ and $f(x-y) + t \wedge r + 0.5 > 0.5 + 0.5 = 1$, which implies that $(xy)_{\min\{t,r\}} q f$ and $(x-y)_{\min\{t,r\}} q f$. Case (ii): If $t \wedge r \leq \frac{1-k}{2}$, then $f(xy) \geq t \wedge r$ and $f(x-y) \geq t \wedge r$. So $(xy)_{\min\{t,r\}} \in f$ and $(x-y)_{\min\{t,r\}} \in f$. Thus in both cases $(xy)_{\min\{t,r\}} \in \vee q f$ and $(x-y)_{\min\{t,r\}} \in \vee q f$. Therefore f is an $(\in, \in \vee q_k)$-fuzzy LA-subring of R. 2) The proof is similar to the proof of 1). 3) Let f be $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subring of R. Assume that there exist $x, y \in R$ and $t \in (\gamma, 1]$, such that

$$\max\{f(xy), \gamma\} < t \leq \min\{f(x), f(y), \delta\}$$

and

$$\max\{f(x-y), \gamma\} < t \leq \min\{f(x), f(y), \delta\}.$$

Thus $\max\{f(xy), \gamma\} < t$ and $\max\{f(x-y), \gamma\} < t$, this implies that $f(xy) < t \leq \gamma$ and $f(x-y) < t \leq \gamma$, which further implies that $(xy)_{\min\{t,s\}} \overline{\in_\gamma \vee q_\delta} f$ with $(x-y)_{\min\{t,s\}} \overline{\in_\gamma \vee q_\delta} f$. As $\min\{f(x), f(y), \delta\} \geq t$, therefore $\min\{f(x), f(y)\} \geq t$ this implies that $f(x) \geq t > \gamma$, $f(y) \geq t > \gamma$, implies that $x_t \in_\gamma f$, $y_s \in_\gamma f$ but $(xy)_{\min\{t,s\}} \overline{\in_\gamma \vee q_\delta} f$ and $(x-y)_{\min\{t,s\}} \overline{\in_\gamma \vee q_\delta} f$ a contradiction to the definition. Hence $\max\{f(xy), \gamma\} \geq \min\{f(x), f(y), \delta\}$ and $\max\{f(x-y), \gamma\} \geq \min\{f(x), f(y), \delta\}$ for all $x, y \in R$. Conversely, assume that there exist $x, y \in R$ and $t, s \in (\gamma, 1]$ such that $x_t \in_\gamma f$, $y_s \in_\gamma f$. By definition we write $f(x) \geq t > \gamma$, $f(y) \geq s > \gamma$, so $\max\{f(xy), \delta\} \geq \min\{f(x), f(y), \delta\}$ and $\max\{f(x-y), \delta\} \geq \min\{f(x), f(y), \delta\}$ this implies that $f(xy) \geq \min\{t, s, \delta\}$ and $f(x-y) \geq \min\{t, s, \delta\}$. Here we have two cases. Case (i): If $\{t, s\} \leq \delta$, then $f(xy) \geq \min\{t, s\} > \gamma$ and $f(x-y) \geq \min\{t, s\} > \gamma$. This implies that $(xy)_{\min\{t,s\}} \in_\gamma f$ and $(x-y)_{\min\{t,s\}} \in_\gamma f$. Case (ii): If $\{t, s\} > \delta$, then $f(xy) + \min\{t, s\} > 2\delta$ and $f(x-y) + \min\{t, s\} > 2\delta$. This implies that $(xy)_{\min\{t,s\}} q_\delta f$ and $(x-y)_{\min\{t,s\}} q_\delta f$. From both cases we get $(xy)_{\min\{t,s\}} \in_\gamma \vee q_\delta f$ and $(x-y)_{\min\{t,s\}} \in_\gamma \vee q_\delta f$ for all x, y in S. Hence f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subring of R. □

Theorem 14. *If f is a fuzzy subset of R, then 1) f is an $(\in, \in \vee q)$-fuzzy ideal of R if and only if $f(xy) \geq \min\{f(x), f(y), 0.5\}$ and $f(x-y) \geq \min\{f(x), f(y), 0.5\}$. 2) f is an $(\in, \in \vee q_k)$-fuzzy ideal of R if and only if $f(xy) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$ and $f(x-y) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$. 3) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideal of S if and only if*

$$\max\{f(xy), \gamma\} \geq \min\{f(x), f(y), \delta\}, \quad \max\{f(x-y), \gamma\} \geq \min\{f(x), f(y), \delta\},$$

where $\gamma, \delta \in [0, 1]$.

Proof. The same as the proof of the Theorem 13. □

Theorem 15. *If f is a fuzzy subset of R, then 1) f is an $(\in, \in \vee q)$-fuzzy generalized bi-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(x), f(z), 0.5\right\}$ for all $x, y, z \in R$. 2) f is an $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}$ for all $x, y, z \in R$. 3) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy generalized bi-ideal of R if and only if $\max\{f\left((xy)z\right), \gamma\} \geq \min\left\{f(x), f(z), \delta\right\}$ for all $x, y, z \in R$.*

Proof. The proof is straightforward. □

Theorem 16. *If f is a fuzzy LA-subring of R, then 1) f is an $(\in, \in \vee q)$-fuzzy bi-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(x), f(z), 0.5\right\}$ for all $x, y, z \in R$. 2) f is an $(\in, \in \vee q_k)$-fuzzy bi-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}$ for all $x, y, z \in R$. 3) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-ideal of R if and only if $\max\{f\left((xy)z\right), \gamma\} \geq \min\left\{f(x), f(z), \delta\right\}$ for all $x, y, z \in R$.*

Proof. The proof is straightforward. □

Theorem 17. *If f is a fuzzy LA-subring of an LA-ring R, then 1) f is an $(\in, \in \vee q)$-fuzzy interior-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(y), 0.5\right\}$ for all $x, y, z \in R$. 2) f is an $(\in, \in \vee q_k)$-fuzzy interior-ideal of R if and only if $f\left((xy)z\right) \geq \min\left\{f(y), \frac{1-k}{2}\right\}$ for all $x, y, z \in R$. 3) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy interior-ideal of R if and only if $\max\{f\left((xy)z\right), \gamma\} \geq \min\left\{f(y), \delta\right\}$ for all $x, y, z \in R$.*

Proof. The proof is straightforward. □

Definition 18. If f is a fuzzy subset of an LA-ring R, then 1) f is called an $(\in, \in \vee q)$-fuzzy quasi-ideal of R, if it satisfies,

$$f(x) \geq \min\left\{\left((f \circ \delta), (\delta \circ f)\right)(x), 0.5\right\},$$

where δ is the fuzzy subset of R mapping every element of R on 1. 2) f is called an $(\in, \in \vee q_k)$-fuzzy quasi-ideal of R, if it satisfies,

$$f(x) \geq \min\left\{\left((f \circ \delta), (\delta \circ f)\right)(x), \frac{1-k}{2}\right\},$$

where δ is the fuzzy subset of R mapping every element of R on 1. 3) f is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy quasi-ideal of R, if it satisfies, $\max\{f(x), \gamma\} \geq \min\left\{\left((f \circ \delta), (\delta \circ f)\right)(x), \delta\right\}$, where δ is the fuzzy subset of R mapping every element of R on 1.

Theorem 19. *Intersection of* $(\in, \in \vee q)$ *(resp.,* $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$*)-fuzzy (LA-subrings, ideals) of an LA-ring R is an* $(\in, \in \vee q)$ *(resp.,* $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$*)-fuzzy(LA-subring, ideal) of R.*

Proof. Let $\{f_i\}_{i \in I}$ be a family of $(\in, \in \vee q)$-fuzzy ideals of R. Let $x, y \in R$. Then

$$(\wedge_{i\in I} f)(xy) = \wedge_{i\in I}(f_i(xy),) \text{ and } (\wedge_{i\in I} f)(x-y) = \wedge_{i\in I}(f_i(x-y)).$$

Since each f_i is an $(\in, \in \vee q)$-fuzzy ideal of R, so

$$f_i(xy) \geq \min\{f_i(x), f_i(y), 0.5\}$$

and

$$f_i(x-y) \geq \min\{f_i(x), f_i(y), 0.5\} \text{ for all } i \in I,$$

thus

$$\begin{aligned}(\wedge_{i\in I}) f(xy) &= \wedge_{i\in I}(f_i(xy)) \geq \wedge_{i\in I}(f_i(x), f_i(y) \wedge 0.5) \\ &= (\wedge_{i\in I} f_i(x), \wedge_{i\in I} f_i(y)) \wedge 0.5 = (\wedge_{i\in I} f_i)(x \wedge y) \wedge 0.5\end{aligned}$$

and

$$\begin{aligned}(\wedge_{i\in I}) f(x-y) &= \wedge_{i\in I}(f_i(x-y)) \geq \wedge_{i\in I}(f_i(x) \wedge f_i(y) \wedge 0.5) \\ &= (\wedge_{i\in I} f_i(x) \wedge f_i(y)) \wedge 0.5 = (\wedge_{i\in I} f_i)(x \wedge y) \wedge 0.5.\end{aligned}$$

Hence $\wedge_{i\in I} f$ is an $(\in, \in \vee q)$-fuzzy ideals of an LA-ring R. Similarly it can easily be proved for other type of fuzzy ideals. □

Theorem 20. *Every* $(\in, \in \vee q)$ *(resp.,* $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$*)-fuzzy generalized bi-ideal of a regular LA-ring R is an* $(\in, \in \vee q)$ *(resp.,* $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$*)-fuzzy bi-ideal of R.*

Proof. Let f be any $(\in, \in \vee q)$- fuzzy generalized bi-ideal of R and let a, b be any elements of R. Then there exists an element $x \in R$ such that $b = (bx)b$. Thus we have $f(ab) = f(a(bx)b) \geq \min\{f(a), f(b), 0.5\}$. This shows that f is an $(\in, \in \vee q)$-fuzzy sub LA-ring of R and so f is an $(\in, \in \vee q)$-fuzzy bi-ideal of R. □

Theorem 21. *Let f be an* $(\in, \in \vee q)$ *(resp.,* $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$*)-fuzzy quasi-ideal of an LA-ring R. Then the set* $f_0 = \{x \in R \mid f(x) > 0\}$ *is a quasi-ideal of R.*

Proof. In order to show that f_0 is a quasi-ideal of R, we have to show that $Rf_0 \cap f_0R \subseteq f_0$. Let $a \in Rf_0 \cap f_0R$. This implies that $a \in Rf_0$ and $a \in f_0R$. So $a = rx$ and $a = yt$ for some $r, t \in R$ and $x, y \in f_0$. Thus $f(x) > 0$ and $f(y) > 0$. Now $f(a) \geq \min\{(f \circ \delta)(a), (\delta \circ f)(a), 0.5\}$. Since

$$(\delta \circ f)(a) = \bigvee_{a=\sum_{i=1}^{n} p_i qi} \left\{ \bigwedge_{i=1}^{n} \{\delta(p_i) \wedge f(q_i)\} \right\} \geq \{\delta(s) \wedge f(x)\} = f(x).$$

Similarly $(f \circ \delta)(a) \geq f(y)$. Thus

$$f(a) \geq \min\{(f \circ \delta)(a), (\delta \circ f)(a), 0.5\} \geq \min\{f(x), f(y), 0.5\} > 0.$$

This implies that $a \in f_0$. Hence f_0 is a quasi-ideal of R. □

Corollary 22. *Every fuzzy quasi-ideal of R is an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy quasi ideal of R.*

Theorem 23. *Every $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy left ideal of R is an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy quasi-ideal of R where we required $k \in (0, 1]$ for $(\in, \in \vee q_k)$-fuzzy subset and $t_1, t_2, \in (\gamma, 1]$ for $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subset.*

Proof. Let $x \in R$, then

$$(\delta \circ f)(x) = \bigvee_{x=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{R(y_i) \wedge f(z_i)\} \right\} = \bigvee_{x=\sum_{i=1}^{n} y_i z_i} f(z_i).$$

This implies that

$$(\delta \circ f)(x) \wedge 0.5 = \left(\bigvee_{x=\sum_{i=1}^{n} y_i z_i} f(z_i) \right) \wedge 0.5 = \bigvee_{x=\sum_{i=1}^{n} y_i z_i} (f(z_i) \wedge 0.5) \leq f(yz) = f(x).$$

Thus $(\delta \circ f)(x) \wedge 0.5 \leq f(x)$. Hence

$$f(x) \geq (\delta \circ f)(x) \wedge 0.5 \geq \min\{(f \circ \delta)(x), (\delta \circ f)(x), 0.5\}.$$

Thus f is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R. Similarly we can show that every $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy right ideal of R is an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy quasi-ideal of R. □

Theorem 24. *If R is an LA-ring with left identity e such that $(xe)R = xR$, then every $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy quasi-ideal of R is an $(\in, \in \vee q)$ (resp., $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$)-fuzzy bi-ideal of R.*

Proof. Suppose f is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R. Now

$$\begin{aligned} f(xy) &\geq (f \circ \delta)(xy) \wedge (\delta \circ f)(xy) \wedge 0.5 \\ &= \left[\bigvee_{xy=\sum_{i=1}^{n} a_i b_i} \left\{ \bigwedge_{i=1}^{n} \{ f(a_i) \wedge \delta(b_i) \} \right\} \right] \\ &\quad \wedge \left[\bigvee_{xy=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{ \delta(p_i) \wedge f(q_i) \} \right\} \right] \wedge 0.5 \\ &\geq [f(x) \wedge \delta(y)] \wedge [\delta(x) \wedge f(y)] \wedge 0.5 \\ &\geq [f(x) \wedge 1] \wedge [1 \wedge f(y)] \wedge 0.5 = f(x) \wedge f(y) \wedge 0.5. \end{aligned}$$

So

$$f(xy) \geq \min\{f(x), f(y), 0.5\}.$$

Also

$$f((xy)z) \geq (f \circ \delta)((xy)z) \wedge (\delta \circ f)((xy)z) \wedge 0.5.$$

Now,

$$(\delta \circ f)((xy)z) = \bigvee_{(xy)z=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \delta(p_i) \wedge f(q_i) \right\}$$

$$= \bigvee_{(xy)z=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{1 \wedge f(q_i)\} \right\} \geq f(z).$$

Then

$$(\delta \circ f)((xy)z) \geq f(z).$$

Also

$$(f \circ \delta)((xy)z) = \bigvee_{(xy)z=\sum_{i=1}^{n} a_i b_i} \left\{ \bigwedge_{i=1}^{n} \{f(a_i) \wedge \delta(b_i)\} \right\}$$

$$= \bigvee_{(xy)z=\sum_{i=1}^{n} a_i b_i} \left\{ \bigwedge_{i=1}^{n} \{f(a_i) \wedge 1\} \right\}.$$

Now,

$$(xy)z = (xy)(ez) = (xe)(yz) \in (xe)R = xR.$$

So $(xy)z = xt$ for some $t \in R$. Then

$$(f \circ \delta)((xy)z) = \bigvee_{xt=\sum a_i b_i} \left\{ \bigwedge_{i=1}^{n} \{f(a) \wedge 1\} \right\} \geq f(x).$$

Thus

$$f((xy)z) \geq f(x) \wedge f(z) \wedge 0.5.$$

Hence f is an $(\in, \in \vee q)$-fuzzy bi-ideal of R. □

3. Lower and Upper Parts of $(\in, \in \vee q)$-Fuzzy Ideals

In this section we identify lower and upper parts of $(\in, \in \vee q)$-fuzzy ideals of LA-rings and explore some interesting properties of LA-rings. At the end of this section, we characterize regular LA-rings by using the properties of $(\in, \in \vee q)$-fuzzy ideals and lower and upper parts of $(\in, \in \vee q)$-fuzzy ideals.

Definition 25. Let f be a fuzzy subset of an LA-ring R. We define the upper part f^{+} and the lower part f^{-} of f as follows, $f^{+}(x) = f(x) \vee 0.5$ and $f^{-}(x) = f(x) \wedge 0.5$.

Theorem 26. *If f and g are fuzzy subsets of an LA-ring R, then*
(1) $(f \wedge g)^{-} = (f^{-} \wedge g^{-})$
(2) $(f \vee g)^{-} = (f^{-} \vee g^{-})$
(3) $(f \circ g)^{-} = (f^{-} \circ g^{-})$

Proof. (1) For all $a \in R$,

$$\begin{aligned}(f \wedge g)^{-}(a) &= (f \wedge g)(a) \wedge 0.5 = f(a) \wedge g(a) \wedge 0.5 = (f(a) \wedge 0.5) \wedge (g(a) \wedge 0.5) \\ &= f^{-}(a) \wedge g^{-}(a) = \left(f^{-} \wedge g^{-}\right)(a).\end{aligned}$$

(2) For all $a \in R$,

$$\begin{aligned}(f \vee g)^{-}(a) &= (f \vee g)(a) \wedge 0.5 = (f(a) \vee g(a)) \wedge 0.5 = (f(a) \wedge 0.5) \vee (g(a) \wedge 0.5) \\ &= f^{-}(a) \vee g^{-}(a) = \left(f^{-} \vee g^{-}\right)(a).\end{aligned}$$

(3) If a is not expressible as $a = bc$ for some $b, c \in R$,
then

$$(f \circ g)(a) = \bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \{f(b_i) \wedge g(c_i)\} \right\} = 0 \wedge 0 = 0.$$

Thus $(f \circ g)^{-}(a) = (f \circ g)(a) \wedge 0.5 = 0$. Since a is not expressible as $a = bc$, so,

$$\begin{aligned}\left(f^{-} \circ g^{-}\right)(a) &= \bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \{f^{-}(b_i) \wedge g-(c_i)\} \right\} \\ &= \bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \{f(b_i) \wedge g(c_i) \wedge 0.5\} \right\} = 0.\end{aligned}$$

Thus in this case $(f\circ g)^{-}=(f^{-}\circ g^{-})$. if a is expressible as $a=xy$ for some $x,y\in R$, then

$$(f\circ g)^{-}(a)=(f\circ g)(a)\wedge 0.5=\bigvee_{a=\sum_{i=1}^{n}x_iy_i}\left\{\bigwedge_{i=1}^{n}\{f(x_i)\wedge g(y_i)\}\right\}\wedge 0.5$$

$$=\bigvee_{a=\sum_{i=1}^{n}x_iy_i}\left\{\bigwedge_{i=1}^{n}\{(f(x_i)\wedge 0.5)\wedge(g(y_i)\wedge 0.5)\}\right\}$$

$$=\bigvee_{a=\sum_{i=1}^{n}x_iy_i}\left\{f^{-}(x_i)\wedge g^{-}(y_i)\right\}=\left(f^{-}\circ g^{-}\right)(a).$$

□

Theorem 27. *If f and g are fuzzy subsets of an LA-ring R, then the following properties hold.*
(1) $(f\wedge g)^{+}=(f^{+}\wedge g^{+})$
(2) $(f\vee g)^{+}=(f^{+}\vee g^{+})$
(3) $(f\circ g)^{+}\geq(f^{+}\circ g^{+})$.
If every element x of R is expressible as $x=bc$, then $(f\circ g)^{+}=(f^{+}\circ g^{+})$.

Proof. (1) For all $a\in R$,

$$(f\wedge g)^{+}(a)=(f\wedge g)(a)\vee 0.5=(f(a)\wedge g(a))\vee 0.5=(f(a)\vee 0.5)\wedge(g(a)\vee 0.5)$$
$$=f^{+}(a)\wedge g^{+}(a)=\left(f^{+}\wedge g^{+}\right)(a).$$

(2) For all $a\in R$,

$$(f\vee g)^{+}(a)=(f\vee g)(a)\vee 0.5=f(a)\vee g(a)\vee 0.5=(f(a)\vee 0.5)\vee(g(a)\vee 0.5)$$
$$=f^{+}(a)\vee g^{+}(a)=\left(f^{+}\vee g^{+}\right)(a).$$

(3) If a is not expressible as $a=bc$ for some $b,c\in R$, then

$$(f\circ g)(a)=\bigvee_{a=\sum_{i=1}^{n}b_ic_i}\left\{\bigwedge_{i=1}^{n}\{f(b_i)\wedge g(c_i)\}\right\}=0\wedge 0=0.$$

Thus $(f \circ g)^{+}(a) = (f \circ g)(a) \vee 0.5 = 0.5$. Since a is not expressible as $a = bc$, so

$$\left(f^{+} \circ g^{+}\right)(a) = \bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ f^{+}(b_i) \wedge g+(c_i) \right\} \right\}$$

$$= \bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ f(b_i) \wedge g(c_i) \vee 0.5 \right\} \right\} = 0.5.$$

Thus in this case $(f \circ g)^{+} = \left(f^{+} \circ g^{+}\right)$. But if a is expressible as $a = xy$ for some $x, y \in R$. So,

$$(f \circ g)^{+}(a) = (f \circ g)(a) \vee 0.5 = \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \left\{ \bigwedge_{i=1}^{n} \left\{ f(x_i) \wedge g(y_i) \right\} \right\} \vee 0.5$$

$$= \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \left\{ \bigwedge_{i=1}^{n} \left\{ (f(x_i) \vee 0.5) \wedge (g(y_i) \vee 0.5) \right\} \right\}$$

$$= \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \left\{ f^{+}(x_i) \wedge g^{+}(y_i) \right\} = \left(f^{+} \circ g^{+}\right)(a).$$

□

Definition 28. Let A be a non-empty subset of an LA-ring R. Then the lower and upper parts of the characteristic function are $C_A^{-} = \begin{cases} 0.5 & \text{if } a \in A \\ 0 & \text{if } a \notin A \end{cases}$ and $C_A^{+} = \begin{cases} 1 & \text{if } a \in A \\ 0.5 & \text{if } a \notin A \end{cases}$.

Theorem 29. *If A and B are non-empty subsets of an LA-ring R, then the following properties hold.*

(1) $(C_A \wedge C_B)^- = C^-_{A\cap B}$
(2) $(C_A \vee C_B)^- = C^-_{A\cup B}$
(3) $(C_A \circ C_B)^- = C^-_{AB}$

Proof. The proof is straightforward. □

Theorem 30. *The lower part of characteristic function C^-_L is an $(\in, \in \vee q)$-fuzzy left ideal of an LA–ring R if and only if L is a left ideal of R.*

Proof. If L is a left ideal of R, then trivially C^-_L is an $(\in, \in \vee q)$-fuzzy left ideal of R.

Conversely, assume that C^-_L is an $(\in, \in \vee q)$-fuzzy left ideal of R. If $y \in L$, then $C^-_L(y) = 0.5$.and $y_{0.5} \in C^-_L$. Since C^-_L is an $(\in, \in \vee q)$-fuzzy left ideal of R, so $(xy)_{0.5} \in \vee q C^-_L$ and $(x-y)_{0.5} \in \vee q C^-_L$, which further implies that $(xy)_{0.5} \in C^-_L$ or $(xy)_{0.5}\, q C^-_L$ and $(x-y)_{0.5} \in C^-_L$ or $(x-y)_{0.5}\, q C^-_L$. Hence $C^-_L(xy) \geq 0.5$ or $C^-_L(xy) + 0.5 > 1$ and $C^-_L(x-y) \geq 0.5$ or $C^-_L(x-y) + 0.5 > 1$. But $C^-_L(xy) + 0.5 > 1$ and $C^-_L(x-y) + 0.5 > 1$ is not possible, because $C^-_L(xy) \leq 0.5$ and $C^-_L(x-y) \leq 0.5$. Thus $C^-_L(xy) \geq 0.5$ and $C^-_L(x-y) \geq 0.5$, which implies that $C^-_L(xy) = 0.5$ and $C^-_L(x-y) = 0.5$. Hence $xy \in L$ and $x - y \in L$. Thus L is a left ideal of R. □

Similarly we can prove that the lower part of characteristic function C^-_R is an $(\in, \in \vee q)$-fuzzy right ideal of R if and only if R is a right ideal of R. Thus the lower part of characteristic function C^-_I is an $(\in, \in \vee q)$-fuzzy two-sided ideal of R if and only if I is a two-sided ideal of R.

Theorem 31. *If Q is a non empty subset of an LA-ring R, then Q is a quasi-ideal of R if and only if the lower part of characteristic function C^-_Q is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R.*

Proof. The proof is straightforward. □

Theorem 32. *If f is an $(\in, \in \vee q)$-fuzzy left (right) ideal of an LA-ring R, then f^- is a fuzzy left (right) ideal of R.*

Proof. Let f be an $(\in, \in \vee q)$-fuzzy left ideal of R. Then for all $a, b \in R$, we have $f(ab) \geq f(b) \wedge 0.5$, and $f(a-b) \geq f(a) \wedge f(b) \wedge 0.5$. This implies that $f(ab) \wedge 0.5 \geq f(b) \wedge 0.5$ and $f(a-b) \wedge 0.5 \geq f(a) \wedge f(b) \wedge 0.5$. So $f^-(ab) \geq f^-(b)$ and $f^-(a-b) \geq f^-(a) \wedge f^-(b)$. Thus f^- is a fuzzy left ideal of R. Similarly if f is an $(\in, \in \vee q)$-fuzzy right ideal of R, then for all $a, b \in R$, we have $f(ab) \geq f(a) \wedge 0.5$, and $f(a-b) \geq f(a) \wedge f(b) \wedge 0.5$. This implies that $f(ab) \wedge 0.5 \geq f(a) \wedge 0.5$ and $f(a-b) \wedge 0.5 \geq f(a) \wedge f(b) \wedge 0.5$.

So $f^{-}(ab) \geq f^{-}(a)$ and $f^{-}(a-b) \geq f^{-}(a) \wedge f^{-}(b)$. Thus f^{-} is a fuzzy right ideal of R. Thus f^{-} is a fuzzy right ideal of R. □

Theorem 33. *For an LA-ring R, the following conditions are equivalent.*
(1) R is regular.
(2) $(f \wedge g)^{-} = (f \circ g)^{-}$ for every $(\in, \in \vee q)$-fuzzy right ideal f and every $(\in, \in \vee q)$-fuzzy left ideal g of R.

Proof. (1) $\Longrightarrow$ (2) Let f be $(\in, \in \vee q)$-fuzzy right ideal and g be $(\in, \in \vee q)$-fuzzy left ideal of R. Now for all $a \in R$, we have

$$\begin{aligned}
(f \circ g)^{-}(a) = (f \circ g)(a) \wedge 0.5 &= \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \left\{ \bigwedge_{i=1}^{n} \{f(x_i) \wedge g(y_i)\} \right\} \wedge 0.5 \\
&= \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \left\{ \bigwedge_{i=1}^{n} \{(f(x_i) \wedge 0.5) \wedge (g(y_i) \wedge 0.5)\} \wedge 0.5 \right\} \\
&\leq \bigvee_{a=\sum_{i=1}^{n} x_i y_i} \{\{f(x_i y_i) \wedge g(x_i y_i)\} \wedge 0.5\} = f(a) \wedge g(a) \wedge 0.5 \\
&= (f \wedge g)(a) \wedge 0.5 = (f \wedge g)^{-}(a).
\end{aligned}$$

So $(f \circ g)^{-} \leq (f \wedge g)^{-}$. Since R is regular, so for each $a \in R$ there exists an element $x \in R$ such that $a = (ax)a$. Thus

$$\begin{aligned}
(f \wedge g)^{-}(a) = (f \wedge g)(a) \wedge 0.5 &\leq (f(ax) \wedge g(a)) \wedge 0.5 \\
&\leq \bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{f(y_i) \wedge g(z_i)\} \right\} \wedge 0.5 \\
&= (f \circ g)(a) \wedge 0.5 = (f \circ g)^{-}(a).
\end{aligned}$$

So $(f \wedge g)^{-} \leq (f \circ g)^{-}$. Thus $(f \wedge g)^{-} = (f \circ g)^{-}$.

(2) $\Longrightarrow$ (1) Let $a \in R$. Then $L = aR$ is a left ideal of R and $R = aR \cup Ra$ is a right ideal of R generated by a. Then by Theorem 30, the lower part

of characteristic functions C_R^- and C_L^- of R and L are $(\in, \in \vee q)$-fuzzy right ideal and $(\in, \in \vee q)$-fuzzy left ideal of R, respectively. Thus we have $C_{RL}^- = (C_R \circ C_L)^- = (C_R \wedge C_L)^- = C_{R \cap L}^-$. Thus $R \cap L = RL$. Hence it follows from Theorem 33 that R is regular. □

Theorem 34. *If R is an LA-ring with left identity e such that $(xe)R = xR$ for all $x \in R$, then the following conditions are equivalent.*

(1) R is regular.

(2) $((h \wedge f) \wedge g)^- \leq ((h \circ f) \circ g)^-$ for every $(\in, \in \vee q)$-fuzzy right ideal h, every $(\in, \in \vee q)$-fuzzy generalized bi-ideal f and for every $(\in, \in \vee q)$-fuzzy left ideal g of R.

(3) $((h \wedge f) \wedge g)^- \leq ((h \circ f) \circ g)^-$ for every $(\in, \in \vee q)$-fuzzy right ideal h, every $(\in, \in \vee q)$-fuzzy bi-ideal f and for every $(\in, \in \vee q)$-fuzzy left ideal g of R.

(4) $((h \wedge f) \wedge g)^- \leq ((h \circ f) \circ g)^-$ for every $(\in, \in \vee q)$-fuzzy right ideal h, every $(\in, \in \vee q)$-fuzzy quasi bi-ideal f and for every $(\in, \in \vee q)$-fuzzy right ideal g of R.

Proof. (1) $\Longrightarrow$ (2) Let h, f and g be any $(\in, \in \vee q)$-fuzzy right ideal, $(\in, \in \vee q)$-fuzzy generalized bi-ideal and for every $(\in, \in \vee q)$-fuzzy right ideal g of R respectively. Let a be any element of R. Since R is regular, so there exists an element $x \in R$ such that $a = (ax)a$.

Hence we have

$$((h \circ f) \circ g)^- (a) = \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{(h \circ f)(y_i) \wedge g(z_i)\} \right\} \right) \wedge 0.5.$$

Now using medial law and the property $(xe)R = xR$ for all $x \in R$, we have $a = (ax)a = (ax)(ea) = (ae)(xa) = a(xa)$.

This implies that

$$\left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{(h \circ f)(y_i) \wedge g(z_i)\} \right\} \right) \wedge 0.5$$

$$\geq (h \circ f)(a) \wedge g(xa) \wedge 0.5$$

$$\geq \left(\bigvee_{a=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{h(p_i) \wedge f(q_i)\} \right\} \right) \wedge (g(a) \wedge 0.5) \wedge 0.5$$

$$\geq (h(ax) \wedge f(a)) \wedge g(a) \wedge 0.5$$

$$\geq (h(a) \wedge 0.5 \wedge f(a)) \wedge g(a) \wedge 0.5$$

$$= (h(a) \wedge f(a)) \wedge g(a) \wedge 0.5 = ((h \wedge f) \wedge g)^{-}(a).$$

Thus, $((h \wedge f) \wedge g)^{-} \leq ((h \circ f) \circ g)^{-}$.

(2) $\Longrightarrow$ (3) $\Longrightarrow$ (4) Straightforward.

(4) $\Longrightarrow$ (1) Let h and g be any $(\in, \in \vee q)$-fuzzy right ideal and any $(\in, \in \vee q)$-fuzzy left ideal of R, respectively. Since δ is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R, by the assumption, we have

$$(h \wedge g)^{-}(a) = (h \wedge g)(a) \wedge 0.5 = ((h \wedge \delta) \wedge g)(a) \wedge 0.5$$

$$= ((h \wedge \delta) \wedge g)^{-}(a) \leq ((h \circ \delta) \circ g)^{-}(a)$$

$$= ((h \circ \delta) \circ g)(a)$$

$$\wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \{(h \circ \delta)(b_i) \wedge g(c_i)\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{h(p_i) \wedge \delta(q_i)\} \wedge g(c_i) \right\} \right\} \right\} \right)$$

$$\wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{h(p_i) \wedge 1\} \wedge g(c_i) \right\} \right\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} h(p_i) \wedge g(c_i) \right\} \right\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{h(p_i) \wedge 0.5\} \wedge g(c_i) \right\} \right\} \right\} \right) \wedge 0.5$$

$$\leq \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} h(p_i q_i) \wedge g(c_i) \right\} \right\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} b_i c_i} \left\{ \bigwedge_{i=1}^{n} \{\{h(b_i) \wedge g(c_i)\}\} \right\} \right) \wedge 0.5$$

$$= (h \circ g)(a) \wedge 0.5 = (h \circ g)^{-}(a).$$

Thus it follows that $(h \wedge g)^{-} \leq (h \circ g)^{-}$ for every $(\in, \in \vee q)$-fuzzy right ideal h and every $(\in, \in \vee q)$-fuzzy left ideal g of R. But $(h \circ g)^{-} \leq (h \wedge g)^{-}$, therefore $(h \circ g)^{-} = (h \wedge g)^{-}$. Hence it follows from Theorem 33 that R is regular. □

Theorem 35. *If R is an LA-ring with left identity e such that $(xe)R = xR$ for all $x \in R$. then the following conditions are equivalent.*

(1) *R is regular.*

(2) *$f^- = ((f \circ \delta) \circ f)^-$ for every $(\in, \in \vee q)$-fuzzy generalized bi-ideal f of R.*

(3) *$f^- = ((f \circ \delta) \circ f)^-$ for every $(\in, \in \vee q)$-fuzzy bi-ideal f of R.*

(4) *$f^- = ((f \circ \delta) \circ f)^-$ for every $(\in, \in \vee q)$-fuzzy quasi-ideal f of R, where δ is the fuzzy subset of R mapping every element of R on 1*

Proof. (1) $\Longrightarrow$ (2) : Let f be an $(\in, \in \vee q)$-fuzzy generalized bi-ideal of R and let a be any element of R. Since R is regular, so there exists an element $x \in R$ such that $a = (ax)a$. Hence we have

$$\begin{aligned}
((f \circ \delta) \circ f)^-(a) &= ((f \circ \delta) \circ f)(a) \wedge 0.5 \\
&= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{(f \circ \delta)(y_i) \wedge f(z_i)\} \right\} \right) \wedge 0.5 \\
&\geq \{(f \circ \delta)(ax) \wedge f(a)\} \wedge 0.5 \\
&= \left\{ \left(\bigvee_{ax=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{f(p_i) \wedge \delta(q_i)\} \right\} \right) \wedge f(a) \right\} \wedge 0.5 \\
&\geq \{(f(a) \wedge \delta(x)) \wedge f(a)\} \wedge 0.5 \\
&= \{(f(a) \wedge 1) \wedge f(a)\} \wedge 0.5 = f(a) \wedge 0.5 = f^-(a).
\end{aligned}$$

Thus $((f \circ \delta) \circ f)^- \geq f^-$. Since f is an $(\in, \in \vee q)$-fuzzy generalized bi-ideal of R. So we have

$$((f \circ \delta) \circ f)^-(a) = ((f \circ \delta) \circ f)(a) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{(f \circ \delta)(y_i) \wedge f(z_i)\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \vee_{y=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{f(p_i) \wedge \delta(q_i)\} \right\} \wedge f(z_i) \right\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \vee_{y=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{f(p_i) \wedge 1\} \right\} \wedge f(z_i) \right\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \left\{ \vee_{y=\sum_{i=1}^{n} p_i q_i} \bigwedge_{i=1}^{n} f(p_i) \wedge f(z_i) \right\} \right\} \right) \wedge 0.5$$

$$\leq \bigvee_{a=\sum_{i=1}^{n} (p_i q_i) z_i} \left\{ \bigwedge_{i=1}^{n} f((p_i q_i) z_i) \wedge 0.5 \right\} = f(a) \wedge 0.5 = f^{-}(a).$$

So, $((f \circ \delta) \circ f)^{-} \leq f^{-}$. Thus $((f \circ \delta) \circ f)^{-} = f^{-}$.

Now (2) $\Longrightarrow$ (3) $\Longrightarrow$ (4) trivially hold.

(4) $\Longrightarrow$ (1) Let A be any quasi-ideal of R. We have

$$(AR)A \subseteq (AR)R \cap (RR)A \subseteq AR \cap RA \subseteq A.$$

Let a be any element of A. Since C_A is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R, we have

$$\left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{\bigwedge_{i=1}^{n} \{(C_A \circ \delta)(y_i) \wedge C_A(z_i)\}\right\}\right) \wedge 0.5$$
$$= ((C_A \circ \delta) \circ C_A)(a) \wedge 0.5$$
$$= ((C_A \circ \delta) \circ C_A)^{-}(a) = C_A^{-}(a) = 0.5.$$

This implies that

$$\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{\bigwedge_{i=1}^{n} \{(C_A \circ \delta)(y_i) \wedge C_A(z_i)\}\right\} \geq 0.5.$$

But

$$\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{\bigwedge_{i=1}^{n} \{(C_A \circ \delta)(y_i) \wedge C_A(z_i)\}\right\} \neq 0.5.$$

Thus

$$\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{\bigwedge_{i=1}^{n} \{(C_A \circ \delta)(y_i) \wedge C_A(z_i)\}\right\} > 0.5.$$

Hence

$$\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{\bigwedge_{i=1}^{n} \{(C_A \circ \delta)(y_i) \wedge C_A(z_i)\}\right\} = 1.$$

This implies that there exist elements b and c of R such that $(C_A \circ \delta)(b) = 1$

and $C_A(c)=1$ with $a=bc$. Thus we have

$$\bigvee_{b=\sum_{i=1}^{n} p_i q_i} \left\{ \bigwedge_{i=1}^{n} \{C_A(p_i) \wedge \delta(q_i)\} \right\} = (C_A \circ \delta)(b) = 1.$$

This implies that there exist elements d and e of R such that $C_A(d)=1$ and $\delta(e)=1$ with $b=de$. Thus $d,c \in A$ and $e \in R$ and so $a=bc=(de)c \in (AR)A$. Therefore, $A \subseteq (AR)A$, and so $A=(AR)A$. Hence R is regular. □

Theorem 36. *If R is an LA-ring with left identity e such that $(xe)R=xR$ for all $x \in R$. then the following conditions are equivalent.*

(1) R is regular.

(2) $(f \wedge g)^{-} \leq (f \circ g)^{-}$ for every $(\in, \in \vee q)$-fuzzy quasi-ideal f and every $(\in, \in \vee q)$-fuzzy left ideal g of R.

(3) $(f \wedge g)^{-} \leq (f \circ g)^{-}$ for every $(\in, \in \vee q)$-fuzzy bi-ideal f and every $(\in, \in \vee q)$-fuzzy left ideal g of R.

(4) $(f \wedge g)^{-} \leq (f \circ g)^{-}$ for every $(\in, \in \vee q)$-fuzzy generalized bi-ideal f and every $(\in, \in \vee q)$-fuzzy left ideal g of R.

Proof. (1) $\Longrightarrow$ (4) Let f and g be any $(\in, \in \vee q)$-fuzzy generalized bi-ideal and any $(\in, \in \vee q)$-fuzzy left ideal of R respectively. Let a be any element of R. Therefore there exist an element $x \in R$ such that $a=(ax)a$. Thus we have

$$(f \circ g)^{-}(a) = (f \circ g)(a) \wedge 0.5 = \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} f(y_i) \wedge g(z_i) \right\} \right) \wedge 0.5.$$

Now, $a=(ax)a=(ax)(ea)=(ae)(xa)=a(xa)$ because $(xe)R=xR$ for all $x \in R$. It follows that

$$\left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} f(y_i) \wedge g(z_i) \right\} \right) \wedge 0.5 \geq f(a) \wedge g(xa) \wedge 0.5$$

$$\begin{aligned}&\geq f(a) \wedge (g(a) \wedge 0.5) \wedge 0.5\\&= (f \wedge g)(a) \wedge 0.5\\&= (f \wedge g)^{-}(a).\end{aligned}$$

So $(f \circ g)^{-} \geq (f \wedge g)^{-}$. (4) $\Longrightarrow$ (3) $\Longrightarrow$ (2) are obvious.

(2) $\Longrightarrow$ (1) Let f be an $(\in, \in \vee q)$-fuzzy right ideal and g be an $(\in, \in \vee q)$-fuzzy left ideal of R. Since every $(\in, \in \vee q)$-fuzzy right ideal of R is an $(\in, \in \vee q)$-fuzzy quasi-ideal of R. So $(f \circ g)^{-} \geq (f \wedge g)^{-}$. Now

$$(f \circ g)^{-}(a) = (f \circ g)(a) \wedge 0.5 = \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{f(y_i) \wedge g(z_i)\} \right\} \right) \wedge 0.5$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{f(y_i) \wedge g(z_i)\} \right\} \wedge 0.5 \right)$$

$$= \left(\bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left\{ \bigwedge_{i=1}^{n} \{(f(y_i) \wedge 0.5) \wedge (g(z_i) \wedge 0.5)\} \right\} \wedge 0.5 \right)$$

$$\leq \bigvee_{a=\sum_{i=1}^{n} y_i z_i} \left(\left\{ \bigwedge_{i=1}^{n} \{f(y_i z_i) \wedge g(y_i z_i)\} \right\} \wedge 0.5 \right)$$

$$= f(a) \wedge g(a) \wedge 0.5$$

$$= (f \wedge g)(a) \wedge 0.5 = (f \wedge g)^{-}(a) . So\, (f \circ g)^{-} \leq (f \wedge g)^{-}.$$

Hence $(f \circ g)^{-} = (f \wedge g)^{-}$ for every $(\in, \in \vee q)$-fuzzy right ideal f of R, and every $(\in, \in \vee q)$-fuzzy left ideal g of R. Thus by Theorem 33, R is regular. □

4. Conclusion

This paper has explored theoretical methods of evaluation to identify the blondeness of the $(\in, \in \vee q)$-fuzzy ideals. A functional approach was used to undertake a characterization of this structure leading to a determination of some interesting LA-ring theoretic properties of the generated structures. Similar types of characterization in section 3, can easily be obtained for $(\in, \in \vee q_k)$-fuzzy ideals as well as $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals. In future work, we will discuss different classes of LA-rings using $(\in, \in \vee q_k)$-fuzzy ideals as well as $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals.

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